Differentiation and Limits   No CALCULATOR!!!!!

Key

1. What is the **minimum value of the slope of the curve** $y = x^5 + x^3 - 2x$?

$y' = m = 5x^4 + 3x^2 - 2$ m has a min where $m' = 0$

$m' = 20x^3 + 6x = 0$

$2x(10x^2+3) = 0$ $x = 0$ $10x^2 + 3 = 0$ no Real. Sol.

he min value of the slope
t $x=0$ is $m = -2$

2. Determine the point on the curve $y = \sqrt{2x+1}$ at which the tangent is parallel to the line $x - 3y = 6$

$x - 3y = 6$

$y = \frac{x}{3} - 2$ $m = \frac{1}{3}$

the tangent is // to $x-3y=6$ so they have the same slope. $m_{tangent} = \frac{1}{3}$

$m_{tangent} = y'$ so, $\frac{1}{3} = \frac{\cancel{2}}{\cancel{2}\sqrt{2x+1}}$

$\sqrt{2x+1} = 3$ $\boxed{x = 4}$ $y = \sqrt{2\cdot 4+1} \boxed{= 3}$

Tangent line is // to $x-3y=6$ at $(4,3)$

3. The function $f(x) = x^4 - 4x^2$ has

a) One local minimum and two local maxima

b) One local minimum and one local maximum

c) Two local minima and no local minimum

d) One local minimum and no local maximum

[✓ e)] Two local minima and one local maximum

f(x) has a local extrem when $f'(x) = 0$ $f' = 4x^3 - 8x = 0$

$4x(x^2-2) = 0$ $x = 0$, $x = \pm\sqrt{2}$

f(x) is a quattic function with + leading coefficient

two min

4. The number of inflection points (change in curvature) of the function in #3 is

a) 0

b) 1

[✓ c) 2]

d) 3

e) 4

A function has an inflection point when f' has a min or max

f' has a min or max where $f'' = 0$

$f'' = 12x^2 - 8 = 0$ $3x^2 - 2 = 0$ $x = \pm\sqrt{\frac{2}{3}}$

↳ two inflection points

5. The maximum value of the function $y = -4x\sqrt{2-x}$ is.....   Round your answer to the nearest hundredth.

y has a max at a value of x that makes $y' = 0$

$y' = -4\sqrt{2-x} + -\overset{2}{\cancel{4}}x\frac{1}{\cancel{2}\sqrt{2-x}}\cdot(-1) = -4\sqrt{2-x} + \frac{2x}{\sqrt{2-x}} = 0$

$4\sqrt{2-x} = \frac{2x}{\sqrt{2-x}}$

$2x = 4(2-x)$

$2x = 8 - 4x$

$x = 4/3$

at $x = 4/3$

$y = -4\cdot\frac{4}{3}\sqrt{2-\frac{4}{3}}$

$\boxed{y = -4.35}$

In questions 6- 9, the position of a particle moving along a horizontal line y = 6 is given by

$s = t^3 - 6t^2 + 12t - 8$

6. For what time interval(s) does the object move to the right?

the object moves to the right were V is +

$V = s' = 3t^2 - 12t + 12 = 3(t^2 - 4t + 4) = 3(t-2)^2 = 0$ $t = 2$ mult. of 2

t | 2
V | +++ 0 +++

min value

So object moves to the right all the time

$[0, \infty)$

7. What is the minimum value of its speed?

See the work above

$V_{min} = 0$

8. For what time interval(s) is its acceleration positive

$a = v' = 6t - 12 = 0$ $t = 2$

t | 2
a | - - - 0 + + +

acc is + for $t \in (0, \infty)$

9. What is the particles location at t = 5? Is the particle speeding up or slowing down at this location?

at $t=5$ $s = 5^3 - 6\cdot 5^2 + 12\cdot 5 - 8$
$s = 27$

At $t=5$ the particle's location is (27, 6)

10. The position of a particle moving on a line is given by s =(-5+(t-2)²)². Answer the following questions and remember NO CALCULATOR.

horizontal line

a) What is the initial position of the particle relative to (0,0)

$S(0) = 1$ So initial position is 1 unit to the right.

b) What is the maximum displacement during the interval -0≤ t ≤ 4? Prove that this is a max not a min.

max displ = 25

t	Displ.
1	16
1.5	22.6
2	25
2.5	22.6
3	16

max

c) Is velocity + or – during the interval (0,2) +

$V = S' = 2(-5+(t-2)^2) \cdot 2(t-2)$
$V = 4(-5+t^2-4t+4)(t-2)$ 2
$= 4(t^2-4t-1)(t-2)$
$= 4t^3 - 8t^2 - 16t^2 + 32t + 4t + 8$
$= 4t^3 - 24t^2 + 28t + 8$

t	0		2		
v	+	+	0	–	–

d) When is the acceleration zero? at $t = 0.71$ $t = 3.29$

e) What is the velocity when the acceleration is zero?

$a = V' = 12t^2 - 48t + 28$

$a = 0$ so $4(3t^2 - 12t + 7) = 0$
$t = 3.29$ sec and $t = 0.71$ sec

e.
at $t = 0.71$
$V = 17.21$ m/s
at $t = 3.29$
$V = -17.2$ m/s

11. The two tangents that can be drawn from point (3,5) to the parabola y = x² have slopes

a) 1 and 5
b) a and 4
[✓ c) 2 and 10]
d) 2 and -1/2
e) 2 and 4

$m_{tangent} = y' = 2x$
at $x = a$ $m_{tangent} = 2a$

$y - 5 = 2a(x-3)$
$y = a^2$

$a^2 - 5 = 2a(a-3)$
$a^2 - 5 = 2a^2 - 6a$
$a^2 - 6a + 5 = 0$
$a = 5$ $a = 1$

$m_{tangent} = $
$= 1$

$m_{tangent} = 2\cdot 1$
$= 2$

12. The function g(x) = $\frac{x^2-9}{3x-9}$ has

$g(x) = \frac{\cancel{(x-3)}(x+3)}{3\cancel{(x-3)}}$

a) A vertical asymptote at x=3
b) a horizontal asymptote at y=1/3
[✓ c) A removable discontinuity at x=3]
d) an infinite discontinuity at x=3
e) No asymptotes or discontinuities

13. Calculate the limit $\lim_{x\to 0} \frac{\sin x}{x^2+3x}$ $= \lim_{x\to 0} \frac{\sin x}{x(x^2+3)} = \lim_{x\to 0} \frac{\sin x}{x} \cdot \lim_{x\to 0} \frac{1}{x^2+3} = 1 \cdot \frac{1}{3} =$

14. $\lim_{x\to\infty} \frac{2x^2+1}{(2-x)(2+x)}$ is $= \lim_{x\to\infty} -\frac{2x^2+1}{x^2-4} = \lim_{x\to\infty} -\frac{\cancel{x^2}(2+\frac{1}{x^2})}{\cancel{x^2}(1-\frac{4}{x^2})} = -\frac{2+0}{1-0} = -$

a. -4
[✓ b. -2]
c. 1
d. 2
e. DNE

15.

Let $f(x)=\begin{cases}\frac{x^2+x}{x} & \text{if } x\neq 0\\ 1 & \text{if } x=0\end{cases}$.

Which of the following statements is (are) true?

I. $f(0)$ exists ✓ $f(0)=1$

II. $\lim_{x\to 0} f(x)$ exists ✓ $\lim_{x\to 0^-} f(x)=\lim_{x\to 0^-}\frac{\cancel{x}(x+1)}{\cancel{x}}=1=\lim_{x\to 0^+} f(x)$

III. f is continuous at $x=0$ ✓ $f(0)=1=\lim_{x\to 0} f(x)=\lim_{x\to 0^+} f(x)$

(A) I only (B) II only (C) I and II only [✓ (D) I, II, and III] (E) none of I, II, or III

16. Calculate the limit $\lim_{x\to 3}\frac{\frac{1}{x^2}-\frac{1}{9}}{x-3}$ $=\lim_{x\to 3}\frac{9-x^2}{9x^2(x-3)}=\lim_{x\to 3}-\frac{\cancel{(x-3)}(x+3)}{9x^2\cancel{(x-3)}}$

$=-\frac{3+3}{9\cdot 3^2}=\boxed{-\frac{6}{81}}$

17. If f(x) = cosx sin3x, then f'(π/6) is equal to

a) ½ b) - √3/2 c) 0 d) 1 [✓ e) -1/2]

$f'(x)=-\sin x\cdot\sin 3x+\cos x\cdot\cos 3x\cdot 3$

$f'(\frac{\pi}{6})=-\sin\frac{\pi}{6}\cdot\sin\frac{\pi}{2}+\cancel{\cos\frac{\pi}{6}\cdot\cos\frac{\pi}{2}\cdot 3}$

$=-0.5\cdot 1=\boxed{-0.5}$

18. If y = f(x²) and f'(x) = $\sqrt{5x-1}$ then dy/dx is equal to:

[✓ a. $2x\sqrt{5x^2-1}$] b. $\sqrt{5x-1}$ c. $2x\sqrt{5x-1}$ d. none of these

$\frac{dy}{dx}=f'(x^2)\cdot 2x=2x\sqrt{5x^2-1}$

19. If f(x) = $\frac{\sin x^2}{x^2}$, then find f'(x). $f'(x)=\frac{x^2\cdot\cos x^2\cdot 2-\sin x^2\cdot 2\cancel{x}}{x^{4}}$

$f'(x)=\frac{2x\cos x^2-2\sin x^2}{x^3}$

20. Suppose f(3) =2, f'(3) = 5 and f''(3) =-2. Then $\frac{d^2}{dx^2}(f^2(x))$ at x=3 is equal to

a. -20 b. 10 c. 20 d. 38 [✓ e. 42]

$\frac{d}{dx}f^2(x)=2f(x)f'(x)$ $\frac{d^2}{dx^2}(f^2(x))=2\cdot f(x)\cdot f''(x)+2f'(x)\cdot f'(x)$ at x=3

$2\cdot 2\cdot(-2)+2\cdot 5^2=42$