4.2 Binomial Distributions

What You SHOULD LEARN

- How to determine if a probability experiment is a binomial experiment
- How to find binomial probabilities using the binomial probability formula
- How to find binomial probabilities using technology, formulas, and a binomial probability table
- How to graph a binomial distribution
- How to find the mean, variance, and standard deviation of a binomial probability distribution

Binomial Experiments ▸ Binomial Probability Formula ▸ Finding Binomial Probabilities ▸ Graphing Binomial Distributions ▸ Mean, Variance, and Standard Deviation

▸ Binomial Experiments

There are many probability experiments for which the results of each trial can be reduced to two outcomes: success and failure. For instance, when a basketball player attempts a free throw, he or she either makes the basket or does not. Probability experiments such as these are called binomial experiments.

DEFINITION

A **binomial experiment** is a probability experiment that satisfies the following conditions.

1. The experiment is repeated for a fixed number of trials, where each trial is independent of the other trials.
2. There are only two possible outcomes of interest for each trial. The outcomes can be classified as a success (S) or as a failure (F).
3. The probability of a success $P(S)$ is the same for each trial.
4. The random variable x counts the number of successful trials.

NOTATION FOR BINOMIAL EXPERIMENTS

Symbol	*Description*
n	The number of times a trial is repeated
$p = P(S)$	The probability of success in a single trial
$q = P(F)$	The probability of failure in a single trial $(q = 1 - p)$
x	The random variable represents a count of the number of successes in n trials: $x = 0, 1, 2, 3, \ldots, n$.

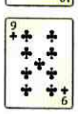
Here is a simple example of a binomial experiment. From a standard deck of cards, you pick a card, note whether it is a club or not, and replace the card. You repeat the experiment five times, so $n = 5$. The outcomes for each trial can be classified in two categories: S = selecting a club and F = selecting another suit. The probabilities of success and failure are

$$p = P(S) = \frac{1}{4} \quad \text{and} \quad q = P(F) = \frac{3}{4}.$$

The random variable x represents the number of clubs selected in the five trials. So, the possible values of the random variable are

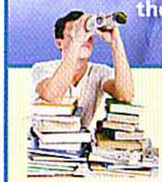
0, 1, 2, 3, 4, and 5.

For instance, if $x = 2$, then exactly two of the five cards are clubs and the other three are not clubs. An example of an experiment with $x = 2$ is shown at the left. Note that x is a discrete random variable because its possible values can be listed.

Trial	Outcome	S or F?
1		F
2		S
3		F
4		F
5		S

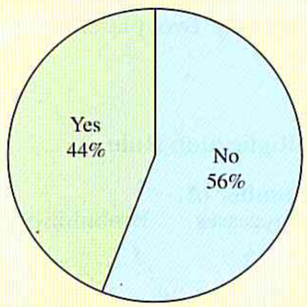
There are two successful outcomes. So, $x = 2$.

PICTURING the WORLD



A recent survey of vehicle owners in the United States asked whether they have cut back on any products or services because of the increase in gas prices. The respondents' answers were either yes or no. (Source: Harris Interactive)

Survey question: Have you cut back on any products or services in order to pay the increased price of gasoline?



Why is this a binomial experiment? Identify the probability of success, p. Identify the probability of failure, q.

EXAMPLE 1

Binomial Experiments

Decide whether the experiment is a binomial experiment. If it is, specify the values of n, p, and q, and list the possible values of the random variable x. If it is not, explain why.

1. A certain surgical procedure has an 85% chance of success. A doctor performs the procedure on eight patients. The random variable represents the number of successful surgeries.
2. A jar contains five red marbles, nine blue marbles, and six green marbles. You randomly select three marbles from the jar, *without replacement*. The random variable represents the number of red marbles.

Solution

1. The experiment is a binomial experiment because it satisfies the four conditions of a binomial experiment. In the experiment, each surgery represents one trial. There are eight surgeries, and each surgery is independent of the others. There are only two possible outcomes for each surgery—either the surgery is a success or it is a failure. Also, the probability of success for each surgery is 0.85. Finally, the random variable x represents the number of successful surgeries.

   $n = 8$

   $p = 0.85$

   $q = 1 - 0.85 = 0.15$

   $x = 0, 1, 2, 3, 4, 5, 6, 7, 8$

2. The experiment is not a binomial experiment because it does not satisfy all four conditions of a binomial experiment. In the experiment, each marble selection represents one trial, and selecting a red marble is a success. When the first marble is selected, the probability of success is 5/20. However, because the marble is not replaced, the probability of success for subsequent trials is no longer 5/20. So, the trials are not independent, and the probability of a success is not the same for each trial.

▸ Try It Yourself 1

Decide whether the following is a binomial experiment. If it is, specify the values of n, p, and q, and list the possible values of the random variable x. If it is not, explain why.

You take a multiple-choice quiz that consists of 10 questions. Each question has four possible answers, only one of which is correct. To complete the quiz, you randomly guess the answer to each question. The random variable represents the number of correct answers.

a. Identify a *trial* of the experiment and what is a "success."
b. Decide if the experiment *satisfies the four conditions* of a binomial experiment.
c. *Make a conclusion* and *identify* n, p, q, and the possible values of x.

Answer: Page A39

Insight

In the binomial probability formula, ${}_nC_x$ determines the number of ways of getting x successes in n trials, regardless of order.

$$ {}_nC_x = \frac{n!}{(n-x)!\,x!} $$

▸ Binomial Probability Formula

There are several ways to find the probability of x successes in n trials of a binomial experiment. One way is to use a tree diagram and the Multiplication Rule. Another way is to use the binomial probability formula.

BINOMIAL PROBABILITY FORMULA

In a binomial experiment, the probability of exactly x successes in n trials is

$$ P(x) = {}_nC_x\, p^x q^{n-x} = \frac{n!}{(n-x)!\,x!} p^x q^{n-x}. $$

EXAMPLE 2

Finding Binomial Probabilities

Microfracture knee surgery has a 75% chance of success on patients with degenerative knees. The surgery is performed on three patients. Find the probability of the surgery being successful on exactly two patients. *(Source: Illinois Orthopaedic and Sportsmedicine Centers)*

Solution

Method 1: Draw a tree diagram and use the Multiplication Rule.

1st Surgery	2nd Surgery	3rd Surgery	Outcome	Number of Successes	Probability
S	S	S	SSS	3	$\frac{3}{4}\cdot\frac{3}{4}\cdot\frac{3}{4}=\frac{27}{64}$
		F	SSF	2	$\frac{3}{4}\cdot\frac{3}{4}\cdot\frac{1}{4}=\frac{9}{64}$
	F	S	SFS	2	$\frac{3}{4}\cdot\frac{1}{4}\cdot\frac{3}{4}=\frac{9}{64}$
		F	SFF	1	$\frac{3}{4}\cdot\frac{1}{4}\cdot\frac{1}{4}=\frac{3}{64}$
F	S	S	FSS	2	$\frac{1}{4}\cdot\frac{3}{4}\cdot\frac{3}{4}=\frac{9}{64}$
		F	FSF	1	$\frac{1}{4}\cdot\frac{3}{4}\cdot\frac{1}{4}=\frac{3}{64}$
	F	S	FFS	1	$\frac{1}{4}\cdot\frac{1}{4}\cdot\frac{3}{4}=\frac{3}{64}$
		F	FFF	0	$\frac{1}{4}\cdot\frac{1}{4}\cdot\frac{1}{4}=\frac{1}{64}$

There are three outcomes that have exactly two successes, and each has a probability of $\frac{9}{64}$. So, the probability of a successful surgery on exactly two patients is $3\left(\frac{9}{64}\right) \approx 0.422$.

Method 2: Use the binomial probability formula.

In this binomial experiment, the values for n, p, q, and x are $n = 3$, $p = \frac{3}{4}$, $q = \frac{1}{4}$, and $x = 2$. The probability of exactly two successful surgeries is

$$ P(2 \text{ successful surgeries}) = \frac{3!}{(3-2)!\,2!}\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right)^1 $$

$$ = 3\left(\frac{9}{16}\right)\left(\frac{1}{4}\right) = 3\left(\frac{9}{64}\right) = \frac{27}{64} \approx 0.422. $$

▸ Try It Yourself 2

A card is selected from a standard deck and replaced. This experiment is repeated a total of five times. Find the probability of selecting exactly three clubs.

a. *Identify* a trial, a success, and a failure.
b. *Identify* n, p, q, and x.
c. Use the *binomial probability formula*.

Answer: Page A39

Study Tip

Recall that $n!$ is read "n factorial" and represents the product of all integers from n to 1. For instance,

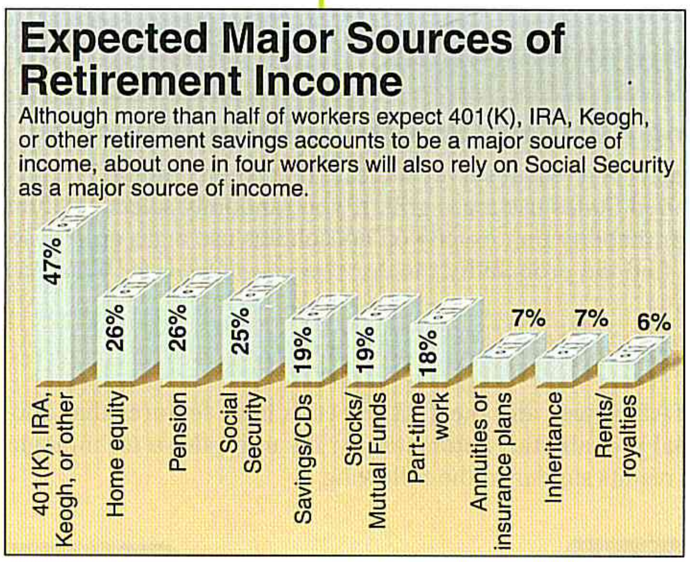
$$ 5! = 5\cdot4\cdot3\cdot2\cdot1 = 120. $$

By listing the possible values of x with the corresponding probability of each, you can construct a **binomial probability distribution.**

EXAMPLE 3

Constructing a Binomial Distribution

In a survey, workers in the United States were asked to name their expected sources of retirement income. The results are shown in the graph. Seven workers who participated in the survey are randomly selected and asked whether they expect to rely on Social Security for retirement income. Create a binomial probability distribution for the number of workers who respond yes.



(Source: The Gallup Organization)

Solution

From the graph, you can see that 25% of working Americans expect to rely on Social Security for retirement income. So, $p = 0.25$ and $q = 0.75$. Because $n = 7$, the possible values of x are 0, 1, 2, 3, 4, 5, 6, and 7.

$$P(0) = {}_7C_0(0.25)^0(0.75)^7 = 1(0.25)^0(0.75)^7 \approx 0.1335$$
$$P(1) = {}_7C_1(0.25)^1(0.75)^6 = 7(0.25)^1(0.75)^6 \approx 0.3115$$
$$P(2) = {}_7C_2(0.25)^2(0.75)^5 = 21(0.25)^2(0.75)^5 \approx 0.3115$$
$$P(3) = {}_7C_3(0.25)^3(0.75)^4 = 35(0.25)^3(0.75)^4 \approx 0.1730$$
$$P(4) = {}_7C_4(0.25)^4(0.75)^3 = 35(0.25)^4(0.75)^3 \approx 0.0577$$
$$P(5) = {}_7C_5(0.25)^5(0.75)^2 = 21(0.25)^5(0.75)^2 \approx 0.0115$$
$$P(6) = {}_7C_6(0.25)^6(0.75)^1 = 7(0.25)^6(0.75)^1 \approx 0.0013$$
$$P(7) = {}_7C_7(0.25)^7(0.75)^0 = 1(0.25)^7(0.75)^0 \approx 0.0001$$

x	$P(x)$
0	0.1335
1	0.3115
2	0.3115
3	0.1730
4	0.0577
5	0.0115
6	0.0013
7	0.0001
	$\Sigma P(x) = 1$

Notice in the table at the left that all the probabilities are between 0 and 1 and that the sum of the probabilities is $1.0001 \approx 1$.

▸ Try It Yourself 3

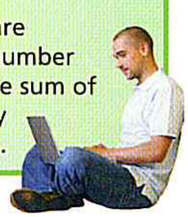
Seven workers who participated in the survey are randomly selected and asked whether they expect to rely on a pension for retirement income. Create a binomial distribution for the number of retirees who respond yes.

a. *Identify* a trial, a success, and a failure.
b. *Identify* n, p, q, and possible values for x.
c. Use the *binomial probability formula* for each value of x.
d. *Use a table* to show that the properties of a probability distribution are satisfied.

Answer: Page A39

Study Tip

When probabilities are rounded to a fixed number of decimal places, the sum of the probabilities may differ slightly from 1.



Finding Binomial Probabilities

In Examples 2 and 3 you used the binomial probability formula to find the probabilities. A more efficient way to find binomial probabilities is to use a calculator or a computer. For instance, you can find binomial probabilities using MINITAB, Excel, and the TI-83/84.

Study Tip

Detailed instructions for using MINITAB, Excel, and the TI-83/84 are shown in the Technology Guide that accompanies this text. For instance, here are instructions for finding a binomial probability on a TI-83/84.

2nd DISTR

0: binompdf(

Enter the values of n, p, and x separated by commas.

ENTER

EXAMPLE 4

Finding a Binomial Probability Using Technology

The results of a recent survey indicate that when grilling, 59% of households in the United States use a gas grill. If you randomly select 100 households, what is the probability that exactly 65 households use a gas grill? Use a technology tool to find the probability. *(Source: Greenfield Online for Weber-Stephens Products Company)*

Solution

MINITAB, Excel, and the TI-83/84 each have features that allow you to find binomial probabilities automatically. Try using these technologies. You should obtain results similar to the following.

MINITAB

Probability Distribution Function

Binomial with n = 100 and p = 0.590000

x	P(X=x)
65.00	0.0391072

TI-83/84

binompdf(100,.59,65)
.0391071795

EXCEL

	A	B	C	D
1	BINOMDIST(65,100,0.59,FALSE)			
2				0.039107

From the displays, you can see that the probability that exactly 65 households use a gas grill is about 0.04.

Try It Yourself 4

The results of a recent survey indicate that 71% of people in the United States use more than one topping on their hot dogs. If you randomly select 250 people, what is the probability that exactly 178 of them will use more than one topping? Use a technology tool to find the probability. *(Source: ICR Survey Research Group for Hebrew International)*

a. *Identify* n, p, and x.
b. *Calculate* the binomial probability.
c. *Write* the result as a sentence.

Answer: Page A39

EXAMPLE 5

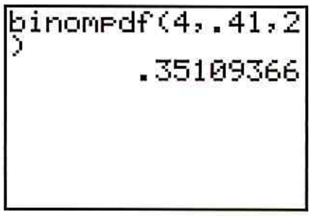
Finding Binomial Probabilities Using Formulas

A survey indicates that 41% of women in the United States consider reading their favorite leisure-time activity. You randomly select four U.S. women and ask them if reading is their favorite leisure-time activity. Find the probability that (1) exactly two of them respond yes, (2) at least two of them respond yes, and (3) fewer than two of them respond yes. *(Source: Louis Harris & Associates)*

Solution

1. Using $n = 4$, $p = 0.41$, $q = 0.59$, and $x = 2$, the probability that exactly two women will respond yes is

$$P(2) = {}_4C_2(0.41)^2(0.59)^2 = 6(0.41)^2(0.59)^2 \approx 0.351.$$



Using a TI-83/84, you can find the probability automatically.

2. To find the probability that at least two women will respond yes, find the sum of $P(2)$, $P(3)$, and $P(4)$.

$$P(2) = {}_4C_2(0.41)^2(0.59)^2 = 6(0.41)^2(0.59)^2 \approx 0.351094$$
$$P(3) = {}_4C_3(0.41)^3(0.59)^1 = 4(0.41)^3(0.59)^1 \approx 0.162654$$
$$P(4) = {}_4C_4(0.41)^4(0.59)^0 = 1(0.41)^4(0.59)^0 \approx 0.028258$$

So, the probability that at least two will respond yes is

$$P(x \geq 2) = P(2) + P(3) + P(4)$$
$$\approx 0.351094 + 0.162654 + 0.028258$$
$$\approx 0.542.$$

3. To find the probability that fewer than two women will respond yes, find the sum of $P(0)$ and $P(1)$.

$$P(0) = {}_4C_0(0.41)^0(0.59)^4 = 1(0.41)^0(0.59)^4 \approx 0.121174$$
$$P(1) = {}_4C_1(0.41)^1(0.59)^3 = 4(0.41)^1(0.59)^3 \approx 0.336822$$

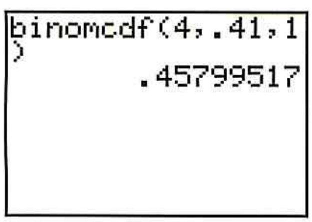
So, the probability that fewer than two will respond yes is

$$P(x < 2) = P(0) + P(1)$$
$$\approx 0.121174 + 0.336822$$
$$\approx 0.458.$$

Study Tip

The complement of "x is at least 2" is "x is less than 2." So, another way to find the probability in part (3) is

$$P(x < 2) = 1 - P(x \geq 2)$$
$$\approx 1 - 0.542$$
$$= 0.458.$$



The cumulative density function (CDF) computes the probability of "x or fewer" successes. The CDF adds the areas for the given x-value and all those to its left.

▸ Try It Yourself 5

A survey indicates that 21% of men in the United States consider fishing their favorite leisure-time activity. You randomly select five U.S. men and ask them if fishing is their favorite leisure-time activity. Find the probability that (1) exactly two of them respond yes, (2) at least two of them respond yes, and (3) fewer than two of them respond yes. *(Source: Louis Harris & Associates)*

a. Determine the appropriate *values of x* for each situation.
b. Find the *binomial probability* for each value of x. Then find the sum, if necessary.
c. *Write* the result as a sentence.

Answer: Page A39

Finding binomial probabilities with the binomial probability formula can be a tedious process. To make this process easier, you can use a binomial probability table. Table 2 in Appendix B lists the binomial probability for selected values of n and p.

EXAMPLE 6

Finding a Binomial Probability Using a Table

About thirty percent of working adults spend less than 15 minutes each way commuting to their jobs. You randomly select six working adults. What is the probability that exactly three of them spend less than 15 minutes each way commuting to work? Use a table to find the probability. *(Source: U.S. Census Bureau)*

Solution A portion of Table 2 in Appendix B is shown here. Using the distribution for $n = 6$ and $p = 0.3$, you can find the probability that $x = 3$, as shown by the highlighted areas in the table.

		p												
n	***x***	**.01**	**.05**	**.10**	**.15**	**.20**	**.25**	**.30**	**.35**	**.40**	**.45**	**.50**	**.55**	**.60**
2	0	.980	.902	.810	.723	.640	.563	.490	.423	.360	.303	.250	.203	.160
	1	.020	.095	.180	.255	.320	.375	.420	.455	.480	.495	.500	.495	.480
	2	.000	.002	.010	.023	.040	.063	.090	.123	.160	.203	.250	.303	.360
3	0	.970	.857	.729	.614	.512	.422	.343	.275	.216	.166	.125	.091	.064
	1	.029	.135	.243	.325	.384	.422	.441	.444	.432	.408	.375	.334	.288
	2	.000	.007	.027	.057	.096	.141	.189	.239	.288	.334	.375	.408	.432
	3	.000	.000	.001	.003	.008	.016	.027	.043	.064	.091	.125	.166	.216
6	0	.941	.735	.531	.377	.262	.178	.118	.075	.047	.028	.016	.008	.004
	1	.057	.232	.354	.399	.393	.356	.303	.244	.187	.136	.094	.061	.037
	2	.001	.031	.098	.176	.246	.297	.324	.328	.311	.278	.234	.186	.138
	3	.000	.002	.015	.042	.082	.132	.185	.236	.276	.303	.312	.303	.276
	4	.000	.000	.001	.006	.015	.033	.060	.095	.138	.186	.234	.278	.311
	5	.000	.000	.000	.000	.002	.004	.010	.020	.037	.061	.094	.136	.187
	6	.000	.000	.000	.000	.000	.000	.001	.002	.004	.008	.016	.028	.047

To explore this topic further, see Activity 4.2 on page 220.

So, the probability that exactly three of the six workers spend less than 15 minutes each way commuting to work is 0.185.

▸ Try It Yourself 6

Forty-five percent of all small businesses in the United States have a Web site. If you randomly select 10 small businesses, what is the probability that exactly four of them have a Web site? Use a table to find the probability. *(Source: Hewlett-Packard Company)*

a. *Identify* a trial, a success, and a failure.
b. Identify n, p, and x.
c. *Use Table 2* in Appendix B to find the binomial probability.
d. *Write* the result as a sentence.

Answer: Page A39

▸ Graphing Binomial Distributions

In Section 4.1, you learned how to graph discrete probability distributions. Because a binomial distribution is a discrete probability distribution, you can use the same process.

EXAMPLE 7

Graphing a Binomial Distribution

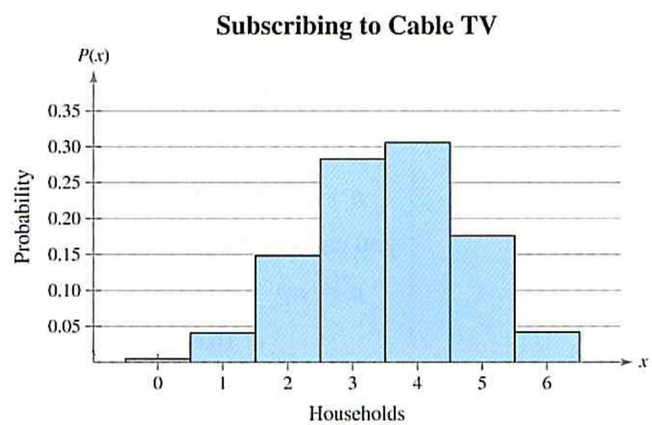
Fifty-nine percent of households in the United States subscribe to cable TV. You randomly select six households and ask each if they subscribe to cable TV. Construct a probability distribution for the random variable x. Then graph the distribution. *(Source: Kagan Research, LLC)*

Solution

To construct the binomial distribution, find the probability for each value of x. Using $n = 6$, $p = 0.59$, and $q = 0.41$, you can obtain the following.

x	0	1	2	3	4	5	6
$P(x)$	0.005	0.041	0.148	0.283	0.306	0.176	0.042

You can graph the probability distribution using a histogram as shown below.



Interpretation From the histogram, you can see that it would be unusual if none or only one of the households subscribes to cable TV because of the low probabilities.

Try It Yourself 7

Sixty-two percent of households in the United States own a computer. You randomly select six households and ask if they own a computer. Construct a probability distribution for the random variable x. Then graph the distribution. *(Source: U.S. Department of Commerce)*

a. *Find* the binomial probability for each value of the random variable x.
b. *Organize* the values of x and their corresponding probability in a binomial distribution.
c. Use a histogram to *graph* the binomial distribution.

Answer: Page A40

Notice in Example 7 that the histogram is skewed left. The graph of a binomial distribution with $p > 0.5$ is skewed left, whereas the graph of a binomial distribution with $p < 0.5$ is skewed right. The graph of a binomial distribution with $p = 0.5$ is symmetric.

Recall that if a probability is 0.05 or less, it is typically considered unusual.

Mean, Variance, and Standard Deviation

Although you can use the formulas learned in Section 4.1 for mean, variance, and standard deviation of a discrete probability distribution, the properties of a binomial distribution enable you to use much simpler formulas.

POPULATION PARAMETERS OF A BINOMIAL DISTRIBUTION

Mean: $\mu = np$

Variance: $\sigma^2 = npq$

Standard deviation: $\sigma = \sqrt{npq}$

EXAMPLE 8

Finding and Interpreting Mean, Variance, and Standard Deviation

In Pittsburgh, Pennsylvania, about 56% of the days in a year are cloudy. Find the mean, variance, and standard deviation for the number of cloudy days during the month of June. Interpret the results and determine any unusual values. *(Source: National Climatic Data Center)*

Solution There are 30 days in June. Using

$$n = 30,\ p = 0.56, \text{ and } q = 0.44$$

you can find the mean, variance, and standard deviation as shown below.

$$\mu = np = 30 \cdot 0.56$$
$$= 16.8$$
$$\sigma^2 = npq = 30 \cdot 0.56 \cdot 0.44$$
$$\approx 7.4$$
$$\sigma = \sqrt{npq} = \sqrt{30 \cdot 0.56 \cdot 0.44}$$
$$\approx 2.7$$

Interpretation On average, there are 16.8 cloudy days during the month of June. The standard deviation is about 2.7 days. Values that are more than two standard deviations from the mean are considered unusual. Because $16.8 - 2(2.7) = 11.4$, a June with 11 cloudy days would be unusual. Similarly, because $16.8 + 2(2.7) = 22.2$, a June with 23 cloudy days would also be unusual.

Try It Yourself 8

In San Francisco, California, 44% of the days in a year are clear. Find the mean, variance, and standard deviation for the number of clear days during the month of May. Interpret the results and determine any unusual values. *(Source: National Climatic Data Center)*

a. *Identify* a success and the values of n, p, and q.
b. *Find the product* of n and p to calculate the mean.
c. *Find the product* of n, p, and q for the variance.
d. *Find the square root* of the variance for the standard deviation.
e. *Interpret* the results.

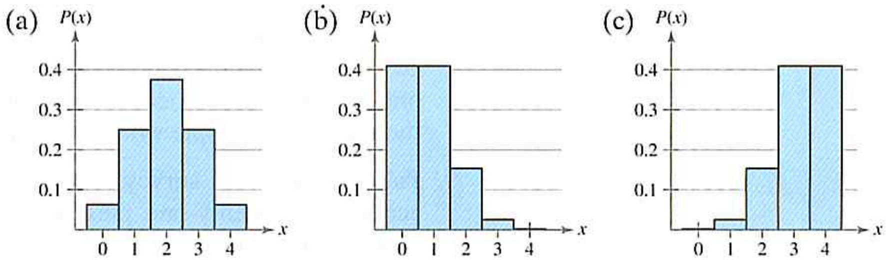
Answer: Page A40

4.2 EXERCISES

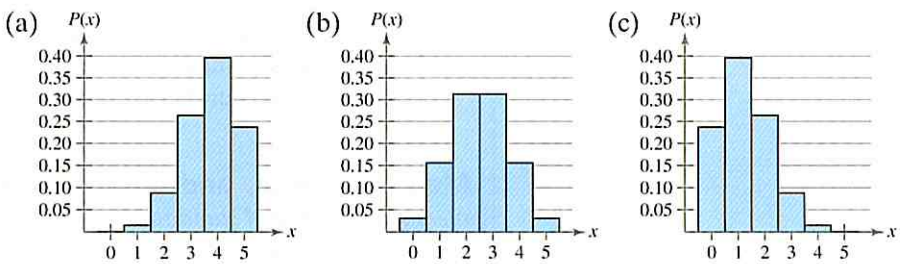
Building Basic Skills and Vocabulary

Graphical Analysis *In Exercises 1 and 2, match the given probabilities with the correct graph. The histograms each represent binomial distributions. Each distribution has the same number of trials n but different probabilities of success p.*

1. $p = 0.20$, $p = 0.50$, $p = 0.80$

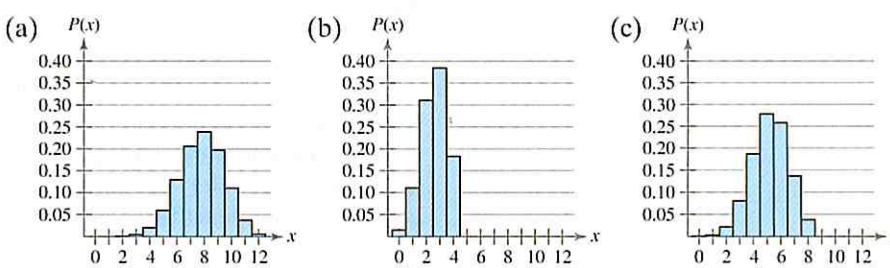


2. $p = 0.25$, $p = 0.50$, $p = 0.75$

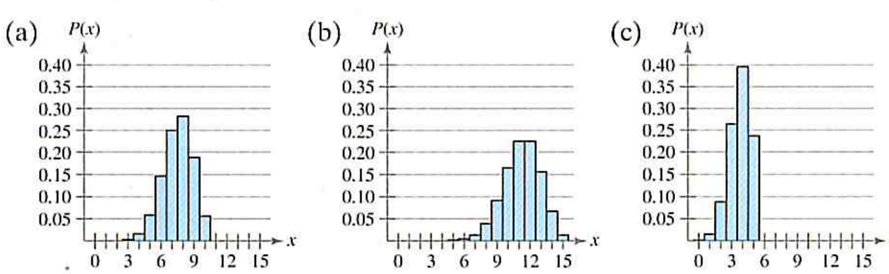


Graphical Analysis *In Exercises 3 and 4, match the given values of n with the correct graph. Each histogram shown represents part of a binomial distribution. Each distribution has the same probability of success p but different numbers of trials n. What happens as the value of n increases and the probability of success remains the same?*

3. $n = 4$, $n = 8$, $n = 12$



4. $n = 5$, $n = 10$, $n = 15$



5. Identify the unusual values of x in each histogram in Exercise 2.

6. Identify the unusual values of x in each histogram in Exercise 4.

Identifying and Understanding Binomial Experiments *In Exercises 7–10, decide whether the experiment is a binomial experiment. If it is, identify a success, specify the values of n, p, and q, and list the possible values of the random variable x. If it is not a binomial experiment, explain why.*

7. **Cyanosis** Cyanosis is the condition of having bluish skin due to insufficient oxygen in the blood. About 80% of babies born with cyanosis recover fully. A hospital is caring for five babies born with cyanosis. The random variable represents the number of babies that recover fully. *(Source: The World Book Encyclopedia)*

8. **Clothing Store Purchases** From past records, a clothing store finds that 26% of the people who enter the store will make a purchase. During a one-hour period, 18 people enter the store. The random variable represents the number of people who do not make a purchase.

9. **Political Polls** A survey asks 1000 adults, "Do tax cuts help or hurt the economy?" Twenty-one percent of those surveyed said tax cuts hurt the economy. Fifteen adults who participated in the survey are randomly selected. The random variable represents the number of adults who think tax cuts hurt the economy. *(Source: Rasmussen Reports)*

10. **Lottery** A state lottery randomly chooses 6 balls numbered from 1 to 40. You choose six numbers and purchase a lottery ticket. The random variable represents the number of matches on your ticket to the numbers drawn in the lottery.

Mean, Variance, and Standard Deviation *In Exercises 11–14, find the mean, variance, and standard deviation of the binomial distribution with the given values of n and p.*

11. $n = 80, p = 0.3$

12. $n = 64, p = 0.85$

13. $n = 124, p = 0.26$

14. $n = 316, p = 0.72$

Using and Interpreting Concepts

Finding Binomial Probabilities *In Exercises 15–24, find the indicated probabilities. If convenient, use technology to find the probabilities.*

15. **Answer Guessing** You are taking a multiple-choice quiz that consists of five questions. Each question has four possible answers, only one of which is correct. To complete the quiz, you randomly guess the answer to each question. Find the probability of guessing (a) exactly three answers correctly, (b) at least three answers correctly, and (c) less than three answers correctly.

16. **Surgery Success** A surgical technique is performed on seven patients. You are told there is a 70% chance of success. Find the probability that the surgery is successful for (a) exactly five patients, (b) at least five patients, and (c) less than five patients.

17. **Baseball Fans** Fifty-nine percent of men consider themselves professional baseball fans. You randomly select 10 men and ask each if he considers himself a professional baseball fan. Find the probability that the number who consider themselves baseball fans is (a) exactly eight, (b) at least eight, and (c) less than eight. *(Source: Gallup Poll)*

18. Favorite Cookie Ten percent of adults say oatmeal raisin is their favorite cookie. You randomly select 12 adults and ask each to name his or her favorite cookie. Find the probability that the number who say oatmeal raisin is their favorite cookie is (a) exactly four, (b) at least four, and (c) less than four. *(Source: WEAREVER)*

19. Vacation Purpose Twenty-one percent of vacationers say the primary purpose of their vacation is outdoor recreation. You randomly select 10 vacationers and ask each to name the primary purpose of his or her vacation. Find the probability that the number who say outdoor recreation is the primary purpose of their vacation is (a) exactly three, (b) more than three, and (c) at most three. *(Source: Travel Industry Association)*

20. Honeymoon Financing Seventy percent of married couples paid for their honeymoon themselves. You randomly select 20 married couples and ask each if they paid for their honeymoon themselves. Find the probability that the number of couples who say they paid for their honeymoon themselves is (a) exactly one, (b) more than one, and (c) at most one. *(Source: Bride's Magazine)*

21. Favorite Nut Fifty-five percent of adults say cashews are their favorite kind of nut. You randomly select 12 adults and ask each to name his or her favorite nut. Find the probability that the number who say cashews are their favorite nut is (a) exactly three, (b) at least four, and (c) at most two. *(Source: Harris Interactive)*

22. Retirement Fourteen percent of workers believe they will need less than $250,000 when they retire. You randomly select 10 workers and ask each how much money he or she thinks they will need for retirement. Find the probability that the number of workers who say they will need less than $250,000 when they retire is (a) exactly two, (b) more than six, and (c) at most five. *(Source: Retirement Corporation of America)*

23. Credit Cards Twenty-eight percent of college students say they use credit cards because of the rewards program. You randomly select 10 college students and ask each to name the reason he or she uses credit cards. Find the probability that the number of college students who say they use credit cards because of the rewards program is (a) exactly two, (b) more than two, and (c) between two and five inclusive. *(Source: Experience.com)*

24. Career Advancement Twenty-four percent of executives say that older workers have blocked their career advancement. You randomly select 12 executives and ask if they feel that older workers have blocked their career advancement. Find the probability that the number who say older workers have blocked their career advancement is (a) exactly four, (b) more than four, and (c) between four and eight inclusive. *(Source: Korn/Ferry International)*

Constructing Binomial Distributions *In Exercises 25–28, (a) construct a binomial distribution, (b) graph the binomial distribution using a histogram, (c) describe the shape of the histogram, find the (d) mean, (e) variance, and (f) standard deviation of the binomial distribution, and (g) interpret the results in the context of the real-life situation. What values of the random variable x would you consider unusual? Explain your reasoning.*

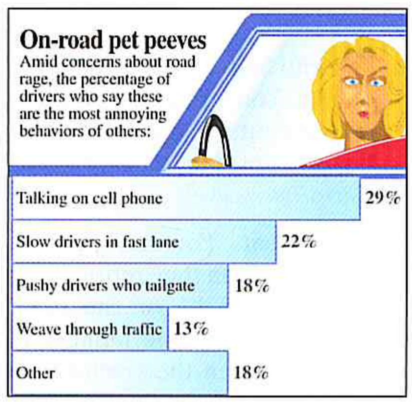
25. Women Baseball Fans Thirty-seven percent of women consider themselves fans of professional baseball. You randomly select six women and ask each if she considers herself a fan of professional baseball. *(Source: Gallup Poll)*

26. No Trouble Sleeping One in four adults says he or she has no trouble sleeping at night. You randomly select five adults and ask each if he or she has no trouble sleeping at night. *(Source: Marist Institute for Public Opinion)*

27. Blood Donors Five percent of people in the United States eligible to donate blood actually do. You randomly select four eligible blood donors and ask if they donate blood. *(Adapted from American Association of Blood Banks)*

28. Blood Types Thirty-eight percent of people in the United States have type O^+ blood. You randomly select five Americans and ask them if their blood type is O^+. *(Source: American Association of Blood Banks)*

29. Road Rage The graph shows the results of a survey of drivers who were asked to name the most annoying habit of other drivers. You randomly select six people who participated in the survey and ask each one of them to name the most annoying habit of other drivers. Let x represent the number who named talking on cell phones as the most annoying habit. *(Source: Hagerty Insurance)*

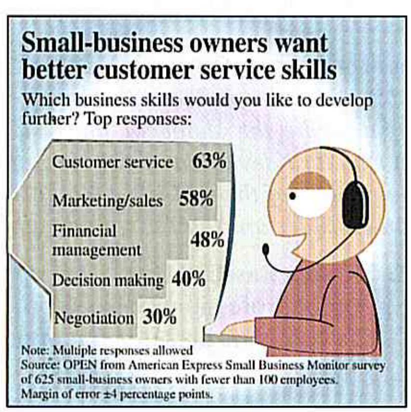


(a) Construct a binomial distribution.

(b) Find the probability that exactly two people will name "talking on cell phones."

(c) Find the probability that at least five people will name "talking on cell phones."

30. Small-Business Owners The graph shows the results of a survey of small-business owners who were asked which business skills they would like to develop further. You randomly select five owners who participated in the survey and ask each one of them which business skills he or she wants to develop further. Let x represent the number who said financial management was the skill they wanted to develop further. *(Source: American Express)*



(a) Construct a binomial distribution.

(b) Find the probability that exactly two owners will say "financial management."

(c) Find the probability that fewer than four owners will say "financial management."

31. Writing Find the mean and standard deviation of the binomial distribution in Exercise 29 and interpret the results in the context of the real-life situation. What values of x would you consider unusual? Explain your reasoning.

32. Writing Find the mean and standard deviation of the binomial distribution in Exercise 30 and interpret the results in the context of the real-life situation. What values of x would you consider unusual? Explain your reasoning.

Extending Concepts

Multinomial Experiments *In Exercises 33 and 34, use the following information.*

A **multinomial experiment** is a probability experiment that satisfies the following conditions.

1. The experiment is repeated a fixed number of times n where each trial is independent of the other trials.
2. Each trial has k possible mutually exclusive outcomes: $E_1, E_2, E_3, \ldots, E_k$.
3. Each outcome has a fixed probability. Therefore, $P(E_1) = p_1$, $P(E_2) = p_2$, $P(E_3) = p_3, \ldots, P(E_k) = p_k$. The sum of the probabilities for all outcomes is

$$p_1 + p_2 + p_3 + \cdots + p_k = 1.$$

4. x_1 is the number of times E_1 will occur; x_2 is the number of times E_2 will occur; x_3 is the number of times E_3 will occur; and so on.
5. The discrete random variable x counts the number of times $x_1, x_2, x_3, \ldots, x_k$ occurs in n independent trials where

$$x_1 + x_2 + x_3 + \cdots + x_k = n.$$

The probability that x will occur is

$$P(x) = \frac{n!}{x_1!x_2!x_3! \cdots x_k!} p_1^{x_1} p_2^{x_2} p_3^{x_3} \ldots p_k^{x_k}.$$

33. Genetics According to a theory in genetics, if tall and colorful plants are crossed with short and colorless plants, four types of plants will result: tall and colorful, tall and colorless, short and colorful, and short and colorless, with corresponding probabilities of $\frac{9}{16}$, $\frac{3}{16}$, $\frac{3}{16}$, and $\frac{1}{16}$. If 10 plants are selected, find the probability that five will be tall and colorful, two will be tall and colorless, two will be short and colorful, and one will be short and colorless.

34. Genetics Another proposed theory in genetics gives the corresponding probabilities for the four types of plants described as $\frac{5}{16}$, $\frac{4}{16}$, $\frac{1}{16}$, and $\frac{6}{16}$. If 10 plants are selected, find the probability that five will be tall and colorful, two will be tall and colorless, two will be short and colorful, and one will be short and colorless.