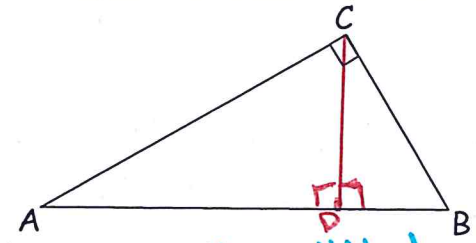
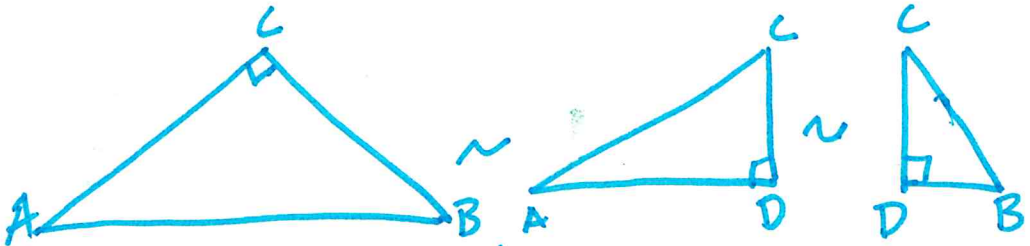
7-4 Similarity in Right Triangles

Theorem 7-3

The altitude to the hypotenuse of a right triangle divides the triangle into two triangles that are similar to the original triangle and to each other.





If $\overline{CD}$ is the altitude, then we have 3 similar $\triangle$s. for $\triangle ABC$

Geometric Mean - a positive number x such that:

$$\frac{a}{x} = \frac{x}{b}$$ (where a and b are constants)

(note; $x = \sqrt{ab}$)

Example 1 - Finding the Geometric Mean

Find the geometric mean of 3 and 12.

a b

Write a proportion: $\frac{3}{x} = \frac{x}{12}$

Cross Product Property: $x^2 = 36$

Find the positive square root: $\sqrt{x^2} = \sqrt{36}$

$x = 6$

✓**Quick Check** - Find the geometric mean of 15 and 20. (No Decimals!)

$\frac{15}{x} = \frac{x}{20} \rightarrow \sqrt{x^2} = \sqrt{300} \rightarrow x = 10\sqrt{3}$

$\sqrt{100} \cdot \sqrt{3}$

$10 : \sqrt{3}$

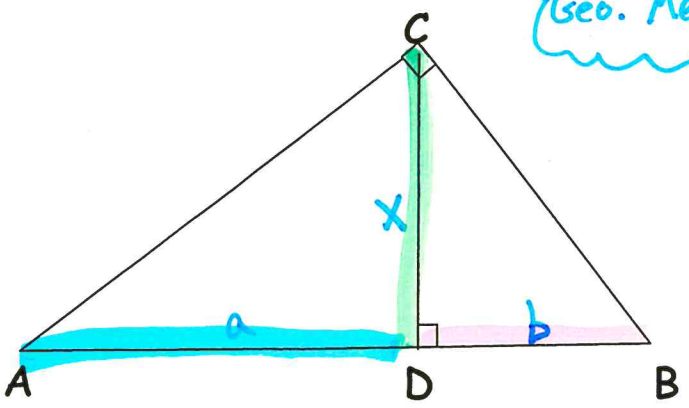
Corollary 1 to Theorem 7-3

The length of the altitude to the hypotenuse (CD) of a right △ is the geometric mean of the lengths of the segments (pieces) of the hypotenuse.

"x" ↑ ↑ "a" & "b"

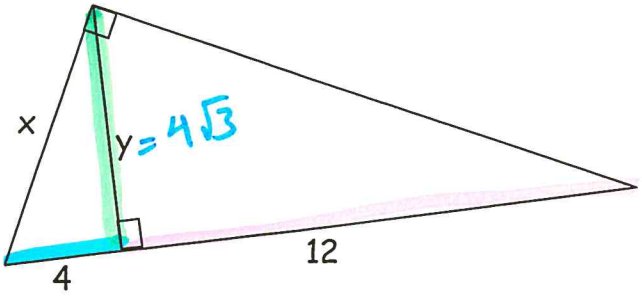
Geo. Mean: $\frac{a}{x} = \frac{x}{b}$

[Altitude = "x" in Geo. Mean]
[Hypot. Pieces = "a" & "b"]



$\frac{AD}{CD} = \frac{CD}{DB}$

Example 2 - Find the value of x and y. y is the altitude's length



$\frac{4}{y} = \frac{y}{12} \rightarrow \sqrt{y^2} = \sqrt{48}$

$\sqrt{16} \cdot \sqrt{3}$

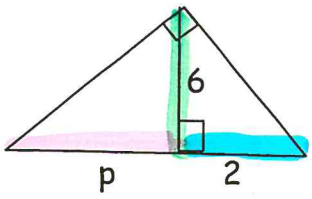
$4 \cdot \sqrt{3}$

$y = 4\sqrt{3}$

Pythag. Thm.

$4^2 + (4\sqrt{3})^2 = x^2 \rightarrow 16 + 48 = x^2 \rightarrow x^2 = 64 \rightarrow x = 8$

Example 3 - Find the value of p.



Sometimes the altitude is known.

$\frac{2}{6} = \frac{6}{p} \rightarrow 2p = 36$

$p = 18$