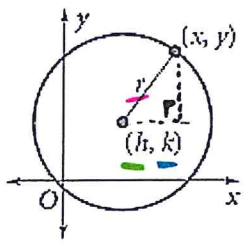
Equations of Circles

In this section, we will learn how to write the equation of a circle if given the center of the circle and the radius. Then we will learn to write the equation of a circle using a point on the circle.

Equation of a Circle:
An equation of a circle with center (h, k) and radius r, is written as:

$$(x-h)^2 + (y-k)^2 = r^2$$



An equation in this form is said to be in Standard Form.

Example 1: Write the standard equation of the circle with the given center and radius.

A) The center is (5, -2) and radius is 7. $h=5$, $k=-2$, $r=7$ $(x-5)^2 + (y+2)^2 = 49$ ← $r^2 = 7^2 = 49$

B) The center is (3, 5) and radius is 6. $h=3$, $k=5$, $r=6$ $(x-3)^2 + (y-5)^2 = 36$ $r^2 = 6^2 = 36$

C) The center is (-2, -1) and the radius is $\sqrt{2}$. $h=-2$, $k=-1$, $r=\sqrt{2}$ $(x+2)^2 + (y+1)^2 = 2$ $r^2 = (\sqrt{2})^2 = 2$

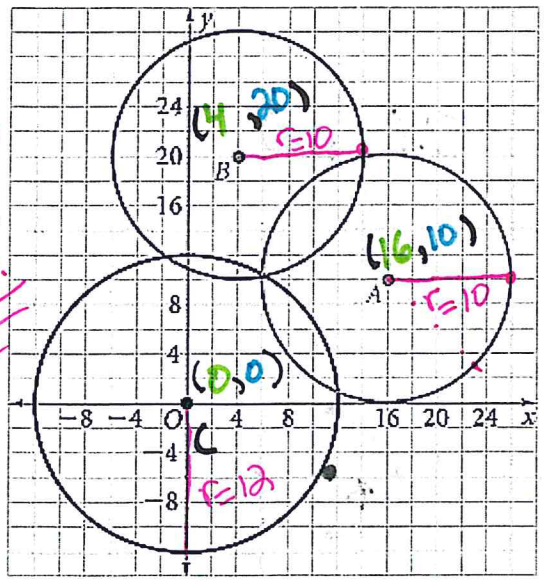
If we have the graph of the circle, we can use it to write the equation of the circle.

1) First, find the center of the circle. (h, k)

2) Next, find the radius of the circle.

3) Then use the center and radius to write the stand. form equation of the circle.

Example 2: Write an equation for each circle below.



Pay attention to the scale!

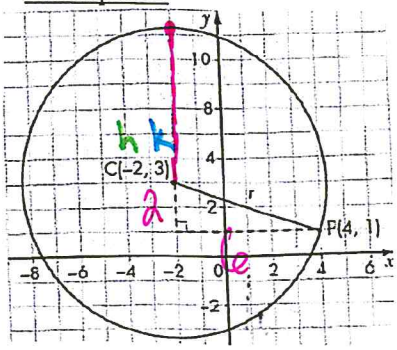
CIRCLE A: $(x-16)^2 + (y-10)^2 = 100$

CIRCLE B: $(x-4)^2 + (y-20)^2 = 100$

CIRCLE C: $(x-0)^2 + (y-0)^2 = 144$

$x^2 + y^2 = 144$

Example 3:



$a^2+b^2=c^2$

(a) Use Pythagorean Theorem to determine the radius of the circle.

$2^2+6^2=r^2$

$4+36=r^2$

$40=r^2$

$r=\sqrt{40}$ → $4, 10$

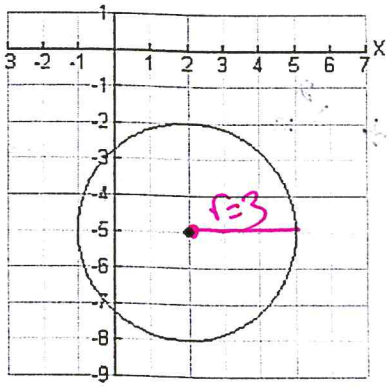
$r=2\sqrt{10}$

(b) Then write the equation of the circle.

$(x+2)^2+(y-3)^2=40$

(c) Do you have to use Pythagorean Theorem to find the radius? How else could you have done that? Dist. Formula $d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

Example 4: Write the equation of the circle below.



Example 5: Graph the given circle.

1. Find the center 2. Find the radius 3.) Plot center AND 4 radii

a.) $(x-4)^2+(y+2)^2=16$ → $r=4$

C: (4, -2)

b.) $(x+3)^2+(y+2)^2=9$ → $r=3$

C: (-3, -2)