4.1 Probability Distributions

What You SHOULD LEARN

- How to distinguish between discrete random variables and continuous random variables
- How to construct a discrete probability distribution and its graph
- How to determine if a distribution is a probability distribution
- How to find the mean, variance, and standard deviation of a discrete probability distribution
- How to find the expected value of a discrete probability distribution

Random Variables ▸ Discrete Probability Distributions ▸ Mean, Variance, and Standard Deviation ▸ Expected Value

▸ Random Variables

The outcome of a probability experiment is often a count or a measure. When this occurs, the outcome is called a random variable.

DEFINITION

A **random variable** x represents a numerical value associated with each outcome of a probability experiment.

The word *random* indicates that x is determined by chance. There are two types of random variables: discrete and continuous.

DEFINITION

A random variable is **discrete** if it has a finite or countable number of possible outcomes that can be listed.

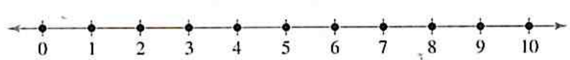
A random variable is **continuous** if it has an uncountable number of possible outcomes, represented by an interval on the number line.

You conduct a study of the number of calls a salesperson makes in one day. The possible values of the random variable x are 0, 1, 2, 3, 4, and so on. Because the set of possible outcomes

$$\{0, 1, 2, 3, \ldots\}$$

can be listed, x is a discrete random variable. You can represent its values as points on a number line.

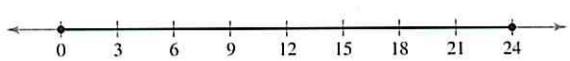
Number of Sales Calls (Discrete)



x can have only whole number values: 0, 1, 2, 3, ...

A different way to conduct the study would be to measure the time (in hours) a salesperson spends making calls in one day. Because the time spent making sales calls can be any number from 0 to 24 (including fractions and decimals), x is a continuous random variable. You can represent its values with an interval on a number line, but you cannot list all the possible values.

Hours Spent on Sales Calls (Continuous)



x can have any value between 0 and 24.

When a random variable is discrete, you can list the possible values it can assume. However, it is impossible to list all values for a continuous random variable.

Study Tip

In most practical applications, discrete random variables represent count data, while continuous random variables represent measured data.

EXAMPLE 1

Discrete Variables and Continuous Variables

Decide whether the random variable x is discrete or continuous. Explain your reasoning.

1. x represents the number of stocks in the Dow Jones Industrial Average that have share price increases on a given day.
2. x represents the volume of water in a 32-ounce container.

Solution

1. The number of stocks whose share price increases can be counted.

   $\{0, 1, 2, 3, \ldots, 30\}$

   So, x is a *discrete* random variable.

2. The amount of water in the container can be any volume between 0 ounces and 32 ounces. So, x is a continuous random variable.

Try It Yourself 1

Decide whether the random variable x is discrete or continuous.

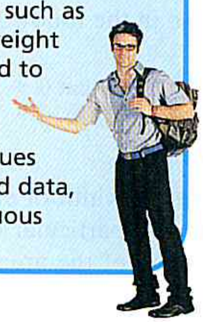
1. x represents the length of time it takes to complete a test.
2. x represents the number of songs played by a band at a rock festival.

a. Decide if x represents counted data or measured data.
b. Make a conclusion and *explain* your reasoning.

Answer: Page A38

Insight

Values of variables such as age, height, and weight are usually rounded to the nearest year, inch, or pound. However, these values represent measured data, so they are continuous random variables.



It is important that you can distinguish between discrete and continuous random variables because different statistical techniques are used to analyze each. The remainder of this chapter focuses on discrete random variables and their probability distributions. You will study continuous distributions later.

Discrete Probability Distributions

Each value of a discrete random variable can be assigned a probability. By listing each value of the random variable with its corresponding probability, you are forming a probability distribution.

DEFINITION

A **discrete probability distribution** lists each possible value the random variable can assume, together with its probability. A probability distribution must satisfy the following conditions.

In Words	*In Symbols*
1. The probability of each value of the discrete random variable is between 0 and 1, inclusive.	$0 \le P(x) \le 1$
2. The sum of all the probabilities is 1.	$\Sigma P(x) = 1$

Because probabilities represent relative frequencies, a discrete probability distribution can be graphed with a relative frequency histogram.

GUIDELINES

Constructing a Discrete Probability Distribution

Let x be a discrete random variable with possible outcomes $x_1, x_2, \ldots, x_n$.

1. Make a frequency distribution for the possible outcomes.
2. Find the sum of the frequencies.
3. Find the probability of each possible outcome by dividing its frequency by the sum of the frequencies.
4. Check that each probability is between 0 and 1, inclusive, and that the sum is 1.

Frequency Distribution

Score, x	Frequency, $P(x)$
1	24
2	33
3	42
4	30
5	21

EXAMPLE 2

Constructing and Graphing a Discrete Probability Distribution

An industrial psychologist administered a personality inventory test for passive-aggressive traits to 150 employees. Individuals were given a score from 1 to 5, where 1 was extremely passive and 5 extremely aggressive. A score of 3 indicated neither trait. The results are shown at the left. Construct a probability distribution for the random variable x. Then graph the distribution using a histogram.

Solution Divide the frequency of each score by the total number of individuals in the study to find the probability for each value of the random variable.

$$P(1) = \frac{24}{150} = 0.16 \qquad P(2) = \frac{33}{150} = 0.22 \qquad P(3) = \frac{42}{150} = 0.28$$

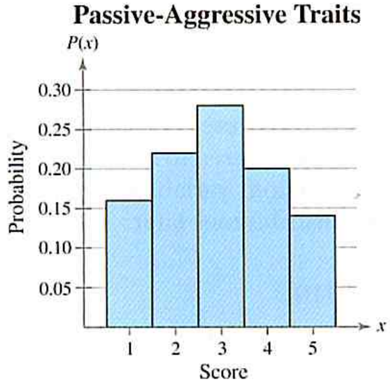
$$P(4) = \frac{30}{150} = 0.20 \qquad P(5) = \frac{21}{150} = 0.14$$

The discrete probability distribution is shown in the following table.

x	1	2	3	4	5
$P(x)$	0.16	0.22	0.28	0.20	0.14

Note that $0 \leq P(x) \leq 1$ and $\Sigma P(x) = 1$.

Passive-Aggressive Traits



The histogram is shown at the left. Because the width of each bar is one, the area of each bar is equal to the probability of a particular outcome. Also, the probability of an event corresponds to the sum of the areas of the outcomes included in the event. For instance, the probability of the event "having a score of 2 or 3" is equal to the sum of the areas of the second and third bars,

$$(1)(0.22) + (1)(0.28) = 0.22 + 0.28 = 0.50.$$

Interpretation You can see that the distribution is approximately symmetric.

▸Try It Yourself 2

A company tracks the number of sales new employees make each day during a 100-day probationary period. The results for one new employee are shown at the left. Construct and graph a probability distribution.

Frequency Distribution

Sales per day, x	Number of days, f
0	16
1	19
2	15
3	21
4	9
5	10
6	8
7	2

a. *Find* the probability of each outcome.
b. *Organize* the probabilities in a probability distribution.
c. *Graph* the probability distribution using a histogram.

Answer: Page A38

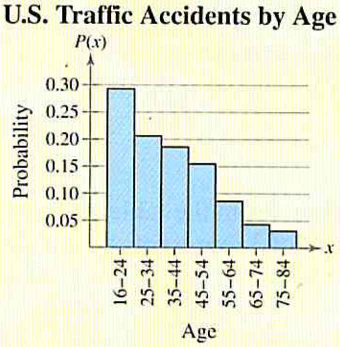
Probability Distribution

Days of rain, x	Probability, $P(x)$
0	0.216
1	0.432
2	0.288
3	0.064

PICTURING the WORLD

In a recent year in the United States, nearly 11 million traffic accidents were reported to the police. A histogram of traffic accidents for various age groups from 16 to 84 is shown. (Source: National Safety Council)

U.S. Traffic Accidents by Age



Estimate the probability that a randomly selected person involved in a traffic accident is in the 16 to 34 age group.

EXAMPLE 3

Verifying Probability Distributions

Verify that the distribution at the left (see page 193) is a probability distribution.

Solution If the distribution is a probability distribution, then (1) each probability is between 0 and 1, inclusive, and (2) the sum of the probabilities equals 1.

1. Each probability is between 0 and 1.
2. $\Sigma P(x) = 0.216 + 0.432 + 0.288 + 0.064$
   $= 1.$

Interpretation Because both conditions are met, the distribution is a probability distribution.

▸ Try It Yourself 3

Verify that the distribution you constructed in Try It Yourself 2 is a probability distribution.

a. Verify that the *probability* of each outcome is between 0 and 1, inclusive.
b. Verify that the *sum* of all the probabilities is 1.
c. Make a *conclusion*.

Answer: Page A38

EXAMPLE 4

Probability Distributions

Decide whether each distribution is a probability distribution.

1.

x	5	6	7	8
$P(x)$	0.28	0.21	0.43	0.15

2.

x	1	2	3	4
$P(x)$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{5}{4}$	-1

Solution

1. Each probability is between 0 and 1, but the sum of the probabilities is 1.07, which is greater than 1. So, it is *not* a probability distribution.
2. The sum of the probabilities is equal to 1, but $P(3)$ and $P(4)$ are not between 0 and 1. So, it is *not* a probability distribution. Probabilities can never be negative or greater than 1.

▸ Try It Yourself 4

Decide whether the distribution is a probability distribution. Explain your reasoning.

1.

x	5	6	7	8
$P(x)$	$\frac{1}{16}$	$\frac{5}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

2.

x	1	2	3	4
$P(x)$	0.09	0.36	0.49	0.06

a. Verify that the *probability* of each outcome is between 0 and 1.
b. Verify that the *sum* of all the probabilities is 1.
c. Make a *conclusion*.

Answer: Page A38

Mean, Variance, and Standard Deviation

You can measure the center of a probability distribution with its mean and measure the variability with its variance and standard deviation. The mean of a discrete random variable is defined as follows.

MEAN OF A DISCRETE RANDOM VARIABLE

The **mean** of a discrete random variable is given by

$$\mu = \Sigma x P(x).$$

Each value of x is multiplied by its corresponding probability and the products are added.

The mean of the random variable represents the "theoretical average" of a probability experiment and sometimes is not a possible outcome. If the experiment were performed many thousands of times, the mean of all the outcomes would be close to the mean of the random variable.

x	$P(x)$
1	0.16
2	0.22
3	0.28
4	0.20
5	0.14

EXAMPLE 5

Finding the Mean of a Probability Distribution

The probability distribution for the personality inventory test for passive-aggressive traits discussed in Example 2 is given at the left. Find the mean score. What can you conclude?

Solution

Use a table to organize your work, as shown below. From the table, you can see that the mean score is approximately 2.9. A score of 3 represents an individual who exhibits neither passive nor aggressive traits. The mean is slightly under 3.

x	$P(x)$	$xP(x)$
1	0.16	$1(0.16) = 0.16$
2	0.22	$2(0.22) = 0.44$
3	0.28	$3(0.28) = 0.84$
4	0.20	$4(0.20) = 0.80$
5	0.14	$5(0.14) = 0.70$
	$\Sigma P(x) = 1$	$\Sigma xP(x) = 2.94$ ← Mean

Interpretation You can conclude that the mean personality trait is neither extremely passive nor extremely aggressive, but is slightly closer to passive.

Try It Yourself 5

Find the mean of the probability distribution you constructed in Try It Yourself 2. What can you conclude?

a. *Find the product* of each random outcome and its corresponding probability.
b. *Find the sum* of the products.
c. *What* can you conclude?

Answer: Page A38

Study Tip

Notice that the mean in Example 5 is rounded to one decimal place. This rounding was done because the mean of a probability distribution should be rounded to one more decimal place than was used for the random variable x. This *round-off rule* is also used for the variance and standard deviation of a probability distribution.

Although the mean of the random variable of a probability distribution describes a typical outcome, it gives no information about how the outcomes vary. To study the variation of the outcomes, you can use the variance and standard deviation of the random variable of a probability distribution.

Study Tip

A shortcut formula for the variance of a probability distribution is

$$\sigma^2 = [\Sigma x^2 P(x)] - \mu^2.$$

STANDARD DEVIATION OF A DISCRETE RANDOM VARIABLE

The **variance** of a discrete random variable is

$$\sigma^2 = \Sigma(x - \mu)^2 P(x).$$

The **standard deviation** is

$$\sigma = \sqrt{\sigma^2} = \sqrt{\Sigma(x - \mu)^2 P(x)}.$$

x	$P(x)$
1	0.16
2	0.22
3	0.28
4	0.20
5	0.14

EXAMPLE 6

Finding the Variance and Standard Deviation

The probability distribution for the personality inventory test for passive-aggressive traits discussed in Example 2 is given at the left. Find the variance and standard deviation of the probability distribution.

Solution From Example 5, you know that before rounding, the mean of the distribution is $\mu = 2.94$. Use a table to organize your work, as shown below.

x	$P(x)$	$x - \mu$	$(x - \mu)^2$	$P(x)(x - \mu)^2$
1	0.16	−1.94	3.764	0.602
2	0.22	−0.94	0.884	0.194
3	0.28	0.06	0.004	0.001
4	0.20	1.06	1.124	0.225
5	0.14	2.06	4.244	0.594
	$\Sigma P(x) = 1$			$\Sigma P(x)(x - \mu)^2 = 1.616$

Variance

So, the variance is

$$\sigma^2 = 1.616 \approx 1.6$$

and the standard deviation is

$$\sigma = \sqrt{\sigma^2} = \sqrt{1.616} \approx 1.3.$$

Interpretation Most of the data values differ from the mean by no more than 1.3 points.

▸ Try It Yourself 6

Find the variance and standard deviation for the probability distribution constructed in Try It Yourself 2.

a. *For each value of x, find the square* of the deviation from the mean and multiply that by the corresponding probability of x.
b. *Find the sum* of the products found in part (a) for the variance.
c. *Take the square root* of the variance for the standard deviation.
d. *Interpret* the results.

Answer: Page A39

Expected Value

The mean of a random variable represents what you would expect to happen for thousands of trials. It is also called the *expected value*.

DEFINITION

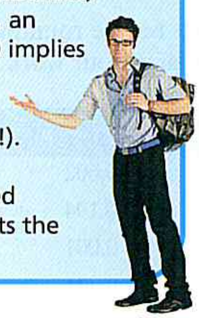
The **expected value** of a discrete random variable is equal to the mean of the random variable.

$$\text{Expected Value} = E(x) = \mu = \sum xP(x)$$

Although probabilities can never be negative, the expected value of a random variable can be negative.

Insight

In most applications, an expected value of 0 has a practical interpretation. For instance, in gambling games, an expected value of 0 implies that a game is a fair game (an unlikely occurrence!). In a profit and loss analysis, an expected value of 0 represents the break-even point.



EXAMPLE 7

Finding an Expected Value

At a raffle, 1500 tickets are sold at $2 each for four prizes of $500, $250, $150, and $75. You buy one ticket. What is the expected value of your gain?

Solution

To find the gain for each prize, subtract the price of the ticket from the prize. For instance, your gain for the $500 prize is

$$\$500 - \$2 = \$498$$

and your gain for the $250 prize is

$$\$250 - \$2 = \$248.$$

Write a probability distribution for the possible gains (or outcomes).

Gain, x	$498	$248	$148	$73	−$2
Probability, $P(x)$	$\frac{1}{1500}$	$\frac{1}{1500}$	$\frac{1}{1500}$	$\frac{1}{1500}$	$\frac{1496}{1500}$

Then, using the probability distribution, you can find the expected value.

$$\begin{aligned} E(x) &= \sum xP(x) \\ &= \$498 \cdot \frac{1}{1500} + \$248 \cdot \frac{1}{1500} + \$148 \cdot \frac{1}{1500} + \$73 \cdot \frac{1}{1500} + (-\$2) \cdot \frac{1496}{1500} \\ &= -\$1.35 \end{aligned}$$

Interpretation Because the expected value is negative, you can expect to lose an average of $1.35 for each ticket you buy.

Try It Yourself 7

At a raffle, 2000 tickets are sold at $5 each for five prizes of $2000, $1000, $500, $250, and $100. You buy one ticket. What is the expected value of your gain?

a. Find the *gain* for each prize.
b. Write a *probability distribution* for the possible gains.
c. Find the *expected value*.
d. *Interpret* the results.

Answer: Page A39

4.1 EXERCISES

Building Basic Skills and Vocabulary

1. What is a random variable? Give an example of a discrete random variable and a continuous random variable. Justify your answer.

2. What is a discrete probability distribution? What are the two conditions that determine a probability distribution?

3. The expected value of an accountant's profit and loss analysis is 0. Explain what this means.

4. What is the significance of the mean of a probability distribution?

True or False? *In Exercises 5–8, determine whether the statement is true or false. If it is false, rewrite it as a true statement.*

5. In most applications, continuous random variables represent counted data, while discrete random variables represent measured data.

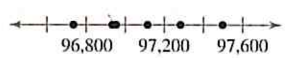
6. For a random variable x the word *random* indicates that the value of x is determined by chance.

7. The mean of a random variable represents the "theoretical average" of a probability experiment and sometimes is not a possible outcome.

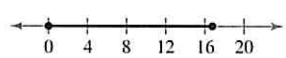
8. The expected value of a discrete random variable is equal to the standard deviation of the random variable.

Graphical Analysis *In Exercises 9–12, decide whether the graph represents a discrete random variable or a continuous random variable. Explain your reasoning.*

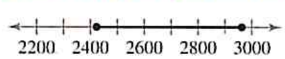
9. The home attendance for football games at a university



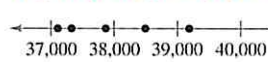
10. The length of time students use a computer each week



11. The annual vehicle-miles driven in the United States. *(Source: U.S. Federal Highway Administration)*



12. The annual traffic fatalities in the United States. *(Source: National Highway Traffic Safety Administration)*



Distinguishing Between Discrete and Continuous Random Variables
In Exercises 13–20, decide whether the random variable x is discrete or continuous. Explain your reasoning.

13. x represents the number of motorcycle accidents in one year in California.

14. x represents the length of time it takes to get to work.

15. x represents the volume of blood drawn for a blood test.

16. x represents the number of rainy days in the month of July in Orlando, Florida.

17. x represents the number of home theater systems sold per month at an electronics store.

18. x represents the tension at which a randomly selected guitar's strings have been strung.

19. x represents the amount of snow (in inches) that fell in Nome, Alaska last winter.

20. x represents the total number of die rolls required for an individual to roll a five.

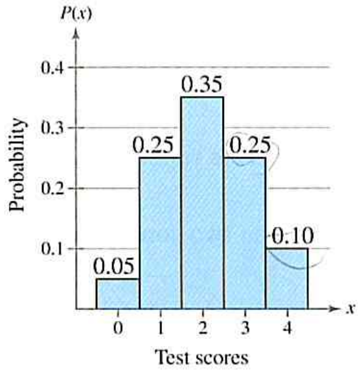


FIGURE FOR EXERCISE 21

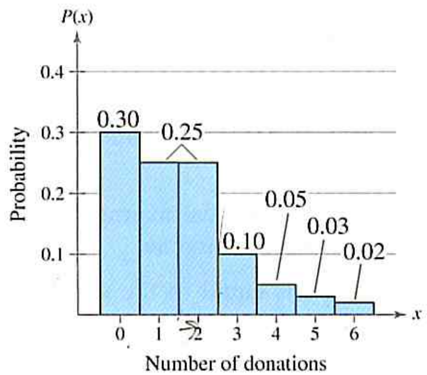


FIGURE FOR EXERCISE 22

Using and Interpreting Concepts

21. Employee Testing A company gave psychological tests to prospective employees. The random variable x represents the possible test scores. Use the histogram to find the probability that a person selected at random from the survey's sample had a test score of (a) more than two and (b) less than four.

22. Blood Donations A survey asked a sample of people how many times they donate blood each year. The random variable x represents the number of donations for one year. Use the histogram to find the probability that a person selected at random from the survey's sample donated blood (a) more than once in a year and (b) less than three times in a year.

Determining a Missing Probability *In Exercises 23 and 24, determine the probability distribution's missing probability value.*

23. Dependent Children A sociologist surveyed the households in a small town. The random variable x represents the number of dependent children in the households.

x	0	1	2	3	4
$P(x)$	0.07	0.20	0.38	?	0.13

24. Dependent Children The sociologist in Exercise 23 surveyed the households in a neighboring town. The random variable x represents the number of dependent children in the households.

x	0	1	2	3	4	5	6
$P(x)$	0.05	?	0.23	0.21	0.17	0.11	0.08

Identifying Probability Distributions *In Exercises 25–28, decide whether the distribution is a probability distribution. If it is not a probability distribution, identify the property (or properties) that are not satisfied.*

25. Tires A mechanic checked the tire pressures on each car that he worked on for one week. The random variable x represents the number of tires that were underinflated.

x	0	1	2	3	4
$P(x)$	0.30	0.25	0.25	0.15	0.05

26. Phone Lines A company recorded the number of phone lines in use per hour during one work day. The random variable x represents the number of phone lines in use.

x	0	1	2	3	4	5	6
$P(x)$	0.135	0.186	0.226	0.254	0.103	0.64	0.032

27. Quality Control A quality inspector checked for imperfections in rolls of fabric for one week. The random variable x represents the number of imperfections found.

x	0	1	2	3	4	5
$P(x)$	$\frac{3}{4}$	$\frac{1}{10}$	$\frac{1}{20}$	$\frac{1}{25}$	$\frac{1}{50}$	$-\frac{1}{100}$

28. Golf Putts A golf tournament director recorded the number of putts needed on a hole for the four rounds of a tournament. The random variable x represents the number of putts needed on the hole.

x	0	1	2	3	4	5
$P(x)$	0.007	0.292	0.394	0.245	0.058	0.004

Constructing Probability Distributions *In Exercises 29–34, (a) use the frequency distribution to construct a probability distribution, find the (b) mean, (c) variance, and (d) standard deviation of the probability distribution, and (e) interpret the results in the context of the real-life situation.*

29. Dogs The number of dogs per household in a small town

Dogs	0	1	2	3	4	5
Households	1491	425	168	48	29	14

30. Cats The number of cats per household in a small town

Cats	0	1	2	3	4	5
Households	1941	349	203	78	57	40

31. Computers The number of computers per household in a small town

Computers	0	1	2	3
Households	300	280	95	20

32. DVDs The number of defects per batch of DVDs inspected

Defects	0	1	2	3	4	5
Batches	95	113	87	64	13	8

33. Overtime Hours The number of overtime hours worked in one week per employee

Overtime hours	0	1	2	3	4	5	6
Employees	6	12	29	57	42	30	16

34. Extracurricular Activities The number of school-related extracurricular activities per student

Activities	0	1	2	3	4	5	6	7
Students	19	39	52	57	68	41	27	17

Finding Expected Value *In Exercises 35–40, use the probability distribution or histogram to find the (a) mean, (b) variance, (c) standard deviation, and (d) expected value of the probability distribution, and (e) interpret the results.*

35. Quiz Students in a class take a quiz with eight questions. The number x of questions answered correctly can be approximated by the following probability distribution.

x	0	1	2	3	4	5	6	7	8
$P(x)$	0.02	0.02	0.06	0.06	0.08	0.22	0.30	0.16	0.08

36. 911 Calls A 911 service center recorded the number of calls received per hour. The number of calls per hour for one week can be approximated by the following probability distribution.

x	0	1	2	3	4	5	6	7
$P(x)$	0.01	0.10	0.26	0.33	0.18	0.06	0.03	0.03

37. Hurricanes The histogram shows the distribution of hurricanes that have hit the U.S. mainland by category, with 1 the weakest level and 5 the strongest. *(Source: National Hurricane Center)*

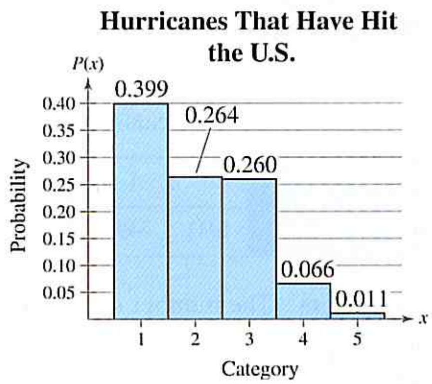


FIGURE FOR EXERCISE 37

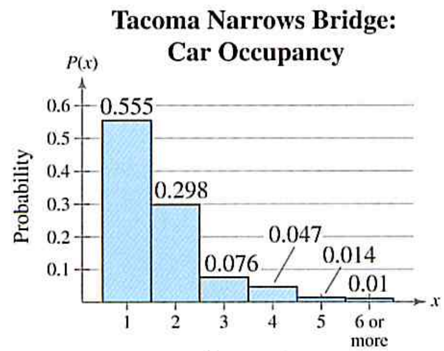


FIGURE FOR EXERCISE 38

38. Car Occupancy The histogram shows the distribution of occupants in cars crossing the Tacoma Narrows Bridge in Washington each week. *(Adapted from Washington State Department of Transportation)*

39. Household Size The histogram shows the distribution of household sizes in the United States for a recent year. *(Adapted from United States Census Bureau)*

40. Car Occupancy The histogram shows the distribution of carpooling by the number of cars per household. *(Adapted from Federal Highway Administration)*

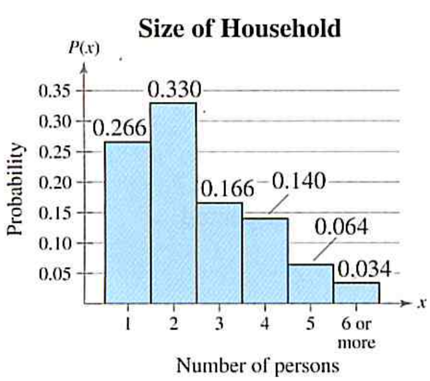


FIGURE FOR EXERCISE 39

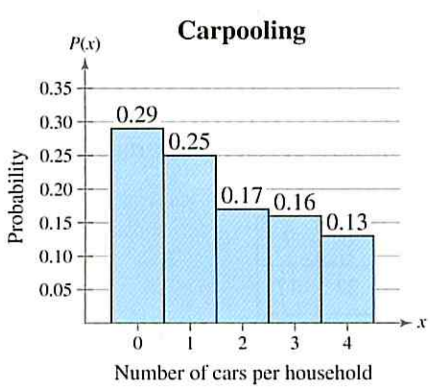


FIGURE FOR EXERCISE 40

41. Finding Probabilities Use the probability distribution you made for Exercise 29 to find the probability of randomly selecting a household that has (a) fewer than two dogs, (b) at least one dog, and (c) between one and three dogs, inclusive.

42. Finding Probabilities Use the probability distribution you made for Exercise 31 to find the probability of randomly selecting a household that has (a) no computers, (b) at least one computer, and (c) between zero and two computers, inclusive.

43. Unusual Values A person lives in a household with three dogs and claims that having three dogs is not unusual. Use the information in Exercise 29 to determine if this person is correct. Explain your reasoning.

44. Unusual Values A person lives in a household with no computer and claims that not having a computer is not unusual. Use the information in Exercise 31 to determine if this person is correct. Explain your reasoning.

Games of Chance *In Exercises 45 and 46, find the expected net gain to the player for one play of the game. If x is the net gain to a player in a game of chance, then $E(x)$ is usually negative. This value gives the average amount per game the player can expect to lose.*

45. In American roulette, the wheel has the 38 numbers

00, 0, 1, 2, . . . , 34, 35, and 36

marked on equally spaced slots. If a player bets \$1 on a number and wins, then the player keeps the dollar and receives an additional 35 dollars. Otherwise, the dollar is lost.

46. A charity organization is selling \$4 raffle tickets as part of a fund-raising program. The first prize is a boat valued at \$3150, and the second prize is a camping tent valued at \$450. The remaining 15 prizes are \$25 gift certificates. The number of tickets sold is 5000.

Extending Concepts

Linear Transformation of a Random Variable *In Exercises 47 and 48, use the following information.*

For a random variable x, a new random variable y can be created by applying a ***linear transformation*** *$y = a + bx$, where a and b are constants. If the random variable x has mean μ_x and standard deviation σ_x, then the mean, variance, and standard deviation of y are given by the following formulas.*

$$\mu_y = a + b\mu_x \qquad \sigma_y{}^2 = b^2\sigma_x{}^2 \qquad \sigma_y = |b|\sigma_x$$

47. The mean annual salary for employees at a company is \$36,000. At the end of the year, each employee receives a \$1000 bonus and a 5% raise (based on salary). What is the new annual salary (including the bonus and raise) for the employees?

48. The mean annual salary for employees at a company is \$36,000 with a variance of 15,202,201. At the end of the year, each employee receives a \$2000 bonus and a 4% raise (based on salary). What is the standard deviation of the new salaries?

Independent and Dependent Random Variables *Two random variables x and y are* ***independent*** *if the value of x does not affect the value of y. If the variables are not independent, they are* ***dependent.*** *A new random variable can be formed by finding the sum or difference of random variables. If a random variable x has mean μ_x and a random variable y has mean μ_y, then the mean of the sum and difference of the variables are given by the following equations.*

$$\mu_{x+y} = \mu_x + \mu_y \qquad \mu_{x-y} = \mu_x - \mu_y$$

If random variables are independent, then the variance and standard deviation of the sum or difference of the random variables can be found. So, if a random variable x has variance $\sigma^2{}_x$ and a random variable y has variance $\sigma^2{}_y$, then the variances of the sum and difference of the variables are given by the following equations. Note that the variance of the difference is the sum of the variances.

$$\sigma^2{}_{x+y} = \sigma^2{}_x + \sigma^2{}_y \qquad \sigma^2{}_{x-y} = \sigma^2{}_x + \sigma^2{}_y$$

In Exercises 49 and 50, the distribution of SAT scores for college-bound male seniors has mean of 1532 and a standard deviation of 312. The distribution of SAT scores for college-bound female seniors has a mean of 1506 and a standard deviation of 304. One male and one female are randomly selected. Assume their scores are independent. *(Source: College Board Online)*

49. What is the average sum of their scores? What is the average difference of their scores?

50. What is the standard deviation of the difference in their scores?