3.3 The Addition Rule

What You SHOULD LEARN

- How to determine if two events are mutually exclusive
- How to use the Addition Rule to find the probability of two events

Mutually Exclusive Events ▸ The Addition Rule ▸ A Summary of Probability

▸ Mutually Exclusive Events

In Section 3.2, you learned how to find the probability of two events, A and B, occurring in sequence. Such probabilities are denoted by $P(A \text{ and } B)$. In this section, you will learn how to find the probability that at least one of two events will occur. Probabilities such as these are denoted by $P(A \text{ or } B)$ and depend on whether the events are mutually exclusive.

DEFINITION

Two events A and B are **mutually exclusive** if A and B cannot occur at the same time.

Study Tip

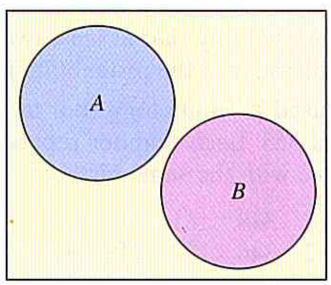
In probability and statistics, the word *or* is usually used as an "inclusive or" rather than an "exclusive or." For instance, there are three ways for "Event *A* or *B*" to occur.

(1) *A* occurs and *B* does not occur.

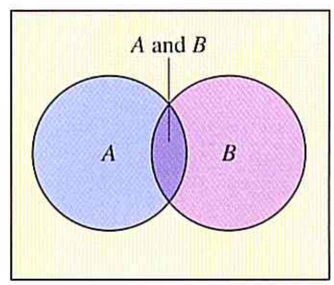
(2) *B* occurs and *A* does not occur.

(3) *A* and *B* both occur.

The Venn diagrams show the relationship between events that are mutually exclusive and events that are not mutually exclusive.



A and *B* are mutually exclusive.



A and *B* are not mutually exclusive.

EXAMPLE 1

Mutually Exclusive Events

Decide if the events are mutually exclusive. Explain your reasoning.

1. Event A: Roll a 3 on a die.
   Event B: Roll a 4 on a die.
2. Event A: Randomly select a male student.
   Event B: Randomly select a nursing major.
3. Event A: Randomly select a blood donor with type O blood.
   Event B: Randomly select a female blood donor.

Solution

1. The first event has one outcome, a 3. The second event also has one outcome, a 4. These outcomes cannot occur at the same time, so the events are mutually exclusive.
2. Because the student can be a male nursing major, the events are not mutually exclusive.
3. Because the donor can be a female with type O blood, the events are not mutually exclusive.

Try It Yourself 1

Decide if the events are mutually exclusive.

1. Event A: Randomly select a jack from a standard deck of cards.
   Event B: Randomly select a face card from a standard deck of cards.
2. Event A: Randomly select a 20-year-old student.
   Event B: Randomly select a student with blue eyes.
3. Event A: Randomly select a vehicle that is a Ford.
   Event B: Randomly select a vehicle that is a Toyota.

a. Decide if one of the following statements is true.

- Events A and B cannot occur at the same time.
- Events A and B have no outcomes in common.
- $P(A \text{ and } B) = 0$

b. Make a conclusion.

Answer: Page A38

Study Tip

By subtracting $P(A \text{ and } B)$ you avoid double counting the probability of outcomes that occur in both A and B

To explore this topic further, see Activity 3.3 on page 170.

The Addition Rule

THE ADDITION RULE FOR THE PROBABILITY OF A OR B

The probability that events A or B will occur, $P(A \text{ or } B)$, is given by

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B).$$

If events A and B are mutually exclusive, then the rule can be simplified to $P(A \text{ or } B) = P(A) + P(B)$. This simplified rule can be extended to any number of mutually exclusive events.

In words, to find the probability one event or the other will occur, add the individual probabilities of each event and subtract the probability they both occur.

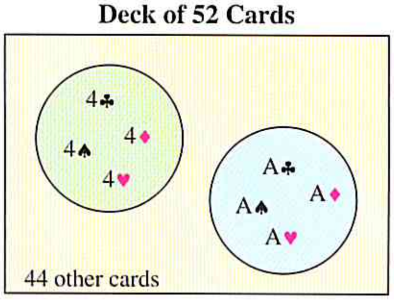
EXAMPLE 2

Using the Addition Rule to Find Probabilities

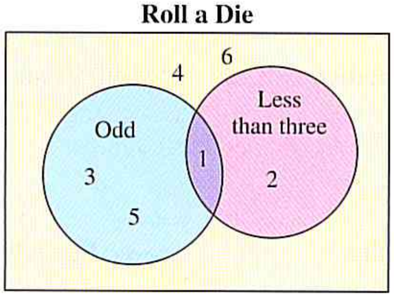
1. You select a card from a standard deck. Find the probability that the card is a 4 or an ace.
2. You roll a die. Find the probability of rolling a number less than three or rolling an odd number.

Solution

Deck of 52 Cards



Roll a Die



1. If the card is a 4, it cannot be an ace. So, the events are mutually exclusive as shown in the Venn diagram. The probability of selecting a 4 or an ace is

$$P(4 \text{ or ace}) = P(4) + P(\text{ace}) = \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{2}{13} \approx 0.154.$$

2. The events are not mutually exclusive because 1 is an outcome of both events as shown in the Venn diagram. So, the probability of rolling a number less than 3 or an odd number is

$$\begin{aligned} P(\text{less than 3 or odd}) &= P(\text{less than 3}) + P(\text{odd}) \\ &\quad - P(\text{less than 3 and odd}) \\ &= \frac{2}{6} + \frac{3}{6} - \frac{1}{6} = \frac{4}{6} = \frac{2}{3} \approx 0.667. \end{aligned}$$

▸ Try It Yourself 2

1. A die is rolled. Find the probability of rolling a 6 or an odd number.
2. A card is selected from a standard deck. Find the probability that the card is a face card or a heart.

a. Decide whether the events are *mutually exclusive.*
b. Find $P(A)$, $P(B)$, and, if necessary, $P(A \text{ and } B)$.
c. Use the *Addition Rule* to find the probability.

Answer: Page A38

EXAMPLE 3

Finding Probabilities of Mutually Exclusive Events

The frequency distribution shows the volume of sales (in dollars) and the number of months a sales representative reached each sales level during the past three years. If this sales pattern continues, what is the probability that the sales representative will sell between $75,000 and $124,999 next month?

Sales volume ($)	Months
0–24,999	3
25,000–49,999	5
50,000–74,999	6
75,000–99,999	7
100,000–124,999	9
125,000–149,999	2
150,000–174,999	3
175,000–199,999	1

Solution To solve this problem, define events A and B as follows.

A = monthly sales between $75,000 and $99,999

B = monthly sales between $100,000 and $124,999

Because events A and B are mutually exclusive, the probability that the sales representative will sell between $75,000 and $124,999 next month is

$$\begin{aligned} P(A \text{ or } B) &= P(A) + P(B) \\ &= \frac{7}{36} + \frac{9}{36} \\ &= \frac{16}{36} \\ &= \frac{4}{9} \approx 0.444. \end{aligned}$$

Try It Yourself 3

Find the probability that the sales representative will sell between $0 and $49,999.

a. *Identify* events A and B.
b. Verify that A and B are *mutually exclusive.*
c. Find the *probability* of each event.
d. Use the *Addition Rule* to find the probability.

Answer: Page A38

EXAMPLE 4

Using the Addition Rule to Find Probabilities

A blood bank catalogs the types of blood, including positive or negative Rh-factor, given by donors during the last five days. The number of donors who gave each blood type is shown in the table. A donor is selected at random.

1. Find the probability that the donor has type O or type A blood.
2. Find the probability that the donor has type B blood or is Rh-negative.

		Blood Type				
		O	**A**	**B**	**AB**	**Total**
RH-factor	**Positive**	156	139	37	12	344
	Negative	28	25	8	4	65
	Total	184	164	45	16	409

Solution

1. Because a donor cannot have type O blood and type A blood, these events are mutually exclusive. So, on the basis of the Addition Rule, the probability that a randomly chosen donor has type O or type A blood is

$$\begin{aligned} P(\text{type O or type A}) &= P(\text{type O}) + P(\text{type A}) \\ &= \frac{184}{409} + \frac{164}{409} \\ &= \frac{348}{409} \\ &\approx 0.851. \end{aligned}$$

2. Because a donor can have type B blood and be Rh-negative, these events are not mutually exclusive. So, on the basis of the Addition Rule, the probability that a randomly chosen donor has type B blood or is Rh-negative is

$$\begin{aligned} P(\text{type B or Rh-neg}) &= P(\text{type B}) + P(\text{Rh-neg}) - P(\text{type B and Rh-neg}) \\ &= \frac{45}{409} + \frac{65}{409} - \frac{8}{409} \\ &= \frac{102}{409} \\ &\approx 0.249. \end{aligned}$$

▸Try It Yourself 4

1. Find the probability that the donor has type B or type AB blood.
2. Find the probability that the donor has type O blood or is Rh-positive.

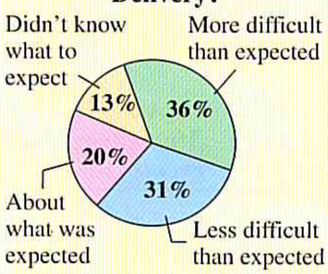
a. Decide if the events are *mutually exclusive.*
b. Use the *Addition Rule.*

Answer: Page A38

PICTURING the WORLD

In a survey conducted by the National Family Organization, new mothers were asked to rate the difficulty of delivering their first child compared with what they expected. (Source: National Family Organization Research for CNS)

How Difficult Was the Delivery?



If you selected a new mother at random and asked her to compare the difficulty of her delivery with what she expected, what is the probability that she would say that it was the same or more difficult than what she expected?

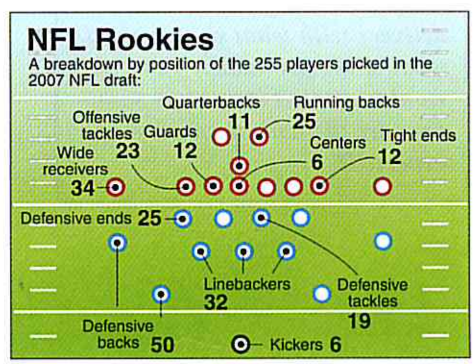
A Summary of Probability

Type of Probability and Probability Rules	In Words	In Symbols
Classical Probability	The number of outcomes in the sample space is known and each outcome is equally likely to occur.	$P(E) = \dfrac{\text{Number of outcomes in event } E}{\text{Number of outcomes in sample space}}$
Empirical Probability	The frequency of outcomes in the sample space is estimated from experimentation.	$P(E) = \dfrac{\text{Frequency of event } E}{\text{Total frequency}} = \dfrac{f}{n}$
Range of Probabilities Rule	The probability of an event is between 0 and 1, inclusive.	$0 \le P(E) \le 1$
Complementary Events	The complement of event E is the set of all outcomes in a sample space that are not included in E, denoted by E'.	$P(E') = 1 - P(E)$
Multiplication Rule	The Multiplication Rule is used to find the probability of two events occurring in a sequence.	$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$ $P(A \text{ and } B) = P(A) \cdot P(B)$ *Independent events*
Addition Rule	The Addition Rule is used to find the probability of at least one of two events occurring.	$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ $P(A \text{ or } B) = P(A) + P(B)$ *Mutually exclusive events*

EXAMPLE 5

Combining Rules to Find Probabilities

Use the graph at the right to find the probability that a randomly selected draft pick is not a running back or a wide receiver.



(Source: NFL.com)

Solution Define events A and B.

A: Draft pick is a running back.
B: Draft pick is a wide receiver.

These events are mutually exclusive, so the probability that the draft pick is a running back or wide receiver is

$$P(A \text{ or } B) = P(A) + P(B) = \tfrac{25}{255} + \tfrac{34}{255} = \tfrac{59}{255} \approx 0.231.$$

By taking the complement of $P(A \text{ or } B)$, you can determine that the probability of randomly selecting a draft pick who is not a running back or wide receiver is

$$1 - P(A \text{ or } B) = 1 - \tfrac{59}{255} = \tfrac{196}{255} \approx 0.769.$$

Try It Yourself 5

Find the probability that a randomly selected draft pick is not a linebacker or a quarterback.

a. Find the *probability* that the draft pick is a linebacker or a quarterback.
b. Find the *complement* of the event.

Answer: Page A38

3.3 EXERCISES

Building Basic Skills and Vocabulary

1. If two events are mutually exclusive, why is $P(A \text{ and } B) = 0$?

2. List examples of

(a) two events that are mutually exclusive.

(b) two events that are not mutually exclusive.

True or False? *In Exercises 3–6, determine whether the statement is true or false. If it is false, explain why.*

3. If two events are mutually exclusive, they have no outcomes in common.

4. If two events are independent, then they are also mutually exclusive.

5. The probability that event A or event B will occur is

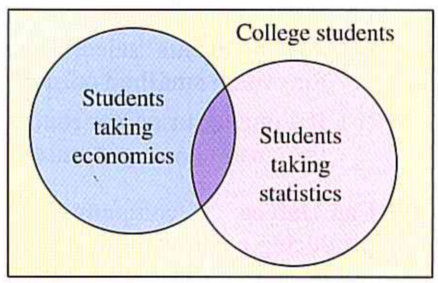
$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ or } B).$$

6. If events A and B are mutually exclusive, then

$$P(A \text{ or } B) = P(A) + P(B).$$

Graphical Analysis *In Exercises 7 and 8, decide if the events shown in the Venn diagram are mutually exclusive. Explain your reasoning.*

7.



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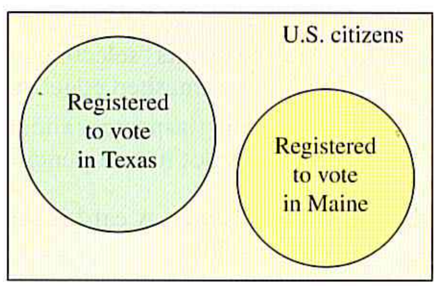


Recognizing Mutually Exclusive Events *In Exercises 9–12, decide if the events are mutually exclusive. Explain your reasoning.*

9. Event A: Randomly select a female worker.
Event B: Randomly select a worker with a college degree.

10. Event A: Randomly select a member of U.S. Congress.
Event B: Randomly select a male U.S. Senator.

11. Event A: Randomly select a person between 18 and 24 years old.
Event B: Randomly select a person between 25 and 34 years old.

12. Event A: Randomly select a person between 18 and 24 years old.
Event B: Randomly select a person earning between \$20,000 and \$29,999.

Using and Interpreting Concepts

13. Audit During a 52-week period, a company paid overtime wages for 18 weeks and hired temporary help for 9 weeks. During 5 weeks, the company paid overtime *and* hired temporary help.

(a) Are the events "selecting a week that contained overtime wages" and "selecting a week that contained temporary help wages" mutually exclusive? Explain.

(b) If an auditor randomly examined the payroll records for only one week, what is the probability that the payroll for that week contained overtime wages or temporary help wages?

14. Newspaper Survey A college has an undergraduate enrollment of 3500. Of these, 860 are business majors and 1800 are women. Of the business majors, 425 are women.

(a) Are the events "selecting a woman student" and "selecting a business major" mutually exclusive? Explain.

(b) If a college newspaper conducts a poll and selects students at random to answer a survey, find the probability that a selected student is a woman or a business major.

15. Carton Defects A company that makes cartons finds the probability of producing a carton with a puncture is 0.05, the probability that a carton has a smashed corner is 0.08, and the probability that a carton has a puncture and has a smashed corner is 0.004.

(a) Are the events "selecting a carton with a puncture" and "selecting a carton with a smashed corner" mutually exclusive? Explain.

(b) If a quality inspector randomly selects a carton, find the probability that the carton has a puncture or has a smashed corner.

16. Can Defects A company that makes soda pop cans finds the probability of producing a can without a puncture is 0.96, the probability that a can does not have a smashed edge is 0.93, and the probability that a can does not have a puncture and does not have a smashed edge is 0.893.

(a) Are the events "selecting a can without a puncture" and "selecting a can without a smashed edge" mutually exclusive? Explain.

(b) If a quality inspector randomly selects a can, find the probability that the can does not have a puncture or does not have a smashed edge.

17. Selecting a Card A card is selected at random from a standard deck. Find each probability.

(a) Randomly selecting a diamond or a 7

(b) Randomly selecting a red suit or a queen

(c) Randomly selecting a 3 or a face card

18. Rolling a Die You roll a die. Find each probability.

(a) Rolling a 6 or a number greater than 4

(b) Rolling a number less than 5 or an odd number

(c) Rolling a 3 or an even number

19. U.S. Age Distribution The estimated percent distribution of the U.S. population for 2015 is shown in the pie chart. Find each probability. *(Source: U.S. Census Bureau)*

(a) Randomly selecting someone under five years old

(b) Randomly selecting someone who is not 65 years or over

(c) Randomly selecting someone who is between 18 and 34 years old

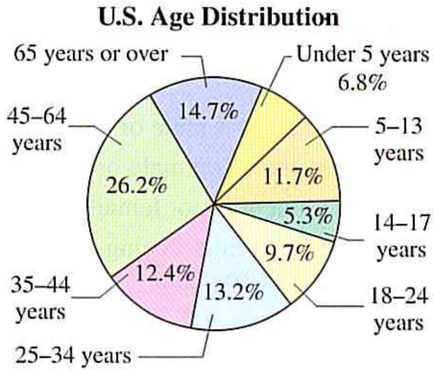


FIGURE FOR EXERCISE 19

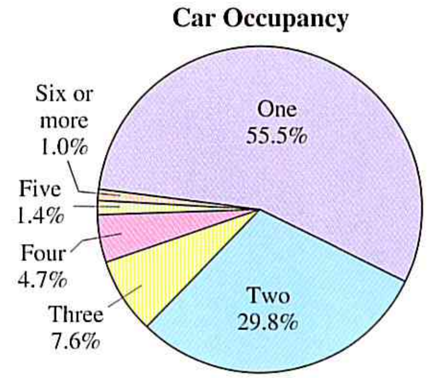


FIGURE FOR EXERCISE 20

20. Tacoma Narrows Bridge The percent distribution of the number of occupants in vehicles crossing the Tacoma Narrows Bridge in Washington is shown in the pie chart. Find each probability. *(Source: Washington State Department of Transportation)*

(a) Randomly selecting a car with two occupants

(b) Randomly selecting a car with two or more occupants

(c) Randomly selecting a car with between two and five occupants, inclusive

21. Education The number of responses to a survey are shown in the Pareto chart. The survey asked 1017 U.S. adults how they feel about the quality of education students receive in kindergarten through grade twelve. Each person gave one response. Find each probability. *(Adapted from Gallup Poll)*

(a) Randomly selecting a person from the sample who is not completely satisfied with the quality of education

(b) Randomly selecting a person from the sample who is somewhat dissatisfied or completely dissatisfied with the quality of education

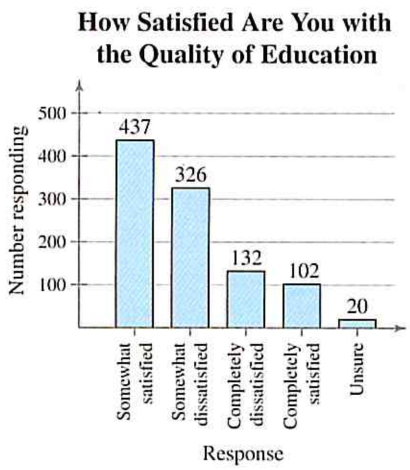


FIGURE FOR EXERCISE 21

22. Movies The number of responses to a survey are shown in the Pareto chart. The survey asked 1005 U.S. adults what they feel is the biggest problem with movies. Each person gave one response. Find each probability. *(Source: Associated Press)*

(a) Randomly selecting a person from the sample who feels the biggest problem with movies is that movies are not as good as they used to be

(b) Randomly selecting a person from the sample who feels the biggest problem with movies is not that movies have too much violence or that the tickets cost too much

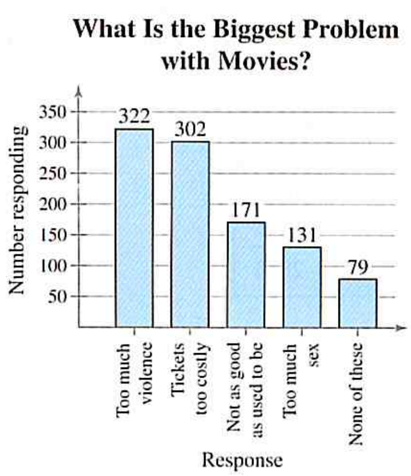


FIGURE FOR EXERCISE 22

23. Nursing Majors The table shows the number of male and female students enrolled in nursing at the University of Oklahoma Health Sciences Center for a recent semester. A student is selected at random. Find the probability of each event. *(Adapted from University of Oklahoma Health Center Office of Admissions and Records)*

	Nursing majors	Non-nursing majors	Total
Males	95	1015	1110
Females	700	1727	2427
Total	795	2742	3537

(a) The student is male or a nursing major.

(b) The student is female or not a nursing major.

(c) The student is not female or a nursing major.

(d) Are the events "being male" and "being a nursing major" mutually exclusive? Explain.

24. Left-Handed People In a sample of 1000 people (525 men and 475 women), 113 are left-handed (63 men and 50 women). The results of the sample are shown in the table. A person is selected at random from the sample. Find the probability of each event.

		Gender		
		Men	**Women**	**Total**
Dominant Hand	**Left**	63	50	113
	Right	462	425	887
	Total	525	475	1000

(a) The person is left-handed or a woman.

(b) The person is right-handed or a man.

(c) The person is not right-handed or a man.

(d) The person is a right-handed woman.

(e) Are the events "being right-handed" and "being a woman" mutually exclusive? Explain.

25. Charity The table shows the results of a survey that asked 2850 people whether they are involved in any type of charity work. A person is selected at random from the sample. Find the probability of each event.

	Frequently	Occasionally	Not at all	Total
Male	221	456	795	1472
Female	207	430	741	1378
Total	428	886	1536	2850

(a) The person is frequently or occasionally involved in charity work.

(b) The person is female or not involved in charity work at all.

(c) The person is male or frequently involved in charity work.

(d) The person is female or not frequently involved in charity work.

(e) Are the events "being female" and "being frequently involved in charity work" mutually exclusive? Explain.

26. Eye Survey The table shows the results of a survey that asked 3203 people whether they wear contacts or glasses. A person is selected at random from the sample. Find the probability of each event.

	Contacts	Glasses	Both	Neither	Total
Male	64	841	177	456	1538
Female	189	427	368	681	1665
Total	253	1268	545	1137	3203

(a) The person wears contacts or glasses.

(b) The person is male or wears both contacts and glasses.

(c) The person is female or wears neither contacts or glasses.

(d) The person is male or does not wear glasses.

(e) Are the events "wearing contacts" and "wearing both contacts and glasses" mutually exclusive? Explain.

Extending Concepts

27. Writing Is there a relationship between independence and mutual exclusivity? To decide, find examples of the following, if possible.

(a) Describe two events that are dependent and mutually exclusive.

(b) Describe two events that are independent and mutually exclusive.

(c) Describe two events that are dependent and not mutually exclusive.

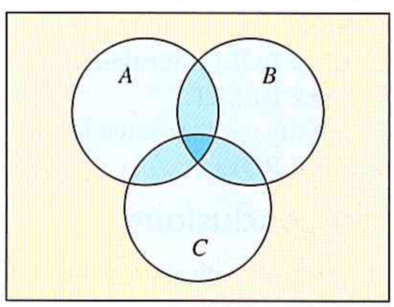
(d) Describe two events that are independent and not mutually exclusive.

Use your results to write a conclusion about the relationship between independence and mutual exclusivity.

Addition Rule for Three Events *The Addition Rule for the probability that events A or B or C will occur, $P(A \text{ or } B \text{ or } C)$, is given by*

$$P(A \text{ or } B \text{ or } C) = P(A) + P(B) + P(C) - P(A \text{ and } B) - P(A \text{ and } C) - P(B \text{ and } C) + P(A \text{ and } B \text{ and } C).$$

In the Venn diagram shown, $P(A \text{ or } B \text{ or } C)$ is represented by the blue areas.



In Exercises 28 and 29, find $P(A \text{ or } B \text{ or } C)$ for the given probabilities.

28. $P(A) = 0.40$, $P(B) = 0.10$, $P(C) = 0.50$,
$P(A \text{ and } B) = 0.05$, $P(A \text{ and } C) = 0.25$, $P(B \text{ and } C) = 0.10$,
$P(A \text{ and } B \text{ and } C) = 0.03$

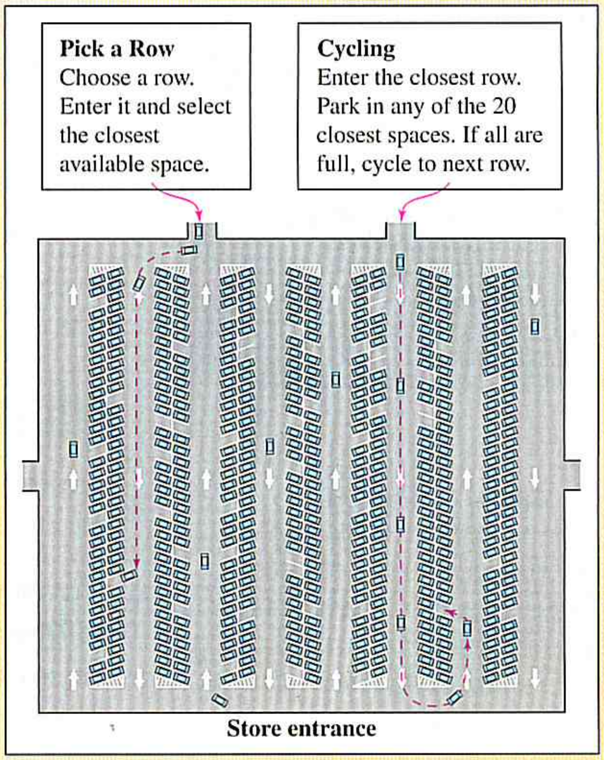
29. $P(A) = 0.38$, $P(B) = 0.26$, $P(C) = 0.14$,
$P(A \text{ and } B) = 0.12$, $P(A \text{ and } C) = 0.03$, $P(B \text{ and } C) = 0.09$,
$P(A \text{ and } B \text{ and } C) = 0.01$

CASE STUDY

Probability and Parking Lot Strategies

The Institute for Operations Research and the Management Sciences (INFORMS) is an international scientific society with more than 12,000 members. It is dedicated to the application of scientific methods to improve decision making, management, and operations. Members of the institute work primarily in business, government, and education. They represent fields as diverse as airlines, health care, law enforcement, the military, the stock market, and telecommunications.

One study published by INFORMS was the result of research conducted by Dr. C. Richard Cassady of Mississippi State University and Dr. John Kobza of Virginia Polytechnic Institute. The parking space study was conducted at a mall that has 4 entrances, 7 rows with 72 spaces each, and directional restrictions. The researchers compared several parking lot strategies to see which strategy saves the most time. The two best strategies are called *Pick a Row* and *Cycling*. The results are shown in the table.



Time or Distance	Pick a Row	Cycling
Time from lot entrance to parking space	37.7 seconds	52.5 seconds
Time from lot entrance to store's door	61.3 seconds	70.7 seconds
Average walking distance to store	257 feet	208 feet

Exercises

1. In a parking lot study, is each parking space equally likely to be empty? Explain your reasoning.
2. According to the results of the study, are you more likely to spend less time using the Pick-a-Row strategy or the Cycling strategy? Explain.
3. According to the results of the study, are you more likely to walk less using the Pick-a-Row strategy or the Cycling strategy? Explain.
4. A key assumption in the study was that the drivers can see which spaces are available as soon as they enter a lane. Why is that important?
5. The parking lot is completely full, and one car leaves. What is the probability that the car was in the first row? Explain your reasoning.
6. A person is leaving from a row that is full. What is the probability that the person was parked in one of the 20 spaces that are closest to the store?
7. Draw a diagram of the parking lot. Color code the parking spaces into three categories of 168 spaces each: most desirable, moderately desirable, and least desirable. Assume that the parking lot is half full. Estimate the probability that you can find a parking space in the most desirable category. Explain your reasoning.