3.4 Additional Topics in Probability and Counting

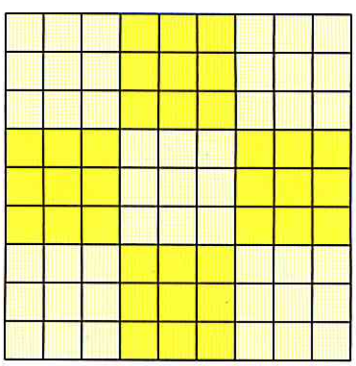
What You SHOULD LEARN

- How to find the number of ways a group of objects can be arranged in order
- How to find the number of ways to choose several objects from a group without regard to order
- How to use counting principles to find probabilities

Study Tip

Notice at the right that as n increases, $n!$ becomes very large. Take some time now to learn how to use the factorial key on your calculator.

Sudoku Number Puzzle



National League Central Division

Chicago Cubs	Cincinnati Reds
Houston Astros	Milwaukee Brewers
Pittsburgh Pirates	St. Louis Cardinals

Permutations ▸ Combinations ▸ Applications of Counting Principles

▸ Permutations

In Section 3.1, you learned that the Fundamental Counting Principle is used to find the number of ways two or more events can occur in sequence. In this section, you will study several other techniques for counting the number of ways an event can occur. An important application of the Fundamental Counting Principle is determining the number of ways that n objects can be arranged in order or in a permutation.

DEFINITION

A **permutation** is an ordered arrangement of objects. The number of different permutations of n distinct objects is $n!$.

The expression $n!$ is read as **n factorial** and is defined as follows.

$$n! = n\cdot(n-1)\cdot(n-2)\cdot(n-3)\cdots 3\cdot 2\cdot 1$$

As a special case, $0! = 1$. Here are several other values of $n!$.

$$1! = 1,\ 2! = 2\cdot 1 = 2,\ 3! = 3\cdot 2\cdot 1 = 6,\ 4! = 4\cdot 3\cdot 2\cdot 1 = 24$$

EXAMPLE 1

Finding the Number of Permutations of n Objects

The objective of a 9×9 Sudoku number puzzle is to fill the grid so that each row, each column, and each 3×3 grid contain the digits 1 to 9. How many different ways can the first row of a blank 9×9 Sudoku grid be filled?

Solution

The number of permutations is $9! = 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1 = 362{,}880$. So, there are 362,880 different ways the first row can be filled.

▸ Try It Yourself 1

The teams in the National League Central Division are listed at the left. How many different final standings are possible?

a. Determine *how many teams*, n, are in the Central Division.
b. Evaluate $n!$.

Answer: Page A38

Suppose you want to choose some of the objects in a group and put them in order. Such an ordering is called a **permutation of n objects taken r at a time.**

PERMUTATIONS OF n OBJECTS TAKEN r AT A TIME

The number of permutations of n distinct objects taken r at a time is

$${}_nP_r = \frac{n!}{(n-r)!},\ \text{where } r \le n.$$

EXAMPLE 2

Finding $_nP_r$

Find the number of ways of forming three-digit codes in which no digit is repeated.

Solution

To form a three-digit code with no repeating digits, you need to select 3 digits from a group of 10, so $n = 10$ and $r = 3$.

$$_nP_r = {}_{10}P_3 = \frac{10!}{(10-3)!} = \frac{10!}{7!} = \frac{10\cdot 9\cdot 8\cdot \cancel{7}\cdot \cancel{6}\cdot \cancel{5}\cdot \cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot \cancel{1}}{\cancel{7}\cdot \cancel{6}\cdot \cancel{5}\cdot \cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot \cancel{1}} = 720$$

So, there are 720 possible three-digit codes that do not have repeating digits.

▸ Try It Yourself 2

A psychologist shows a list of eight activities to her subject. How many ways can the subject pick a first, second, and third activity?

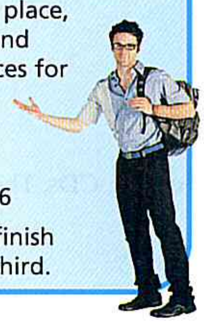
a. Find the quotient of $n!$ and $(n - r)!$. (List the factors and divide out.)
b. Write the result as a sentence. *Answer: Page A38*

Insight

Notice that the Fundamental Counting Principle can be used in Example 3 to obtain the same result. You can see that there are 43 choices for first place, 42 choices for second place, and 41 choices for third place. You can then conclude that there are

$43 \cdot 42 \cdot 41 = 74{,}046$

ways the cars can finish first, second, and third.



EXAMPLE 3

Permutations of *n* Objects Taken *r* at a Time

Forty-three race cars started the 2007 Daytona 500. How many ways can the cars finish first, second, and third? *(Source: NASCAR.com)*

Solution You need to select three race cars from a group of 43, so $n = 43$ and $r = 3$. Because the order is important, the number of ways the cars can finish first, second, and third is

$$_nP_r = {}_{43}P_3 = \frac{43!}{(43-3)!} = \frac{43!}{40!} = 43\cdot 42\cdot 41 = 74{,}046.$$

▸ Try It Yourself 3

The board of directors for a company has 12 members. One member is the president, another is the vice president, another is the secretary, and another is the treasurer. How many ways can can these positions be assigned?

a. *Identify* the total number of objects n and the number of objects r being chosen in order.
b. *Evaluate* $_nP_r$. *Answer: Page A38*

Study Tip

The letters *AAAABBC* can be rearranged in 7! orders, but many of these are not distinguishable. The number of distinguishable orders is

$$\frac{7!}{4!\cdot 2!\cdot 1!} = \frac{7\cdot 6\cdot 5}{2} = 105.$$

You may want to order a group of n objects in which some of the objects are the same. For instance, consider a group of letters consisting of four As, two Bs, and one C. How many ways can you order such a group? Using the previous formula, you might conclude that there are $_7P_7 = 7!$ possible orders. However, because some of the objects are the same, not all of these permutations are *distinguishable.* How many distinguishable permutations are possible? The answer can be found using the following formula.

DISTINGUISHABLE PERMUTATIONS

The number of **distinguishable permutations** of n objects, where n_1 are of one type, n_2 are of another type, and so on is

$$\frac{n!}{n_1!\cdot n_2!\cdot n_3!\cdots n_k!}, \text{ where } n_1 + n_2 + n_3 + \cdots + n_k = n.$$

EXAMPLE 4

Distinguishable Permutations

A building contractor is planning to develop a subdivision. The subdivision is to consist of 6 one-story houses, 4 two-story houses, and 2 split-level houses. In how many distinguishable ways can the houses be arranged?

Solution There are to be 12 houses in the subdivision, 6 of which are of one type (one-story), 4 of another type (two-story), and 2 of a third type (split-level). So, there are

$$\frac{12!}{6! \cdot 4! \cdot 2!} = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6!}{6! \cdot 4! \cdot 2!}$$
$$= 13{,}860 \text{ distinguishable ways.}$$

Interpretation There are 13,860 distinguishable ways to arrange the houses in the subdivision.

Try It Yourself 4

The contractor wants to plant six oak trees, nine maple trees, and five poplar trees along the subdivision street. The trees are to be spaced evenly apart. In how many distinguishable ways can they be planted?

a. *Identify* the total number of objects, n, and the number of each type of object in the group, n_1, n_2, and n_3.

b. *Evaluate* $\dfrac{n!}{n_1! \cdot n_2! \cdots n_k!}$.

Answer: Page A38

▸ Combinations

You want to buy three CDs from a selection of five CDs. There are 10 ways to make your selections.

ABC, ABD, ABE,
ACD, ACE,
ADE,
BCD, BCE,
BDE,
CDE

In each selection, order does not matter (ABC is the same set as BAC). The number of ways to choose r objects from n objects without regard to order is called the number of **combinations of n objects taken r at a time.**

Insight

You can think of a combination of n objects chosen r at a time as a permutation of n objects in which the r selected objects are alike and the remaining $n - r$ (not selected) objects are alike.

COMBINATION OF n OBJECTS TAKEN r AT A TIME

A combination is a selection of r objects from a group of n objects without regard to order and is denoted by $_nC_r$. The number of combinations of r objects selected from a group of n objects is

$$_nC_r = \frac{n!}{(n-r)!r!}.$$

EXAMPLE 5

Finding the Number of Combinations

A state's department of transportation plans to develop a new section of interstate highway and receives 16 bids for the project. The state plans to hire four of the bidding companies. How many different combinations of four companies can be selected from the 16 bidding companies?

Solution The state is selecting four companies from a group of 16, so $n = 16$ and $r = 4$. Because order is not important, there are

$$
\begin{aligned}
{}_nC_r = {}_{16}C_4 &= \frac{16!}{(16-4)!4!} \\
&= \frac{16!}{12!4!} \\
&= \frac{16 \cdot 15 \cdot 14 \cdot 13 \cdot 12!}{12! \cdot 4!} \\
&= 1820 \text{ different combinations.}
\end{aligned}
$$

Interpretation There are 1820 different combinations of four companies that can be selected from the 16 bidding companies.

▸ Try It Yourself 5

The manager of an accounting department wants to form a three-person advisory committee from the 20 employees in the department. In how many ways can the manager form this committee?

a. *Identify* the number of objects in the group n and the number of objects to be selected r.
b. *Evaluate* ${}_nC_r$.
c. *Write* the results as a sentence. *Answer: Page A38*

The table summarizes the counting principles.

Principle	Description	Formula
Fundamental Counting Principle	If one event can occur in m ways and a second event can occur in n ways, the number of ways the two events can occur in sequence is $m \cdot n$.	$m \cdot n$
Permutation	The number of different ordered arrangements of n distinct objects	$n!$
	The number of permutations of n distinct objects taken r at a time, where $r \leq n$	${}_nP_r = \frac{n!}{(n-r)!}$
	The number of distinguishable permutations of n objects where n_1 are of one type, n_2 are of another type, and so on	$\frac{n!}{n_1! \cdot n_2! \cdots n_k!}$
Combinations	The number of combinations of r objects selected from a group of n objects without regard to order	${}_nC_r = \frac{n!}{(n-r)!r!}$

PICTURING the WORLD

The largest lottery jackpot ever, $390 million, was won in the Mega Millions lottery. When the Mega Millions jackpot was won, five numbers were chosen from 1 to 56 and one number, the Mega Ball, was chosen from 1 to 46. The winning numbers are shown below.

If you buy one ticket, what is the probability that you will win the Mega Millions lottery?

Applications of Counting Principles

EXAMPLE 6

Finding Probabilities

A student advisory board consists of 17 members. Three members serve as the board's chair, secretary, and webmaster. Each member is equally likely to serve any of the positions. What is the probability of selecting at random the three members that hold each position?

Solution

There is one favorable outcome and there are

$$_{17}P_3 = \frac{17!}{(17-3)!} = \frac{17!}{14!} = 17 \cdot 16 \cdot 15 = 4080$$

ways the three positions can be filled. So, the probability of correctly selecting the three members that hold each position is

$$P(\text{selecting the three members}) = \frac{1}{4080} \approx 0.0002.$$

Try It Yourself 6

A student advisory board consists of 20 members. Two members serve as the board's chair and secretary. Each member is equally likely to serve either of the positions. What is the probability of selecting at random the two members that hold each position?

a. Find the *number of ways* the two positions can be filled.
b. Find the *probability* of correctly selecting the two members.

Answer: Page A38

EXAMPLE 7

Finding Probabilities

You have 11 letters consisting of one M, four Is, four Ss, and two Ps. If the letters are randomly arranged in order, what is the probability that the arrangement spells the word *Mississippi*?

Solution There is one favorable outcome and there are

$$\frac{11!}{1! \cdot 4! \cdot 4! \cdot 2!} = 34{,}650 \qquad \text{11 letters with 1, 4, 4, and 2 like letters}$$

distinguishable permutations of the given letters. So, the probability that the arrangement spells the word *Mississippi* is

$$P(\text{Mississippi}) = \frac{1}{34{,}650} \approx 0.000029.$$

Try It Yourself 7

You have 6 letters consisting of one L, two Es, two Ts, and one R. If the letters are randomly arranged in order, what is the probability that the arrangement spells the word *letter*?

a. *Find* the number of favorable outcomes and the number of distinguishable permutations.
b. *Divide* the number of favorable outcomes by the number of distinguishable permutations.

Answer: Page A38

EXAMPLE 8

Finding Probabilities

Find the probability of being dealt five diamonds from a standard deck of playing cards. (In poker, this is a diamond flush.)

Solution The possible number of ways of choosing 5 diamonds out of 13 is ${}_{13}C_5$. The number of possible five-card hands is ${}_{52}C_5$. So, the probability of being dealt 5 diamonds is

$$P(\text{diamond flush}) = \frac{{}_{13}C_5}{{}_{52}C_5} = \frac{1287}{2{,}598{,}960} \approx 0.0005.$$

▸ Try It Yourself 8

Find the probability of being dealt five diamonds from a standard deck of playing cards that also includes two jokers. In this case, the joker is considered to be a wild card that can be used to represent any card in the deck.

a. Find the *number of ways* of choosing 5 diamonds out of 15.
b. *Find* the number of possible five-card hands.
c. *Divide* the result of part a by the result of part b.

Answer: Page A38

EXAMPLE 9

Finding Probabilities

A food manufacturer is analyzing a sample of 400 corn kernels for the presence of a toxin. In this sample, three kernels have dangerously high levels of the toxin. If four kernels are randomly selected from the sample, what is the probability that exactly one kernel contains a dangerously high level of the toxin?

Solution

The possible number of ways of choosing one toxic kernel out of three toxic kernels is ${}_3C_1$. The possible number of ways of choosing 3 nontoxic kernels from 397 nontoxic kernels is ${}_{397}C_3$. So, using the Fundamental Counting Principle, the number of ways of choosing one toxic kernel and three nontoxic kernels is

$$\begin{aligned} {}_3C_1 \cdot {}_{397}C_3 &= 3 \cdot 10{,}349{,}790 \\ &= 31{,}049{,}370. \end{aligned}$$

The number of possible ways of choosing 4 kernels from 400 kernels is ${}_{400}C_4 = 1{,}050{,}739{,}900$. So, the probability of selecting exactly 1 toxic kernel is

$$P(1 \text{ toxic kernel}) = \frac{{}_3C_1 \cdot {}_{397}C_3}{{}_{400}C_4} = \frac{31{,}049{,}370}{1{,}050{,}739{,}900} \approx 0.0296.$$

▸ Try It Yourself 9

A jury consists of five men and seven women. Three are selected at random for an interview. Find the probability that all three are men.

a. *Find* the product of the number of ways to choose three men from five and the number of ways to choose zero women from seven.
b. *Find* the number of ways to choose 3 jury members from 12.
c. *Divide* the result of part a by the result of part b.

Answer: Page A38

3.4 EXERCISES

Building Basic Skills and Vocabulary

1. When you calculate the number of permutations of n distinct objects taken r at a time, what are you counting? Give an example.

2. When you calculate the number of combinations of r objects taken from a group of n objects, what are you counting? Give an example.

True or False? *In Exercises 3–6, determine whether the statement is true or false. If it is false, rewrite it as a true statement.*

3. A combination is an ordered arrangement of objects.

4. The number of different ordered arrangements of n distinct objects is $n!$.

5. If you divide the number of permutations of 11 objects taken 3 at a time by 3!, you will get the number of combinations of 11 objects taken 3 at a time.

6. ${}_7C_5 = {}_7C_2$

In Exercises 7–14, perform the indicated calculation.

7. ${}_7P_3$

8. ${}_{14}P_2$

9. ${}_7C_4$

10. ${}_8P_6$

11. ${}_{24}C_6$

12. $\dfrac{{}_8C_4}{{}_{12}C_6}$

13. $\dfrac{{}_6P_2}{{}_{10}P_4}$

14. $\dfrac{{}_8C_3}{{}_{12}C_3}$

In Exercises 15–18, decide if the situation involves permutations, combinations, or neither. Explain your reasoning.

15. The number of ways 15 people can line up in a row for concert tickets

16. The number of ways a four-member committee can be chosen from 10 people

17. The number of ways 2 captains can be chosen from 28 players on a lacrosse team.

18. The number of four-letter passwords that can be created when no letter can be repeated.

Using and Interpreting Concepts

19. Space Shuttle Menu Space shuttle astronauts each consume an average of 3000 calories per day. One meal normally consists of a main dish, a vegetable dish, and two different desserts. The astronauts can choose from 10 main dishes, 8 vegetable dishes, and 13 desserts. How many different meals are possible? *(Source: NASA)*

20. Skiing Eight people compete in a downhill ski race. Assuming that there are no ties, in how many different orders can the skiers finish?

21. Security Code In how many ways can the letters A, B, C, D, E, and F be arranged for a six-letter security code?

22. Starting Lineup The starting lineup for a softball team consists of 10 players. How many different batting orders are possible using the starting lineup?

23. Lottery Number Selection A lottery has 52 numbers. In how many different ways can 6 of the numbers be selected? (Assume that order of selection is not important.)

24. Assembly Process There are four processes involved in assembling a certain product. These processes can be performed in any order. Management wants to find which order is the least time-consuming. How many different orders will have to be tested?

25. Tree Planting A landscaper wants to plant four oak trees, eight maple trees, and six poplar trees along the border of a lawn. The trees are to be spaced evenly apart. In how many distinguishable ways can they be planted?

26. Experimental Group In order to conduct an experiment, 4 subjects are randomly selected from a group of 20 subjects. How many different groups of four subjects are possible?

27. Letters In how many distinguishable ways can the letters in the word *statistics* be written?

28. Jury Selection From a group of 40 people, a jury of 12 people is selected. In how many different ways can a jury of 12 people be selected?

Word Jumble *In Exercises 29–34, do the following.*

(a) Find the number of distinguishable ways the letters can be arranged.

(b) There is one arrangement that spells an important term used throughout the course. Find the term.

(c) If the letters are randomly arranged in order, what is the probability that the arrangement spells the word from part b?

29. palmes

30. nevte

31. etre

32. ediman

33. unoppolati

34. sidtbitoiurn

35. Horse Race A horse race has 12 entries. Assuming that there are no ties, what is the probability that the three horses owned by one person finish first, second, and third?

36. Pizza Toppings A pizza shop offers nine toppings. No topping is used more than once. What is the probability that the toppings on a pizza are pepperoni, onions, and mushrooms?

37. Jukebox You look over the songs on a jukebox and determine that you like 15 of the 56 songs.

(a) What is the probability that you like the next three songs that are played? Assume a song cannot be repeated.

(b) What is the probability that you do not like the next three songs that are played? Assume a song cannot be repeated.

38. Officers The offices of president, vice president, secretary, and treasurer for an environmental club will be filled from a pool of 14 candidates. Six of the candidates are members of the debate team.

(a) What is the probability that all of the offices are filled by members of the debate team?

(b) What is the probability that none of the offices are filled by members of the debate team?

39. Employee Selection Four sales representatives for a company are to be chosen to participate in a training program. The company has eight sales representatives, two in each of four regions. In how many ways can the four sales representatives be chosen if (a) there are no restrictions and (b) the selection must include a sales representative from each region? (c) What is the probability that the four sales representatives chosen to participate in the training program will be from only two of the four regions if they are chosen at random?

40. License Plates In a certain state, each automobile license plate number consists of two letters followed by a four-digit number. How many distinct license plate numbers can be formed if (a) there are no restrictions and (b) the letters O and I are not used? (c) What is the probability of selecting at random a license plate that ends in an even number?

41. Password A password consists of two letters followed by a five-digit number. How many passwords are possible if (a) there are no restrictions and (b) none of the letters or digits can be repeated? (c) What is the probability of guessing the password in one trial if there are no restrictions?

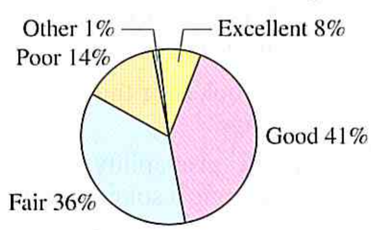
42. Area Code An area code consists of three digits. How many area codes are possible if (a) there are no restrictions and (b) the first digit cannot be a 1 or a 0? (c) What is the probability of selecting an area code at random that ends in an odd number if the first digit cannot be a 1 or a 0?

43. Repairs In how many orders can three broken computers and two broken printers be repaired if (a) there are no restrictions, (b) the printers must be repaired first, and (c) the computers must be repaired first? (d) If the order of repairs has no restrictions and the order of repairs is done at random, what is the probability that a printer will be repaired first?

44. Defective Units A shipment of 10 microwave ovens contains two defective units. In how many ways can a restaurant buy three of these units and receive (a) no defective units, (b) one defective unit, and (c) at least two nondefective units? (d) What is the probability of the restaurant buying at least two defective units?

Financial Shape *In Exercises 45–48, use the pie chart, which shows how U.S. adults rate their financial shape. (Source: Pew Research Center)*

Rate Your Financial Shape



45. Suppose 4 people are chosen at random from a group of 1200. What is the probability that all four would rate their financial shape as excellent? (Make the assumption that the 1200 people are represented by the pie chart.)

46. Suppose 10 people are chosen at random from a group of 1200. What is the probability that all 10 would rate their financial shape as poor? (Make the assumption that the 1200 people are represented by the pie chart.)

47. Suppose 80 people are chosen at random from a group of 500. What is the probability that none of the 80 people would rate their financial shape as fair? (Make the assumption that the 500 people are represented by the pie chart.)

48. Suppose 55 people are chosen at random from a group of 500. What is the probability that none of the 55 people would rate their financial shape as good? (Make the assumption that the 500 people are represented by the pie chart.)

49. Probability In a state lottery, you must select 5 numbers (in any order) out of 40 correctly to win the top prize.

(a) How many ways can 5 numbers be chosen from 40 numbers?

(b) You purchase one lottery ticket. What is the probability of winning the top prize?

50. Probability A company that has 200 employees chooses a committee of 15 to represent employee retirement issues. When the committee was formed, none of the 56 minority employees were selected.

(a) Use a technology tool to find the number of ways 15 employees can be chosen from 200.

(b) Use a technology tool to find the number of ways 15 employees can be chosen from 144 nonminorities.

(c) If the committee was chosen randomly (without bias), what is the probability that it contained no minorities?

(d) Does your answer to part (c) indicate that the committee selection was biased? Explain your reasoning.

51. Cards You are dealt a hand of five cards from a standard deck of playing cards. Find the probability of being dealt a hand consisting of

(a) four-of-a-kind.

(b) a full house, which consists of 1 three-of-a-kind and 1 two-of-a-kind.

(c) three-of-a-kind. (The other two cards are different from each other.)

(d) two clubs and one of each other three suits.

52. Warehouse A warehouse employs 24 workers on first shift and 17 workers on second shift. Eight workers are chosen at random to be interviewed about the work environment. Find the probability of choosing

(a) all first-shift workers.

(b) all second-shift workers.

(c) six first-shift workers.

(d) four second-shift workers.

Extending Concepts

NBA Draft Lottery *In Exercises 53–58, use the following information. The National Basketball Association (NBA) uses a lottery to determine which team gets the first pick in its annual draft. The teams eligible for the lottery are the 14 non-playoff teams. Fourteen Ping-Pong balls numbered 1 through 14 are placed in a drum. Each of the 14 teams is assigned several of the possible four-number combinations that correspond to the numbers on the Ping-Pong balls, such as 3, 8, 10, and 12. Four balls are then drawn out to determine the first pick in the draft. The order in which the balls are drawn is not important. All of the four-number combinations are assigned to the 14 teams by computer except for one four-number combination. When this four-number combination is drawn, the balls are put back in the drum and another drawing takes place. For instance, if Team A has been assigned the four-number combination 3, 8, 10, 12 and the balls shown at the left are drawn, then Team A wins the first pick.*

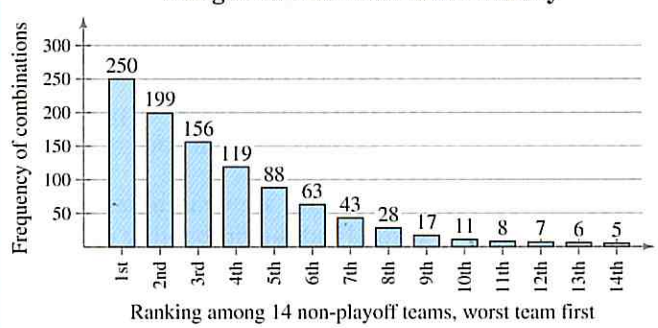
After the first pick of the draft is determined, the process continues to choose the teams that will select second and third picks. The remaining order of the draft is determined by the number of losses of each team.

53. In how many ways can 4 of the numbers 1 to 14 be selected if order is *not* important? How many sets of 4 numbers are assigned to the 14 teams?

54. In how many ways can four of the numbers be selected if order is important?

In the Pareto chart, the number of combinations assigned to each of the 14 teams is shown. The team with the most losses (the worst team) gets the most chances to win the lottery. So, the worst team receives the greatest frequency of four-number combinations, 250. The team with the best record of the 14 non-playoff teams has the fewest chances, with 5 four-number combinations.

Frequency of Four-Number Combinations Assigned in the NBA Draft Lottery



55. For each team, find the probability that the team will win the first pick.

56. What is the probability that the team with the worst record will win the second pick, given that the team with the best record, ranked 14th, wins the first pick?

57. What is the probability that the team with the worst record will win the third pick, given that the team with the best record, ranked 14th, wins the first pick and the team ranked 2nd wins the second pick?

58. What is the probability that neither the first- nor the second-worst team will get the first pick?

Uses & Abuses

Uses

Probability affects decisions when the weather is forecast, when marketing strategies are determined, when medications are selected, and even when players are selected for professional sports teams. Although intuition is often used for determining probabilities, you will be better able to assess the likelihood that an event will occur by applying the rules of classical probability and empirical probability.

For instance, suppose you work for a real estate company and are asked to estimate the likelihood that a particular house will sell for a particular price within the next 90 days. You could use your intuition, but you could better assess the probability by looking at sales records for similar houses.

Abuses

One common abuse of probability is thinking that probabilities have "memories." For example, if a coin is tossed eight times, the probability that it will land heads up all eight times is only about 0.004. However, if the coin has already been tossed seven times and has landed heads up each time, the probability that it will land heads up on the eighth time is 0.5. Each toss is independent of all other tosses. The coin does not "remember" that it has already landed heads up seven times.

Ethics

A human resources director for a company with 100 employees wants to show that her company is an equal opportunity employer of women and minorities. There are 40 women employees and 20 minority employees in the company. Nine of the women employees are minorities. Despite this fact, the director reports that 60% of the company is either a woman or a minority. If one employee is selected at random, the probability that the employee is a woman is 0.4 and the probability that the employee is a minority is 0.2. This does not mean, however, that the probability that a randomly selected employee is a woman or a minority is 0.4 + 0.2 or 0.6, because nine employees belong to both groups. In this case, it would be ethically incorrect to omit this information from her report because these individuals would have been counted twice.

EXERCISES

1. ***Assuming Probability Has a "Memory"*** A "Daily Number" lottery has a three-digit number from 000 to 999. You buy one ticket each day. Your number is 389.

- **a.** What is the probability of winning next Tuesday and Wednesday?
- **b.** You won on Tuesday. What's the probability of winning on Wednesday?
- **c.** You didn't win on Tuesday. What's the probability of winning on Wednesday?

2. ***Adding Probabilities Incorrectly*** A town has a population of 500 people. Suppose that the probability that a randomly chosen person owns a pickup is 0.25 and the probability that a randomly chosen person owns an SUV is 0.30. What can you say about the probability that a randomly chosen person owns a pickup or an SUV? Could this probability be 0.55? Could it be 0.60? Explain your reasoning.

3 CHAPTER SUMMARY

What did you learn?	EXAMPLE(S)	REVIEW EXERCISES
Section 3.1		
How to identify the sample space of a probability experiment and to identify simple events	1, 2	1–4
How to use the Fundamental Counting Principle to find the number of ways two or more events can occur	3, 4	5, 6
How to distinguish among classical probability, empirical probability, and subjective probability	5–8	7–12
How to find the probability of the complement of an event and how to find other probabilities using tree diagrams and the Fundamental Counting Principle	9–11	13–16
Section 3.2		
How to find conditional probabilities	1	17, 18
How to distinguish between independent and dependent events	2	19, 20
How to use the Multiplication Rule to find the probability of two events occurring in sequence $P(A \text{ and } B) = P(A) \cdot P(B \mid A)$ if events are dependent $P(A \text{ and } B) = P(A) \cdot P(B)$ if events are independent	3–5	21, 22
Section 3.3		
How to determine if two events are mutually exclusive	1	23, 24
How to use the Addition Rule to find the probability of two events $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ $P(A \text{ or } B) = P(A) + P(B)$ if events are mutually exclusive	2–5	25–34
Section 3.4		
How to find the number of ways a group of objects can be arranged in order and the number of ways to choose several objects from a group without regard to order $_nP_r = \frac{n!}{(n-r)!}$, $\frac{n!}{n_1! \cdot n_2! \cdot n_3! \cdots n_k!}$, $_nC_r = \frac{n!}{(n-r)!r!}$	1–5	35–38
How to use counting principles to find probabilities	6–9	39–42

3 REVIEW EXERCISES

Section 3.1

In Exercises 1–4, identify the sample space of the probability experiment and determine the number of outcomes in the event. Draw a tree diagram if it is appropriate.

1. *Experiment:* Tossing four coins
   Event: Getting three heads
2. *Experiment:* Rolling 2 six-sided dice
   Event: Getting a sum of 4 or 5
3. *Experiment:* Choosing a month of the year
   Event: Choosing a month that begins with the letter J
4. *Experiment:* Guessing the gender of the three children in a family
   Event: The family has two boys

In Exercises 5 and 6, use the Fundamental Counting Principle.

5. A student must choose between 7 classes to take at 8:00 A.M., 4 classes to take at 9:00 A.M., and 3 classes to take at 10:00 A.M. How many ways can the student arrange the schedule?
6. The state of Virginia's license plate has three letters followed by four digits. Assuming that any letter or digit can be used, how many different license plates are possible?

In Exercises 7–12, classify the statement as an example of classical probability, empirical probability, or subjective probability.

7. On the basis of prior counts, a quality control officer says there is a 0.05 probability that a randomly chosen part is defective.
8. The probability of randomly selecting five cards of the same suit (a flush) from a standard deck is about 0.0005.
9. The chance that Corporation A's stock price will fall today is 75%.
10. The probability of a person from the United States being left-handed is 11%.
11. The probability of rolling 2 six-sided dice and getting a sum greater than nine is $\frac{1}{6}$.
12. The chance that a randomly selected person in the United States is between 15 and 24 years old is about 14%. *(Source: U.S. Census Bureau)*

In Exercises 13 and 14, the table shows the approximate distribution of the sizes of firms for 2004. Use the table to determine the probability of the event. (Source: U.S. Small Business Administration)

Number of Employees	0 to 4	5 to 9	10 to 19	20 to 99	100 or more
Percent of Firms	60.8%	17.7%	10.8%	8.9%	1.8%

13. What is the probability that a randomly selected firm will have at least 10 employees?
14. What is the probability that a randomly selected firm will have fewer than 20 employees?

Telephone Numbers *The telephone numbers for a region of a state have an area code of 570. The next seven digits represent the local telephone numbers for that region. A local telephone number cannot begin with a 0 or 1. Your cousin lives within the given area code.*

15. What is the probability of randomly generating your cousin's telephone number?

16. What is the probability of not randomly generating your cousin's telephone number?

Section 3.2

In Exercises 17 and 18, the list shows the results of a study on the use of plus/minus grading at North Carolina State University. It shows the percents of graduate and undergraduate students who received grades with pluses and minuses (for example, C+, A−, etc.). (Source: North Carolina State University)

- *Of all students who received one or more plus grades, 92% were undergraduates and 8% were graduates.*
- *Of all students who received one or more minus grades, 93% were undergraduates and 7% were graduates.*

17. Find the probability that a student is an undergraduate student, given that the student received a plus grade.

18. Find the probability that a student is a graduate student, given that the student received a minus grade.

In Exercises 19 and 20, decide whether the events are independent or dependent.

19. Tossing a coin four times, getting four heads, and tossing it a fifth time and getting a head

20. Taking a driver's education course and passing the driver's license exam

In Exercises 21 and 22, find the probability of the sequence of events.

21. You are shopping, and your roommate has asked you to pick up toothpaste and dental rinse. However, your roommate did not tell you which brands to get. The store has eight brands of toothpaste and five brands of dental rinse. What is the probability that you will purchase the correct brands of both products?

22. Your sock drawer has 18 folded pairs of socks, with 8 pairs of white, 6 pairs of black, and 4 pairs of blue. What is the probability, without looking in the drawer, that you will first select and remove a black pair, then select either a blue or a white pair?

Section 3.3

In Exercises 23 and 24, decide if the events are mutually exclusive.

23. Event A: Randomly select a red jelly bean from a jar.

Event B: Randomly select a yellow jelly bean from the same jar.

24. Event A: Randomly select a person who loves cats.

Event B: Randomly select a person who owns a dog.

25. A random sample of 250 working adults found that 37% access the Internet at work, 44% access the Internet at home, and 21% access the Internet at both work and home. What is the probability that a person in this sample selected at random accesses the Internet at home or at work?

26. A sample of automobile dealerships found that 19% of automobiles sold are silver, 22% of automobiles sold are sports utility vehicles (SUVs), and 16% of automobiles sold are silver SUVs. What is the probability that a randomly chosen sold automobile from this sample is silver or an SUV?

In Exercises 27–30, determine the probability.

27. A card is randomly selected from a standard deck. Find the probability that the card is between 4 and 8 (inclusive) or is a club.

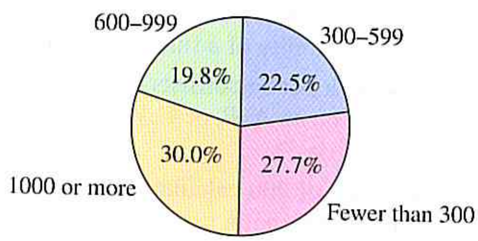
28. A card is randomly selected from a standard deck. Find the probability that the card is red or a queen.

29. A 12-sided die, numbered 1–12, is rolled. Find the probability that the roll results in an odd number or a number less than 4.

30. An eight-sided die, numbered 1–8, is rolled. Find the probability that the roll results in an even number or a number greater than 6.

In Exercises 31 and 32, use the pie chart, which shows the percent distribution of the number of students in traditional U.S. secondary schools. (Adapted from U.S. National Center for Education Statistics)

Students in Secondary Schools

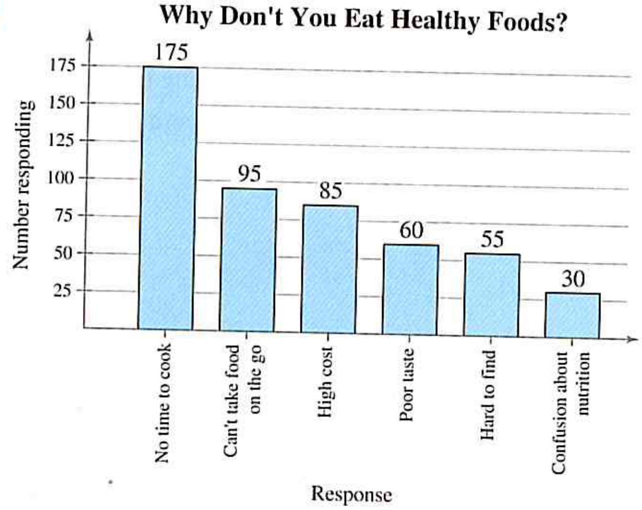


31. Find the probability of randomly selecting a school with 600 or more students.

32. Find the probability of randomly selecting a school with between 300 and 999 students, inclusive.

In Exercises 33 and 34, use the Pareto chart, which shows the results of a survey in which 500 adults were asked why they don't always eat healthy foods.

Why Don't You Eat Healthy Foods?



33. Find the probability of randomly selecting an adult from the sample who feels that healthy foods have poor taste or are hard to find.

34. Find the probability of randomly selecting an adult from the sample who doesn't always eat healthy foods because he or she has no time to cook or is confused about nutrition.

Section 3.4

In Exercises 35–38, use combinations and permutations.

35. Fifteen cyclists enter a race. In how many ways can they finish first, second, and third?

36. Five players on a basketball team must choose a player on the opposing team to defend. In how many ways can they choose their defensive assignments?

37. A literary magazine editor must choose 4 short stories for this month's issue from 17 submissions. In how many ways can the editor choose this month's stories?

38. An employer must hire 2 people from a list of 13 applicants. In how many ways can the employer choose to hire 2 people?

In Exercises 39–42, use counting principles to find the probability.

39. In poker, a full house consists of a three-of-a-kind and a two-of-a-kind. Find the probability of a full house consisting of three kings and two queens.

40. A security code consists of three letters followed by one digit. The first letter cannot be an A, B, or C. What is the probability of guessing the security code in one trial?

41. A batch of 200 calculators contains three defective calculators. What is the probability that a sample of three calculators will have

(a) no defective calculators?

(b) all defective calculators?

(c) at least one defective calculator?

(d) at least one nondefective calculator?

42. A batch of 350 raffle tickets contains four winning tickets. You buy four tickets. What is the probability that you have

(a) no winning tickets?

(b) all of the winning ticket?

(c) at least one winning ticket?

(d) at least one nonwinning ticket?

3 CHAPTER QUIZ

Take this quiz as you would take a quiz in class. After you are done, check your work against the answers given in the back of the book.

1. The table shows the number (in thousands) of earned degrees conferred in the United States in the year 2004 by level and gender. *(Source: National Center for Education Statistics)*

		Gender		
		Male	**Female**	**Total**
Level of Degree	**Associate**	260	405	665
	Bachelor's	595	804	1399
	Master's	230	329	559
	Doctorate	25	23	48
	Total	1110	1561	2671

A person who earned a degree in the year 2004 is randomly selected. Find the probability of selecting someone who

(a) earned a bachelor's degree.
(b) earned a bachelor's degree given that the person is a female.
(c) earned a bachelor's degree given that the person is not a female.
(d) earned an associate degree or a bachelor's degree.
(e) earned a doctorate given that the person is a male.
(f) earned a master's degree or is a female.
(g) earned an associate degree and is a male.
(h) is a female given that the person earned a bachelor's degree.

2. Decide if the events are mutually exclusive. Then decide if the events are independent or dependent. Explain your reasoning.

Event A: A golfer scoring the best round in a four-round tournament
Event B: Losing the golf tournament

3. A shipment of 150 television sets contains 3 defective units. Determine how many ways a vending company can buy three of these units and receive (a) no defective units, (b) all defective units, and (c) at least one good unit.

4. In Exercise 3, find the probability of the vending company receiving (a) no defective units, (b) all defective units, and (c) at least one good unit.

5. The access code for a warehouse's security system consists of six digits. The first digit cannot be 0 and the last digit must be even. How many different codes are available?

6. From a pool of 30 candidates, the offices of president, vice president, secretary, and treasurer will be filled. In how many different ways can the offices be filled?

Putting It All Together

REAL Statistics — Real Decisions

You work for the company that runs the Powerball lottery. Powerball is a lottery game in which five white balls are chosen from a drum containing 55 balls and one red ball is chosen from a drum containing 42 balls. To win the jackpot, a player must match all five white balls and the red ball. Other winners and their prizes are also shown in the table.

Working in the public relations department, you handle many inquiries from the media and from lottery players. You receive the following e-mail.

> *You list the probability of matching only the red ball as* 1/69. *I know from my statistics class that the probability of winning is the ratio of the number of successful outcomes to the total number of outcomes. Could you please explain why the probability of matching only the red ball is* 1/69?

Your job is to answer this question, using the probability techniques you have learned in this chapter to justify your answer. In answering the question, assume only one ticket is purchased.

POWERBALL

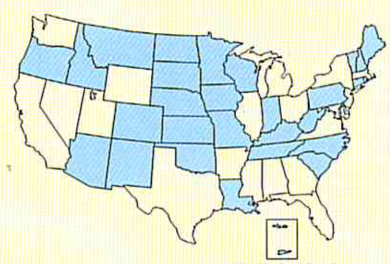
www.musl.com

Powerball Winners and Prizes

Match	Prize	Approximate Probability
5 white, 1 red	Jackpot	1/146,107,962
5 white	$100,000	1/3,563,609
4 white, 1 red	$5,000	1/584,432
4 white	$100	1/14,254
3 white, 1 red	$100	1/11,927
3 white	$7	1/291
2 white, 1 red	$7	1/745
1 white	$4	1/127
1 red	$3	1/69

(Source: Multi-State Lottery Association)

Where Is Powerball Played?
Powerball is played in 29 states, Washington, D.C., and the U.S. Virgin Islands



(Source: Multi-State Lottery Association)

Exercises

1. *How Would You Do It?*

(a) How would you investigate the question about the probability of matching only the red ball?

(b) What statistical methods taught in this chapter would you use?

2. *Answering the Question*

Write an explanation that answers the question about the probability of matching only the red ball. Include in your explanation any probability formulas that justify your explanation.

3. *Another Question*

You receive another question asking how the overall probability of winning a prize in the Powerball lottery is determined. The overall probability of winning a prize in the Powerball lottery is 1/37. Write an explanation that answers the question and include any probability formulas that justify your explanation.