Instructions: "I recoil with dismay and horror at this lamentable plague of functions which do not have derivatives." -- *Charles Hermite (1822-1901)*

Find each integral:

1.) $\int(4x+22)e^{x^2+11x-4}dx$ $= 2\int(2x+11)e^{x^2+11x-4}dx$

$u = x^2+11x-4$

$du = 2x+11$

$= 2\int e^u\,du$

$= 2e^u + C$

$\boxed{= 2e^{x^2+11x-4} + C}$

3.) $\int \sin^5(x)\cos(x)\,dx$ $= \int u^5 du$

$u = \sin x$

$du = \cos x\,dx$

$= \frac{u^6}{6} + C$

$\boxed{= \frac{(\cos x)^6}{6} + C}$

2.) $\int 120(3x+1)^4dx$ $= 40\int 3(3x+1)^4 dx$

$u = 3x+1$

$du = 3dx$

$= 40\int u^4 du$

$= 40\frac{u^5}{5} + C$

$\boxed{= 8(3x+1)^5 + C}$

4.) $\int \frac{8\sec^2(x)}{3+2\tan(x)}dx$ $= 4\int \frac{2\sec^2 x\,dx}{3+2\tan x}$ $= 4\int \frac{1}{u}du$

$u = 3+2\tan x$

$du = 2\sec^2 x\,dx$

$= 4\ln|u| + C$

$\boxed{= 4\ln(3+2\tan x) + C}$

5.) The rate at which a company is making profit is modeled by the function $r(t) = 3t^2 - 6t + 4$ where t is measured in weeks and $r(t)$ is measured in thousands of dollars per week. What is the average rate at which the company is making profits over the first four weeks? <u>Include units.</u>

$r(t) = 3t^2 - 6t + 4$

$r_{avg} = \frac{1}{4-0}\int_0^4 r(t)\,dt = \frac{1}{4}\int_0^4 3t^2 - 6t + 4\,dt$

$r_{avg} = \frac{1}{4}\left(t^3 - 3t^2 + 4t\right)\Big|_0^4$

$= \frac{1}{4}(4)^3 - \frac{3}{4}4^2 + 4 - \left(\frac{1}{4}(0)^3 - \frac{3}{4}(0^2)\ 4\,0\right)$

$= 16 - 12 + 4 = 8$

$\boxed{r_{avg} = 8 \text{ thousand } \$/\text{week}}$

Multiple Choice:

$u = 10x \quad du = 10\,dx$

6.) $\int_0^{\frac{\pi}{10}} \sin(10x)\,dx = \frac{1}{10}\int_0^{\frac{\pi}{10}} 10\sin(10x)\,dx = \frac{1}{10}\int_0^{\frac{\pi}{10}} \sin(u)\,du = \frac{1}{10}(-\cos u) = \frac{1}{10}(-\cos(10x))\Big|_0^{\frac{\pi}{10}}$

(A) $-\frac{1}{5}$ (B) $-\frac{1}{10}$ (C) 0 (D) $\frac{1}{10}$ (E) $\frac{1}{5}$

$= \frac{1}{10}(-\cos\pi) - \frac{1}{10}(-\cos 0)$

$\frac{1}{10} + \frac{1}{10} = \frac{2}{10} = \frac{1}{5}$

7.) The average value of $cos(x)$ on the interval $[1,5]$ is

(A) $\frac{\sin 1 - \sin 5}{4}$

(B) $\frac{\sin 1 + \sin 5}{6}$

(C) $\frac{\sin 1 - \sin 5}{6}$

(D) $\frac{\sin 5 - \sin 1}{4}$

(E) $\frac{\sin 5 + \sin 1}{4}$

$avg = \frac{1}{5-1}\int_1^5 \cos x\,dx$

$avg = \frac{1}{4}\sin x\Big|_1^5$

$= \frac{\sin 5 - \sin 1}{4}$

$x^3+1 = u \quad du = 3x^2\,dx$

8.) $\int \frac{3x^2}{\sqrt{x^3+1}}\,dx = \int \frac{du}{\sqrt{u}} = \frac{u^{1/2}}{1/2} + C = 2\sqrt{x^3+1} + C$

(A) $2\sqrt{x^3+1} + C$

(B) $\frac{3}{2}\sqrt{x^3+1} + C$

(C) $\sqrt{x^3+1} + C$

(D) $\ln(\sqrt{x^3+1}) + C$

(E) $\ln(x^3+1) + C$

9.) Since 1850, near the beginning of the industrial revolution, the rate of species extinction on earth has followed the function $R(t) = 2.386(1.04)^t$ where R is measured in the amount of species that go extinct per year and t is measured in years since 1850. Explain the meaning of the following integral expression. Include what it means, the units of measurement, and the time interval.

$$\frac{1}{163}\int_0^{163} 2.386(1.04)^t\,dt$$ = average number of species that go extinct per year from 1850 to 2013.