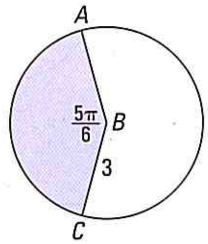
QUESTIONS

Covering the Reading



In 1 and 2, use circle B at the left, in which $m\angle ABC = \frac{5\pi}{6}$ radians.

1. Compute the area of sector ABC.

2. Compute the length of $\widehat{AC}$.

In 3 and 4, consider a circle with radius 8 cm and a central angle of $\frac{3\pi}{4}$.

3. Find the length of the arc cut off by this angle.

4. Find the area of the sector determined by this angle.

In 5 and 6, repeat Questions 3 and 4 if the central angle has measure 48°.

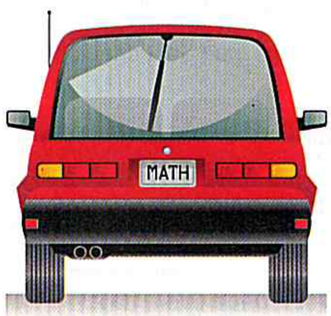
7. Refer to Example 1. Find the area of the sector exactly.

8. The Circular Sector Area Formula is of the form $A = kr^2$. When the central angle of the sector has measure θ radians, what is the value of k?

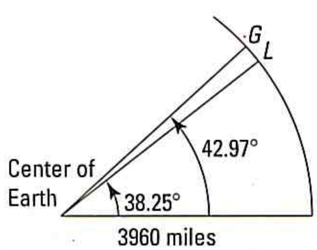
9. Refer to Example 2. Find the length of the arc of the sector to the nearest tenth of a millimeter.

10. Refer to Example 3. To the nearest centimeter, how far does the tip of the arm travel in one minute?

Applying the Mathematics



11. The windshield wiper at the back of a hatchback has an 18" blade mounted on a 10" arm, as shown at the left. If the wiper turns through an angle of 124°, what area is swept clean?



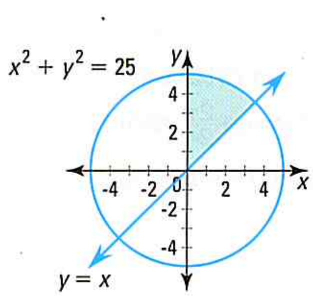
12. The diagram at the left shows a cross section of Earth. G represents Grand Rapids, MI (longitude 85.67 degrees W and latitude 42.97 degrees N) and L represents Louisville, KY (85.77 degrees W, 38.25 degrees N). Because their longitudes are close, assume that Grand Rapids is directly north of Louisville. If the radius of Earth is about 3,960 miles, estimate the air distance from Grand Rapids to Louisville.

13. Suppose you ride a bike with wheels 22" in diameter so that the wheels rotate 150 revolutions per minute.

a. Find the number of inches traveled during each revolution.

b. How many inches are traveled each minute?

c. Use your answer from part **b** to find the speed, in miles per hour, that you are traveling. (Hint: Write the units in each step as you multiply by appropriate conversion factors. Cancel units until mph remains.)



14. If a sector in a circle with a central angle of $\frac{\pi}{6}$ has an area of 3π square meters, what is the radius of the circle?

15. At the left, the circle $x^2 + y^2 = 25$ and the line $y = x$ are graphed. Find the area of the shaded sector.

Home

Answers

Name ______________________

LESSON MASTER 13-1

Questions on SPUR Objectives
See pages 863–865 for objectives.

Skills Objective A

In 1–4, give exact values.

1. $\csc \frac{\pi}{4}$ ______________

2. $\cot \frac{\pi}{2}$ ______________

3. $\sec(-240°)$ ______________

4. $\csc 75°$ ______________

In 5–8, evaluate to the nearest hundredth.

5. $\cot 4$ ______________

6. $\sec 70°$ ______________

7. $\csc(-35°)$ ______________

8. $(\csc 154°)^{-1}$ ______________

In 9–12, let $\sin\theta = 0.34$ where $0 \le \theta \le \frac{\pi}{2}$. Evaluate to the nearest hundredth.

9. $\csc\theta$ ______________

10. $\sec\theta$ ______________

11. $\cot\theta$ ______________

12. $\sec(\pi + \theta)$ ______________

Properties Objective E

In 13 and 14, tell if the function with the given equation is *even, odd,* or *neither.*

13. $f(\theta) = \sec\theta$ ______________

14. $g(\theta) = \csc\theta$ ______________

In 15–18, *true or false*. Let *f* be the tangent function and *g* be the cotangent function.

15. *f* and *g* have the same domain. ______________

16. *f* and *g* have the same range. ______________

17. *f* and *g* have the same period. ______________

18. The graphs of $y = f(x)$ and $y = g(x)$ have the same asymptotes. ______________

19. Identify all points of discontinuity of the graph of $y = \csc x$.

____

20. Find all values of x such that $\cot x = \tan x$.

____

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Home

Answers

Name

LESSON MASTER **4-4**

Questions on SPUR Objectives
See pages 303–307 for objectives.

Properties Objective D

1. The point (1, 0) is rotated about the origin such that $\cos\theta = -\frac{8}{17}$.

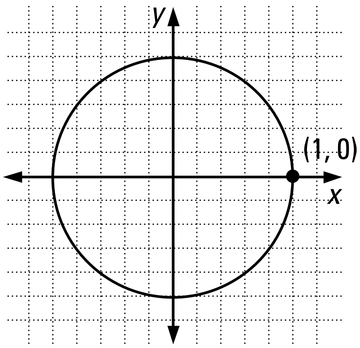
a. In what quadrant(s) could $R_\theta(1, 0)$ lie?

____

b. Justify your answer to part a by graphing $R_\theta(1, 0)$ on the unit circle at the right

c. Find all possible values of $\sin\theta$.

____



Properties Objective E

2. If $\sin\theta = \frac{\sqrt{17}}{7}$, find all possible values for the following.

a. $\cos\theta$ ______________

b. $\tan\theta$ ______________

3. If $\cos\theta = 0.68$, evaluate the following.

a. $\cos(-\theta)$ ______________

b. $\cos(\pi - \theta)$ ______________

4. If $\sin\theta = -0.368$, and $\pi < \theta < \frac{3\pi}{2}$, evaluate the following.

a. $\sin(\pi + \theta)$ ______________

b. $\sin\left(\frac{\pi}{2} - \theta\right)$ ______________

c. $\cos(-\theta)$ ______________

d. $\tan(\pi - \theta)$ ______________

5. *True or false.* $\tan(k \cdot \pi + \theta) = \tan\theta$ for all integers k. Justify your answer.

____

____

____

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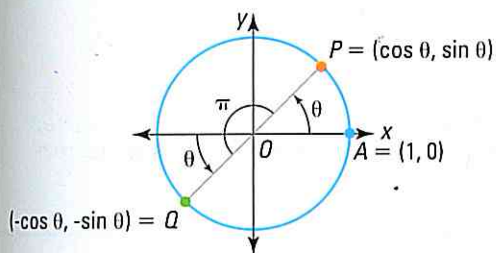
Sines, Cosines, and Tangents of $\theta + \pi$

The final type of identity in this lesson has to do with adding π to the argument θ of a circular function. Geometrically, this is akin to going halfway around the unit circle.



Let $A = (1, 0)$. Again, let $P = R_\theta(1, 0) = (\cos\theta, \sin\theta)$. Now let Q be the image of P under R_π. Because R_π maps (a, b) to $(-a, -b)$, Q has coordinates $(-\cos\theta, -\sin\theta)$. But Q is also the image of A under a rotation of $\pi + \theta$. So Q also has coordinates $(\cos(\pi + \theta), \sin(\pi + \theta))$. Equating the two ordered pairs for Q proves the first two parts of the following theorem.

Half-Turn Theorem
For all θ, measured in radians,

$$\cos(\pi + \theta) = -\cos\theta,$$
$$\sin(\pi + \theta) = -\sin\theta,$$
and
$$\tan(\pi + \theta) = \tan\theta.$$

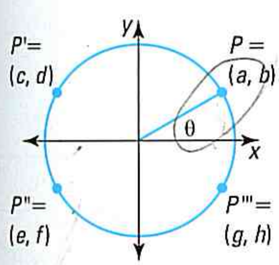
The third equation follows by dividing the second equation by the first.

In using these identities, it is not enough to memorize the theorems' statements. You should also be able to use the unit circle to do a visual check of your answers, or to derive a property if you forget one.

QUESTIONS

Covering the Reading

1. *True or false.* For all θ, $\cos^2\theta + \sin^2\theta = 1$.
2. **a.** If $\cos\theta = \frac{24}{25}$, what are two possible values of $\sin\theta$?
   b. Draw a picture to justify your answers to part **a**.
3. Write your answers to Activity 1.
4. **a.** *True or false.* $\cos 4 = \cos(-4)$.
   b. Use both a unit circle and a calculator to justify your answer to part **a**.



In 5–7, refer to the figure at the left. $P = R_\theta(1, 0)$, $P' = r_{y\text{-axis}}(P)$, $P'' = R_\pi(P)$, and $P''' = r_{x\text{-axis}}(P)$. Identify the coordinates equal to each expression.

5. $\cos(\pi - \theta)$
6. $\sin(\pi + \theta)$
7. Write an expression for $\tan(-\theta)$.
8. Show your work for Activity 2.

In 9 and 10, θ is measured in degrees and $\sin\theta = .4$. Evaluate without using a calculator.

9. $\sin(-\theta)$
10. $\sin(180° - \theta)$

11. Show the answer to Activity 3.

In 12 and 13, θ is measured in radians and cos θ = .2. Evaluate without using a calculator.

12. $\cos(\pi + \theta)$

13. $\sin\left(\frac{\pi}{2} - \theta\right)$

In 14 and 15, tan θ = *k* and θ is measured in radians. Evaluate.

14. $\tan(-\theta)$

15. $\tan(\pi - \theta)$

16. Write the version of the Half-Turn Theorem for θ measured in degrees.

Applying the Mathematics

In 17 and 18, *true or false.* If true, state the theorem that supports your answer. If false, use the unit circle to show that it is false.

17. $\sin\left(\frac{-2\pi}{3}\right) = \sin\left(\frac{2\pi}{3}\right)$

18. $\cos\left(\frac{\pi}{6}\right) = \cos\left(\frac{5\pi}{6}\right)$

19. Give a counterexample to prove that $\sin(\pi - \theta) = \sin \pi - \sin \theta$ is *not* an identity.

20. Given that $\sin\left(\frac{\pi}{10}\right) = \frac{\sqrt{5} - 1}{4}$, find each value.

a. $\sin\left(\frac{9\pi}{10}\right)$ **b.** $\sin\left(-\frac{\pi}{10}\right)$ **c.** $\sin\left(\frac{11\pi}{10}\right)$ **d.** $\cos\left(\frac{4\pi}{10}\right)$

21. Prove: For all θ, $\cos\theta + \cos(\pi - \theta) = 0$.

Review

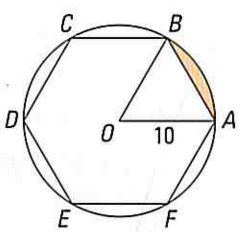
In 22–24, without using a calculator, give exact values for the following.

22. $\sin \frac{\pi}{2}$

23. $\cos 5\pi$

24. $\tan\left(\frac{\pi}{2} + \frac{\pi}{2}\right)$ *(Lesson 4-3)*

25. Refer back to the Ferris wheel in Lesson 4-3. How high is your seat off the ground if you are 2 seats away from a seat being loaded? *(Lesson 4-3)*



26. At the left *ABCDEF* is a regular hexagon inscribed in a circle with radius 10. What is the area of the shaded region shown? *(Lesson 4-2)*

27. Change 135° to radians. *(Lesson 4-1)*

28. Find an equation for the image of the graph of $y = x^2$ under the scale change $(x, y) \to \left(\frac{1}{2}x, 5y\right)$ *(Lesson 3-5)*

Exploration

29. Use a calculator to explore whether the statement $\frac{\sin^2\theta}{1 + \cos\theta} = 1 - \cos\theta$ is an identity. Justify your answer, either by providing a counterexample or by using properties of sines and cosines to prove the statement.

CHAPTER REVIEW

Questions on SPUR Objectives

SPUR stands for **S**kills, **P**roperties, **U**ses, and **R**epresentations. The Chapter Review questions are grouped according to the SPUR Objectives for this chapter.

SKILLS DEAL WITH THE PROCEDURES USED TO GET ANSWERS.

Objective A: *Convert between degrees, radians, and revolutions.* *(Lesson 4-1)*

In 1 and 2, a rotation is given. **a.** Convert to degrees. **b.** Convert to radians.

1. $\frac{1}{5}$ revolution counterclockwise

2. $\frac{2}{3}$ revolution clockwise

In 3 and 4, convert to degrees without using a calculator.

3. $-\frac{\pi}{6}$ radians **4.** $\frac{7\pi}{12}$ radians

In 5 and 6, convert to radians without using a calculator.

5. 135° **6.** 300°

In 7 and 8, tell how many revolutions are represented by each rotation.

7. $\frac{2\pi}{3}$ **8.** 540°

Objective B: *Find lengths of circular arcs, and areas of sectors.* *(Lesson 4-2)*

9. Find the length of the arc of a 120° central angle in a circle with radius 9 inches.

10. The arc of a central angle of $\frac{\pi}{6}$ radians has a length of 3π feet. Find the radius of the circle.

11. Find the area of the sector whose central angle is $\frac{\pi}{6}$ radians in a circle whose radius is 10 meters.

12. The area of a sector in a circle of radius 6 inches is 48 square inches. Find the measure of the central angle to the nearest tenth of a degree.

Objective C: *Find sines, cosines, and tangents of angles.* *(Lessons 4-3, 4-5)*

In 13–15, give exact values.

13. $\cos\left(\frac{\pi}{4}\right)$

14. $\sin\left(\frac{\pi}{3}\right)$

15. $\tan\left(\frac{\pi}{6}\right)$

In 16–18, approximate to the nearest thousandth.

16. tan 1.1

17. sin .0926

18. cos .4563

In 19–21, evaluate to the nearest hundredth.

19. sin 3

20. cos (-42.2°)

21. tan 151°

In 22–25, give exact values.

22. $\cos\left(\frac{5\pi}{4}\right)$ **23.** sin 210°

24. $\tan\left(-\frac{\pi}{4}\right)$ **25.** $\sin\left(\frac{9\pi}{2}\right)$

26. Give three values of θ from -2π to 2π such that $\cos\theta = 1$.

In 27 and 28, let $P = R_\theta(1, 0)$. Find the coordinates of P when θ is the following.

27. 7π

28. $\frac{3}{5}$ of a revolution clockwise

29. Solve $\sin x = -\frac{1}{2}$ exactly in the interval $-\frac{\pi}{2} \le x \le \frac{5\pi}{2}$.

PROPERTIES DEAL WITH THE PRINCIPLES BEHIND THE MATHEMATICS.

Objective D: *Apply the definitions of the sine, cosine, and tangent functions.* *(Lessons 4-3, 4-4, 4-6)*

30. For the sine function identify each.

a. domain **b.** range

31. For what values of θ is $\tan\theta$ undefined?

In 32 and 33, let $f(x) = \cos x$. *True or false.*

32. f is an even function.

33. The maximum value of f is 1.

34. In what interval(s) between 0 and 2π are both the cosine and tangent functions negative?

35. *Multiple choice.* For what values of θ is $\sin\theta < 0$ and $\cos\theta > 0$?

(a) $0 < \theta < \frac{\pi}{2}$ (b) $\frac{\pi}{2} < \theta < \pi$

(c) $\pi < \theta < \frac{3\pi}{2}$ (d) $\frac{3\pi}{2} < \theta < 2\pi$

Objective E: *Apply theorems about sines, cosines, and tangents.* *(Lessons 4-4, 4-5)*

36. Why is the statement that for all θ, $\sin(\pi - \theta) = \sin\theta$ called the Supplements Theorem?

In 37–40, given $\sin\theta = k$, without using a calculator, find each.

37. $\cos\left(\frac{\pi}{2} - \theta\right)$

38. $\sin(\pi - \theta)$

39. $\sin(\theta - \pi)$

40. $\sin(\pi + \theta)$

41. Use theorems about sines and cosines to prove that $-\sin\left(\frac{\pi}{2} - \theta\right) = \cos(\pi - \theta)$.

In 42–44, *true or false*. Justify your answer.

42. For all θ, $\cos(\theta + 3\pi) = \cos\theta$.

43. For all θ, $\sin(\theta + 6\pi) = \sin\theta$.

44. For all θ, $\cos^2\theta + \sin^2\theta = \tan^2\theta$.

45. If $\cos\theta = \frac{1}{4}$, without using a calculator find all possible values of each.

a. $\sin\theta$ **b.** $\tan\theta$

Objective F: *Identify the amplitude, period, frequency, phase shift, and other properties of circular functions.* *(Lessons 4-7, 4-8, 4-9)*

In 46–49, give, if it exists, **a.** the period, **b.** the amplitude, and **c.** the phase shift.

46. $\frac{y}{5} = \sin\frac{x}{2}$

47. $y = 2\cos(3\pi x)$

48. $y = 2\cos\left(x - \frac{\pi}{3}\right)$

49. $h(\theta) = \frac{1}{2}\tan(2\theta)$

50. Identify each for the function given by the equation $y = -4\cos\frac{x}{3}$.

a. amplitude **b.** period **c.** frequency

51. State **a.** the maximum and **b.** the minimum values of the function $f(t) = 10 + 5\sin 2t$.

52. Suppose the transformation $(x, y) \to \left(2x + 1, \frac{y}{3} - 1\right)$ is applied to the function $y = \sin x$.

a. State an equation for the image.

b. Find the amplitude, period, phase shift, and vertical shift of the image.

53. Let $S(x, y) = \left(\frac{x}{3}, -2y\right)$ and $T(x, y) = (x + 6, y)$.

a. Find the image of $y = \cos x$ under the composite transformation $T \circ S$.

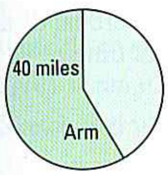
b. Find a single transformation that maps $y = \cos x$ to the function in part **a.**

CHAPTER FOUR

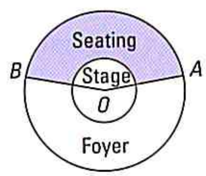
USES DEAL WITH APPLICATIONS OF MATHEMATICS IN REAL SITUATIONS.

Objective G: *Solve problems involving lengths of arcs or areas of sectors.* *(Lesson 4-2)*

54. A radar screen represents a circle of radius 40 miles. If the arm shown below makes 25 revolutions per minute, what area is mapped by the radar in each second?



55. A theater is planned as shown. The internal radius of the building is $AO = 26$ meters and $m\angle AOB = 160°$. If the stage area has a radius of 5 meters, find the area of the seating section.



Objective H: *Use equations of circular functions to solve problems about real phenomena.* *(Lessons 4-7, 4-8, 4-9)*

56. An alternating current I in amps of a circuit at time t in seconds ($t > 0$) is given by the formula $I = 40 \cos 60\pi t$.

a. Find the maximum and minimum values of the current.

b. How many times per second does the current reach its maximum value?

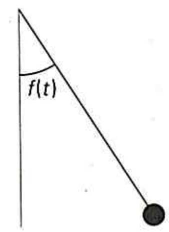
57. The voltage V in volts in a circuit after t seconds ($t > 0$) is given by $V = 120 \cos 60\pi t$.

a. Find the first time the voltage is 100.

b. Find three times at which the voltage is maximized.

58. A certain sound wave has equation $y = 60 \sin 20\pi t$. Give an equation of a sound wave with twice the frequency and half as loud as this one.

59. A simple pendulum is shown at the right. The angular displacement from vertical (in radians) as a function of time t in seconds is given by



$$f(t) = \frac{1}{2}\sin\left(2t + \frac{\pi}{2}\right).$$

a. What is the initial angular displacement?

b. What is the frequency of f?

c. How long will it take for the pendulum to make 5 complete swings?

Objective I: *Find equations of circular functions to model periodic phenomena.* *(Lessons 4-10)*

60. Suppose the height h in meters of a tide at Greenwich Mean Time t is given in the table below.

t	0	5	6	12	18	24
h	3.7	0.1	0.2	3.6	0.3	3.7

a. Fit a sine wave to these data.

b. What is the period of the sine wave?

c. What is the amplitude of the sine wave?

61. The length of the day from sunrise to sunset in a city at 30° N latitude (such as Baton Rouge, Louisiana, or Cairo, Egypt) is given in the table below.

a. Find an equation using the sine function to model these data.

b. What is the period of the function used to model these data?

c. Predict the length of the day on December 21, the winter solstice, at 30° N latitude.

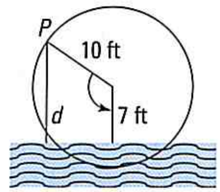
Date	Jan 1	Feb 1	Mar 1	Apr 1	May 1	June 1	July 1	Aug 1	Sept 1	Oct 1	Nov 1	Dec 1	Jan 1
Days after January 1	0	31	59	90	120	151	181	212	243	273	304	334	365
Length in hours	10.25	10.77	11.53	12.47	13.32	13.93	14.05	13.58	12.77	11.88	11.00	10.37	10.25

Source: *The Weather Almanac*

 CHAPTER FOUR

62. The figure below shows a waterwheel rotating at 4 revolutions per minute. The distance d of point P from the surface of the water as a function of time t in seconds can be modeled by a sine wave with equation of the form

$$\frac{d-k}{b} = \sin\left(\frac{t-h}{a}\right).$$



a. What is the amplitude of the distance function?

b. What is the period of the distance function?

c. If point P emerges from the water just as you start a stopwatch, write an equation for the distance function.

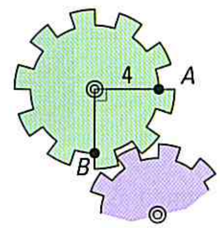
d. Approximately when does point P first reach its highest point?

63. A gear with a 4 cm radius rotates counterclockwise at a rate of 120 revolutions per minute. The gear starts with point A on the tooth level with the center of the wheel as shown below.

a. Write an equation to give the vertical distance y that point A is above or below the starting position at time t.

b. How far above or below the starting position will point A be after 5 minutes?

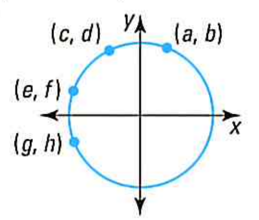
c. Write an equation for the vertical distance h point A is above point B at time t.



REPRESENTATIONS DEAL WITH PICTURES, GRAPHS, OR OBJECTS THAT ILLUSTRATE CONCEPTS.

Objective J: *Use the unit circle to find values of sines, cosines and tangents.* *(Lesson 4-3)*

In 64–67, refer to the unit circle at the right. Which letter could represent the value given?



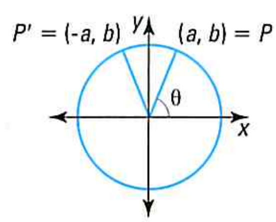
64. sin 70°

65. cos (-160°)

66. $\cos\left(\frac{11\pi}{18}\right)$

67. $\sin\left(\frac{26\pi}{9}\right)$

In 68 and 69, let P' be the reflection image of P over the y-axis, as shown in the unit circle at the right. State the value of the following.



68. $\sin(\pi - \theta)$

69. $\tan(-\theta)$

70. Draw a picture using a unit circle to show why the cosine function is an even function.

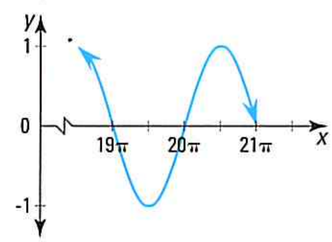
Objective K: *Draw or interpret graphs of the parent sine, cosine, and tangent functions.* *(Lesson 4-6)*

71. Consider the function $f(x) = \sin x$.

a. Sketch a graph of f without using an automatic grapher.

b. What is the period of f?

72. Below is part of the graph of a function f. Which of the following could be an equation for f: $f(x) = \cos x$ or $f(x) = \sin x$? Justify your answer.



73. Consider the graphs of $S(x) = \sin x$ and $C(x) = \cos x$. What translation maps
   a. S to C? b. C to S?

74. a. Use an automatic grapher to draw
$$y = \cos x$$
$$y = \cos(-x)$$
and $$y = \cos(\pi - x)$$
on the same set of axes.
   b. Describe the relations among the curves.
   c. What theorems do the relations between these three graphs represent?

75. Let $g(\theta) = \tan\theta$.
   a. Sketch a graph of $y = g(\theta)$.
   b. State its period.
   c. Write equations for two of the asymptotes of g.

Objective L: *Graph transformation images of circular functions.* *(Lessons 4-7, 4-8, 4-9)*

In 76–79, sketch one cycle of the graph without using an automatic grapher.

76. $y = \frac{1}{2}\sin\left(\frac{1}{2}x\right)$

77. $y = 8\cos(\pi x)$.

78. $y = \tan\left(x - \frac{\pi}{2}\right) + 3$

79. $y = \sin(x + 4) - 1$

80. a. Write an equation for the image of $y = \sin x$ under a phase shift of $\frac{\pi}{6}$.
   b. Check by graphing.

81. a. Write an equation for the image of $y = \cos x$ under a phase shift of $-\frac{4\pi}{3}$.
   b. Check by graphing.

In 82 and 83, **a.** sketch a graph of the function. **b.** State the period and maximum value if it exists.

82. $\frac{y-1}{2} = \cos(x - \pi)$

83. $y = 6 - 5\sin(4x + 2)$

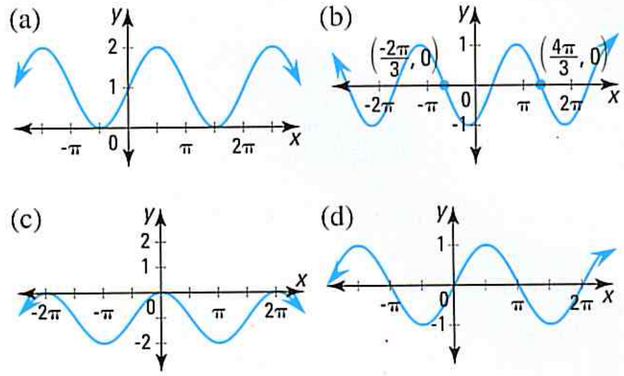
Objective M: State equations for graphs of circular functions. *(Lessons 4-7, 4-8, 4-9)*

In 84–86, match each equation with its graph.

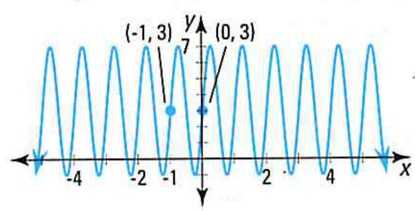
84. $y = \sin\left(x - \frac{\pi}{3}\right)$

85. $y = 1 + \sin x$

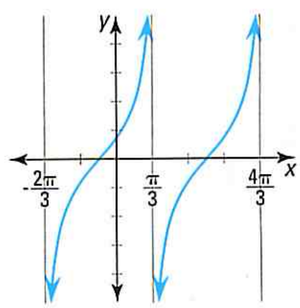
86. $y = \cos\left(x - \frac{\pi}{2}\right)$



87. Give an equation for the sine wave below.



88. Find an equation for the graph below, which is a translation image of the graph of $y = \tan x$.



CHAPTER REVIEW

Questions on SPUR Objectives

SPUR stands for **S**kills, **P**roperties, **U**ses, and **R**epresentations. The Chapter Review questions are grouped according to the SPUR Objectives for this chapter.

SKILLS DEAL WITH THE PROCEDURES USED TO GET ANSWERS.

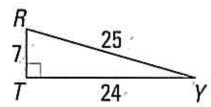
Objective A: *Find sines, cosines, and tangents of angles.* *(Lesson 5-1)*

1. Refer to triangle TRY below. Find each.

a. $\sin R$

b. $\sin Y$

c. $\cos R$



In 2–4, approximate to the nearest thousandth.

2. $\tan 42°$ **3.** $\sin 16°40'$ **4.** $\cos 82.13°$

In 5–7, find the exact value without using a calculator.

5. $\cos 60°$ **6.** $\sin 45°$ **7.** $\tan 30°$

Objective B: *Evaluate inverse trigonometric functions.* *(Lessons 5-3, 5-5, 5-6)*

In 8–10, evaluate without a calculator. State angle measures in degrees.

8. $\sin^{-1}\left(\frac{1}{2}\right)$ **9.** Arctan 1 **10.** $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

In 11–13, give the value to the nearest tenth of a radian.

11. Arcsin 0.1895 **12.** $\tan^{-1}10$

13. $\cos^{-1}(-0.8753)$

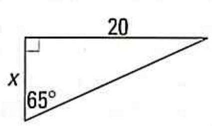
In 14 and 15, evaluate without a calculator.

14. $\cos\left(\text{Arccos}\left(\frac{4}{5}\right)\right)$ **15.** $\sin^{-1}\left(\sin\left(\frac{3\pi}{4}\right)\right)$

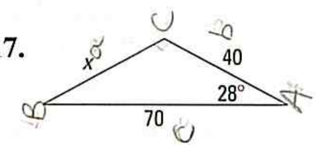
Objective C: *Use trigonometry to find lengths, angles, or areas.* *(Lessons 5-1, 5-2, 5-4)*

In 16 and 17, find x.

16.



17.



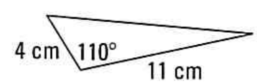
18. Find, to the nearest tenth of a degree, the measures of the angles of a 5-12-13 triangle.

19. Find, to the nearest tenth of a degree, the measure of the largest angle in a triangle whose sides are 4, 5, and 6 cm.

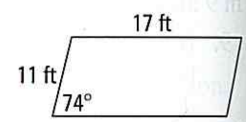
20. In $\triangle XYZ$, $m\angle X = 48°$, $m\angle Y = 68°$, and $y = 10$. Find x.

21. In $\triangle ABC$, $a = 3$, $b = 5$, and $m\angle A = 20.7°$. Find all possible values of $m\angle B$ to the nearest tenth of a degree.

22. Calculate the area of the triangle.



23. Find the area of the parallelogram.



Objective D: *Solve trigonometric equations.* *(Lessons 5-3, 5-5, 5-6, 5-7)*

In 24–26, find θ, where $0° < \theta < 90°$, to the nearest hundredth.

24. $\tan\theta = 1.5$ **25.** $\sin\theta = 0.32$

26. $\cos\theta = 0.1234$

27. Find the exact degree measure such that $\sin\theta = \frac{\sqrt{3}}{2}$ given $0 \le \theta \le \frac{\pi}{2}$.

28. Give the number of solutions to the equation for $0 \le x \le 2\pi$. Justify your reasoning.

a. $8 \sin x = 5$ **b.** $5 \sin x = 8$

c. $5 \tan x = 8$

In 29–32, solve given that $0 \le \theta \le 2\pi$.

29. $\sin\theta = -0.34$ **30.** $\cos\theta = \frac{7}{15}$

31. $\sin^2\theta + 3\sin\theta + 1 = 0$

32. $\tan\theta = -0.4$

In 33–38, describe the general solution.

33. $\cos x = 0.24$

34. $4 \sin x = -1$

35. $3 \tan \theta - 5 = 0$

36. $\cos (3x) = 0.724$

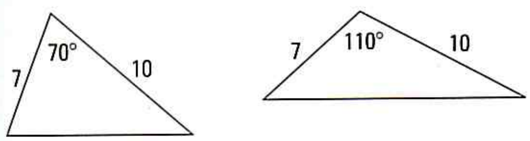
37. $\sin (2\pi x) = -0.341$

38. $2 \cos^2(3x) - \cos (3x) = 1$

PROPERTIES DEAL WITH THE PRINCIPLES BEHIND THE MATHEMATICS.

Objective E: *Interpret the Law of Sines, Law of Cosines, and related theorems.* *(Lessons 5-2, 5-4)*

39. Explain why the two triangles below have the same area.



40. Explain how the Pythagorean Theorem follows from the Law of Cosines.

41. In $\triangle EFG$, $m\angle E = 35°$, $EF = 4$, and $FG = 9$. Tom claims that there is exactly one triangle satisfying these conditions. Karen claims that there are two. Who is correct? Why?

42. Applying the Law of Sines sometimes lead to a situation with two answers. Explain why.

Objective F: *State properties of inverse trigonometric functions.* *(Lessons 5-3, 5-5, 5-6)*

43. For what values of θ is the following statement true? If $k = \sin \theta$, then $\theta = \sin^{-1} k$.

44. *Multiple choice.* If $\sin 5x = a$ and $-\frac{\pi}{2} \le 5x \le \frac{\pi}{2}$, then x equals __?__.

(a) $5 \sin^{-1} a$ (b) $\sin^{-1}\left(\frac{a}{5}\right)$

(c) $\sin^{-1} (5a)$ (d) $\frac{1}{5} \sin^{-1} a$

45. Why must the domain of $y = \cos x$ be restricted in order to define $y = \cos^{-1} x$?

46. State the domain and the range of $y = \sin^{-1} x$.

47. *True or false.* The function $y = \text{Arctan}\, x$ has period π.

48. Explain why $\cos^{-1}\left(\cos\left(-\frac{\pi}{4}\right)\right) \ne -\frac{\pi}{4}$.

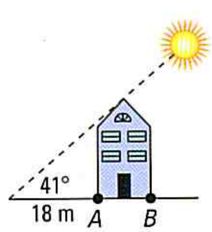
$x^2 = 40^2 + 70^2 + (2(40)(70)) \cdot \cos(28)$

USES DEAL WITH APPLICATIONS OF MATHEMATICS IN REAL SITUATIONS.

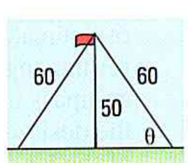
Objective G: *Solve problems using trigonometric ratios in right triangles.* *(Lesson 5-1)*

49. A ladder against a wall makes a 75° angle with the ground. If the base of the ladder is 4 feet from the wall, find the length of the ladder.

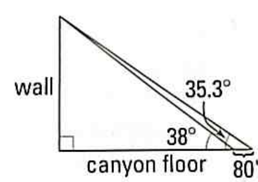
50. A school building casts a shadow 18 m long when the elevation of the sun is 41° as shown at the right. How high is the building if $AB = 16$ m?



51. Guy wires 60 feet long are to be used to steady a flagpole 50 feet tall. What acute angle θ will the wires make with the ground?



52. Two sightings 80' apart are taken to the top of a canyon wall. The angles of elevation are shown. How high is the top above the canyon floor?

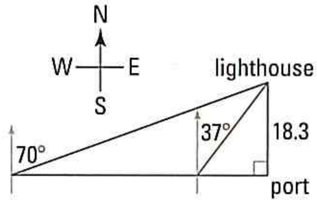


53. From ancient to modern times, carpenters and masons have used a 3-4-5 right triangle to mark right angles. Suppose that each measurement could be as much as 3% off. One worst case scenario occurs when the sides are each 3% longer and the hypotenuse is 3% shorter.

a. Find the measures of the three angles of a 3-4-5 triangle.

b. Calculate the measure of the largest angle for the worst case scenario described above. Do you think the 3-4-5 triangle method is accurate enough?

54. A tanker sails due west from port at 1 P.M. at a steady speed. At 2:15 P.M., the bearing of a lighthouse from the ship is 37° (from north).



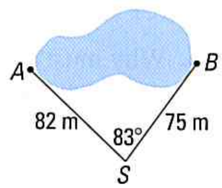
a. If the lighthouse is 18.3 miles due north of the port, how far out to sea is the ship?

b. Find the speed of the ship.

c. At what time will the lighthouse lie on a bearing of 70° from the ship, assuming that the speed of the ship is constant?

Objective H: *Solve problems involving the Laws of Sines and Cosines.* *(Lessons 5-2, 5-4)*

55. A team of surveyors measuring from A to B across a pond places a vertical stake at S, a point from which they can measure distances to A and B across dry ground. The measures are shown in the diagram. Find AB.

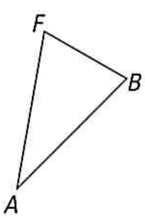


56. a. Find the length of the longer diagonal of a parallelogram with one angle of 39° and sides of 8.2 cm and 10.1 cm.

b. Find the length of the shorter diagonal of this parallelogram.

57. An airport controller notes from radar that one jet 12° west of south and 20 miles from the airport is flying toward a private plane which is 27 miles directly south of the airport. How far apart are the planes?

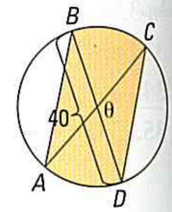
58. Forest ranger towers at A and B are 10 miles apart. The ranger in A spots a fire 10° east of north. She calls the ranger at B, who locates the fire as 30° north of west with respect to B.



a. If B is directly northeast of A, find the distance of the fire from A.

b. Find the distance of the fire from B.

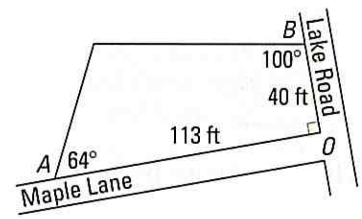
59. In the diagram at the right, $\overline{AB} \parallel \overline{CD}$. The circle has diameter 40 mm and $AB = CD = 35$ mm.



a. Find θ to the nearest tenth of a degree.

b. Find the shaded area.

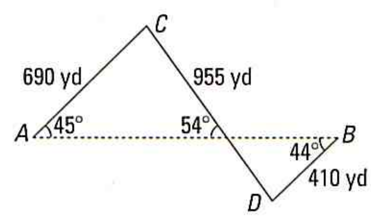
60. An irregularly shaped plot of land has certain known dimensions. It borders two roads which meet at right angles. Frontage on Lake Road measures 40' and frontage on Maple Lane measures 113'. Property lines run away from the roads at angles of 64° and 100°, as shown.



a. Find the lengths of the other two sides of the plot of land.

b. What is the area of the property?

61. In order to sail against the wind, a sailor must take an indirect path—what is called "tacking." Suppose a boat sails from point A to an upwind point B following the path shown below, where $AC = 690$ yd, $CD = 955$ yd, $DB = 410$ yds. What is the actual distance between A and B? Round your answer to the nearest yard.



Objective I: *Write and solve equations for phenomena described by trigonometric and circular functions.* *(Lesson 5-3, 5-4, 5-5, 5-6, 5-7)*

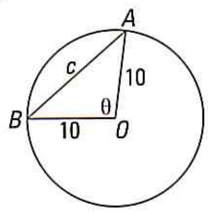
62. On a compass used to draw arcs, the leg which holds the pencil measures 6.2". Find a formula for the angle θ at the top of the compass in terms of the radius of the desired circle and the length of the leg.

63. The vertical displacement d of a mass oscillating at the end of a spring is given by the equation $d = 3 \cos(\pi t)$ where t is time in seconds.

a. Solve this equation for t.

b. At what time does d first equal 2 cm?

64. c, the length of chord $\overline{AB}$ in ⊙O, depends on the magnitude of central angle θ.



a. Use the Law of Cosines to write an equation for c in terms of θ.

b. Solve the equation of part **a** for θ.

c. What is an appropriate domain for c?

d. Use your equation to find the value of θ when $c = 15$.

65. The measured voltage E in a circuit at t seconds ($t \geq 0$) is given by $E = 20 \cos(4\pi t)$. Find, to the nearest .01 second, the first five times that $E = 16$.

66. New Orleans has about 14 hours of daylight at the summer solstice and 9.3 hours at the winter solstice. An equation for number of hours h of daylight as a function of days d after March 21 is given by

$$h = 11.65 + 2.35 \sin\left(\frac{\frac{d}{365}}{2\pi}\right).$$

a. Find the next four times New Orleans has 13.6 hours of daylight. Give your answer in days after March 21.

b. Does New Orleans have a day in the calendar year before March 21 with 10 hours of daylight?

67. The distance from ground level to a paddle on an old wooden mill wheel can be modeled by $d = 11 + 15 \sin(\pi(t - 3))$, where t = time in minutes.

a. Graph the function. Why does it dip below the t-axis?

b. On your graph mark the first two times ($t > 0$) that the paddle is 5' above the ground.

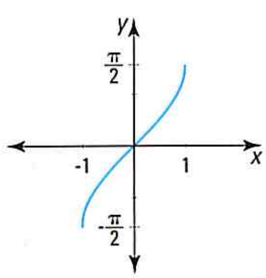
c. Find the first two times the paddle is 5' above the ground by solving $5 = 11 + 15 \sin(\pi(t - 3))$.

REPRESENTATIONS DEAL WITH PICTURES, GRAPHS, OR OBJECTS THAT ILLUSTRATE CONCEPTS.

Objective J: *Graph or identify graphs of inverse trigonometric functions.* *(Lessons 5-3, 5-5, 5-6)*

68. Graph $y = \cos x$ and $y = \cos^{-1} x$ for $0 \leq x \leq \pi$ on the same set of axes.

69. *Multiple choice.* Which function is graphed at the right?



(a) $y = \tan^{-1} x$

(b) $y = \cos^{-1} x$

(c) $y = \sin x$

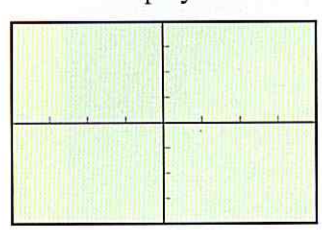
(d) $y = \sin^{-1} x$

70. *True or false.* The graph of $y = \tan^{-1} x$ has asymptotes.

71. Sarah graphed $y = \cos^{-1} x$ on her calculator in degree mode. Her screen looked like the one drawn below. She thought her calculator was broken, but G. J. fixed it in 30 seconds.

a. What might G. J. have done to display the graph?

b. What would the display look like?



$-360 \leq x \leq 360$, x-scale = 90
$-4 \leq y \leq 4$, y-scale = 1

CHAPTER REVIEW

Questions on SPUR Objectives

SPUR stands for **S**kills, **P**roperties, **U**ses, and **R**epresentations. The Chapter Review questions are grouped according to the SPUR Objectives for this chapter.

SKILLS DEAL WITH THE PROCEDURES USED TO GET ANSWERS.

Objective A: *Multiply matrices, when possible.* *(Lesson 11-1)*

In 1–4, use the following matrices.

$A = \begin{bmatrix} 1/2 & 3 \\ 5 & -1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 5 & -1 \\ -1 & 0 & 1 \end{bmatrix}$

$C = \begin{bmatrix} x \\ 3 \end{bmatrix}$ $D = \begin{bmatrix} 1 & -7 \end{bmatrix}$

1. *Multiple choice.* Which of the following products is not possible?

(a) AC (b) CD

(c) BA (d) A^2C

2. Find AB.

3. Find DC.

4. Find A^2.

Objective B: *Use matrices to solve systems of equations.* *(Lesson 11-7)*

5. Solve for x: $\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ 1 \end{bmatrix} = \begin{bmatrix} 7 \\ -4 \end{bmatrix}$.

6. Consider the system $\begin{cases} x + 3y = 11 \\ 2x - y = 8 \end{cases}$.

a. Represent the system by a matrix equation.

b. Find the inverse of the coefficient matrix in part **a**.

c. Multiply on the left both sides of the matrix equation by the answer in part **b**.

d. What is the solution to the system of equations?

In 7 and 8, solve the given system of equations using matrices.

7. $\begin{cases} 5x + 2y = 6 \\ 10x - 10y = -9 \end{cases}$

8. $\begin{cases} 8x + 3y = 10 \\ x + 8y = -14 \end{cases}$

9. Solve the system $\begin{cases} x + 2y + 3z = 10 \\ 3x + y + 2z = 13, \\ 2x + 3y + z = 13 \end{cases}$

given that

$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 3 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} -\frac{5}{18} & \frac{7}{18} & \frac{1}{18} \\ \frac{1}{18} & -\frac{5}{18} & \frac{7}{18} \\ \frac{7}{18} & \frac{1}{18} & -\frac{5}{18} \end{bmatrix}.$$

Objective C: *Find the inverse of a 2 × 2 matrix.* *(Lesson 11-7)*

In 10 and 11, find the inverse of the given matrix.

10. $\begin{bmatrix} 2 & 1 \\ -5 & 4 \end{bmatrix}$

11. $\begin{bmatrix} \cos 40° & -\sin 40° \\ \sin 40° & \cos 40° \end{bmatrix}$

12. Find $\begin{bmatrix} \frac{2}{9} & \frac{1}{9} \\ -\frac{1}{3} & \frac{4}{3} \end{bmatrix}^{-1}$.

13. *Multiple choice.* Tell which two of the following matrices have no inverses.

(a) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} -2 & 4 \\ -6 & 8 \end{bmatrix}$

(e) $\begin{bmatrix} -2 & 4 \\ -1 & 2 \end{bmatrix}$

 CHAPTER ELEVEN

PROPERTIES DEAL WITH THE PRINCIPLES BEHIND THE MATHEMATICS.

Objective D: *Apply properties of matrices and matrix multiplication.* *(Lessons 11-1, 11-3, 11-7)*

In 14 and 15, an equation relating the matrices M, N, and P is given. The matrix M has dimensions 4×2, and the matrix P has dimensions 4×5. Give the dimensions of N.

14. $MN = P$

15. $PN = M$

16. *Multiple choice.* Which of the following statements about 2×2 matrices R, S, and T is false?

(a) RS is also a 2×2 matrix.

(b) $(RS)T = R(ST)$

(c) There is a matrix I such that $RI = IR = R$

(d) $RS = SR$

17. Name three types of transformations that can be represented by 2×2 matrices.

Objective E: *Apply the Addition and Double Angle Formulas.* *(Lessons 11-5, 11-6)*

In 18 and 19, state the exact value.

18. $\sin 105°$ **19.** $\cos \frac{5\pi}{12}$

In 20 and 21, A and B are acute angles with $\sin A = .8$ and $\cos B = .5$. Find each.

20. $\cos(A - B)$ **21.** $\sin 2A$

In 22 and 23, simplify.

22. $\cos(\pi - \theta)$ **23.** $\sin\left(\theta - \frac{\pi}{2}\right)$

24. Give a formula for $\cos 4\theta$ in terms of $\cos \theta$ only.

In 25–27, write as the sine or cosine of a single argument.

25. $2 \sin A \cos A$ **26.** $\cos\frac{\pi}{5}\cos\frac{\pi}{3} + \sin\frac{\pi}{5}\sin\frac{\pi}{3}$

27. $2\cos^2 25° - 1$

In 28–29, explain how the formula was derived.

28. $\sin(\alpha + \beta)$ **29.** $\cos 2\theta$

USES DEAL WITH APPLICATIONS OF MATHEMATICS IN REAL SITUATIONS.

Objective F: *Use a matrix to organize information.* *(Lessons 11-1)*

In 30 and 31, use the production matrix P and the cost matrix C shown here.

$P =$

	Melons	Lettuce (heads)
Farm A	1000	10,000
Farm B	2000	10,000
Farm C	500	20,000
Farm D	500	5,000

$C =$

	Cost to Produce	Cost to Consumer
Melons	.45	.90
Lettuce	.65	1.00

30. a. Calculate PC. **b.** What does PC represent?

c. Find the total cost of producing melons and lettuce on Farm C.

d. What element of PC contains this cost?

31. Find the total cost to the consumer of the melons and lettuce produced on Farm A.

In 32 and 33, use the following matrix of Foreign Exchange rates.

$= F$

	United Kingdom (pound)	Canada (dollar)	Japan (yen)	United States (dollar)
1975	2.22	0.98	0.0034	1.00
1980	2.32	0.86	0.0044	1.00
1985	1.30	0.73	0.0042	1.00
1990	1.78	0.86	0.0069	1.00
1995	1.58	0.73	0.0106	1.00

32. a. Describe the data given by each row of F.

b. How many U.S. dollars were 1000 Japanese yen worth in 1995?

33. a. If a tourist entered the U.S. in 1995 with 150 pounds U.K., 212 dollars Canadian, and 100 dollars U.S., what is the U.S. equivalent of that cash?

b. *True or false.* To compare the total cash value of the tourist in part **a** at five-year intervals between 1975 and 1995, one would look at the product matrix FC. (C is shown at the right.)

$$C = \begin{bmatrix} 150 \\ 212 \\ 0 \\ 100 \end{bmatrix}$$

REPRESENTATIONS DEAL WITH PICTURES, GRAPHS, OR OBJECTS THAT ILLUSTRATE CONCEPTS.

Objective G: *Represent reflections, rotations, scale changes, and size changes as matrices.* *(Lesson 11-2, 11-3, 11-4)*

34. Write the matrix representing r_x, reflection over the x-axis.

35. Write the matrix for the scale change $T: (x, y) \to \left(x, \frac{1}{3}y\right)$.

36. Write the matrix which represents a 90° rotation counterclockwise about the origin.

In 37 and 38, describe the transformation represented by the matrix.

37. $\begin{bmatrix} 11 & 0 \\ 0 & 11 \end{bmatrix}$

38. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

39. Tell what rotation is represented by $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$.

40. Give the rotation matrix for R_{32}.

41. Give exact values of the matrix for R_{120}.

42. If $T(1, 0) = (q, t)$, $T(0, 1) = (e, z)$, and T can be represented by a 2×2 matrix, what is this matrix?

43. A transformation which can be represented by a matrix takes (0, 0) to (0, 0), (1, 0) to (5, 0), and (0, 1) to (0, -1).

a. What is the matrix for the transformation?

b. What is the image of (1, 1)?

Objective H: *Represent composites of transformations as matrix products.* *(Lessons 11-3, 11-4)*

44. A figure is rotated 90° clockwise, then reflected over the y-axis.

a. Write a matrix product for the composite transformation.

b. Compute the matrix product from part **a**.

45. Tell, in words, what composition of transformations is represented by

$$\begin{bmatrix} \cos 32° & -\sin 32° \\ \sin 32° & \cos 32° \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}.$$

46. The matrix representing R_{135} is

$$A = \begin{bmatrix} -\sqrt{2}/2 & -\sqrt{2}/2 \\ \sqrt{2}/2 & -\sqrt{2}/2 \end{bmatrix}.$$

a. Compute A^2.

b. What transformation does A^2 represent?

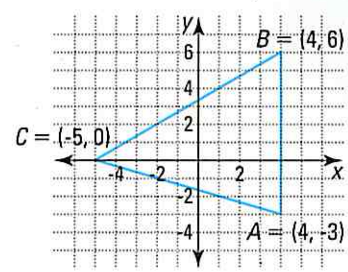
47. Let Y be the matrix representing r_y.

a. Compute Y^3.

b. What transformation does Y^3 represent?

Objective I: *Use matrices to find the image of a figure under a transformation.* *(Lessons 11-2, 11-3, 11-4)*

48. Refer to $\triangle ABC$ below.



a. Find a matrix for the image $\triangle A'B'C'$ under the transformation represented by $\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$.

b. Draw the image.

49. a. Find a matrix for the vertices of the image of the square having opposite vertices (0, 0) and (1, 1) under the transformation with matrix $\begin{bmatrix} 7 & 0 \\ -1 & 1 \end{bmatrix}$.

b. Draw the square and its image on the same axes.

50. Let $A = (7, 0)$, $B = (0, 2)$, and $C = (-1, -1)$.

a. Describe $(r_x \circ R_{90})(\triangle ABC)$ as the product of three matrices.

b. Find a single matrix for the image triangle.

51. A two-dimensional figure has been rotated counterclockwise around the origin by 75°. Write the matrix for the transformation which returns the figure to its original orientation.

CHAPTER REVIEW

Questions on SPUR Objectives

SPUR stands for **S**kills, **P**roperties, **U**ses, and **R**epresentations. The Chapter Review questions are grouped according to the SPUR Objectives for this chapter.

SKILLS DEAL WITH THE PROCEDURES USED TO GET ANSWERS.

Objective A: *Evaluate the reciprocal trigonometric functions.* (Lesson 13-1)

In 1–3, give exact values.

1. $\csc \frac{\pi}{2}$ **2.** $\cot\left(-\frac{\pi}{4}\right)$ **3.** $\sec 390°$

In 4–6, evaluate to the nearest hundredth.

4. $\sec 28°$ **5.** $\csc 3.7$ **6.** $\cot 237°$

Objective B: *Perform operations with complex numbers in polar or trigonometric form.* (Lesson 13-7)

In 7 and 8, determine z_1z_2. Leave your answer in the form of the original numbers.

7. $z_1 = 5(\cos 32° + i \sin 32°)$,
$z_2 = 3(\cos 157° + i \sin 157°)$

8. $z_1 = [7, 0]$, $z_2 = \left[1.3, \frac{\pi}{4}\right]$

In 9 and 10, given z_1 and z_1z_2, determine z_2. Leave your answer in the form of the original numbers.

9. $z_1 = 9\left(\cos \frac{\pi}{5} + i \sin \frac{\pi}{5}\right)$,
$z_1z_2 = 18\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right)$

10. $z_1 = \left[2, \frac{\pi}{6}\right]$, $z_1z_2 = \left[5, \frac{7\pi}{6}\right]$

In 11 and 12, a complex number is given. **a.** Give the absolute value. **b.** Give an argument.

11. $-10 + 12i$ **12.** $\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}$

Objective C: *Represent complex numbers in different forms.* (Lessons 13-6, 13-7)

In 13–15, write the complex number in polar form. Use an argument θ in the interval $0° \le \theta < 360°$.

13. $-\sqrt{3} + i$ **14.** $2 + 5i$

15. $\frac{1}{8}\left(\cos \frac{\pi}{5} + i \sin \frac{\pi}{5}\right)$

In 16 and 17, write the complex number in $a + bi$ form.

16. $3(\cos(-120°) + i \sin(-120°))$ **17.** $\left[4, \frac{2\pi}{5}\right]$

18. Give two different representations of the complex number $[-2, 45°]$ in trigonometric form.

Objective D: *Find powers and roots of complex numbers.* (Lesson 13-8)

In 19 and 20, find z^n for the given z and n.

19. $z = \sqrt{2} + \sqrt{2}\,i$, $n = 5$

20. $z = 3(\cos 240° + i \sin 240°)$, $n = 4$

In 21 and 22, find the indicated roots.

21. the 4 fourth roots of $256(\cos 12° + i \sin 12°)$

22. the 6 sixth roots of -2

PROPERTIES DEAL WITH THE PRINCIPLES BEHIND THE MATHEMATICS.

Objective E: *Apply properties of the reciprocal trigonometric functions.* (Lesson 13-1)

In 23 and 24, consider the function $f(x) = \csc x$.

23. What is the period of f?

24. For what values is f undefined?

In 25 and 26, *true or false*.

25. $\sec \theta = \frac{1}{\csc \theta}$, for all θ.

26. The function $f(x) = \cot x$ is an even function.

Objective F: *Prove trigonometric identities.* *(Lessons 13-2, 13-3)*

27. Prove the identity $(1 - \cos^2 x)(1 + \cot^2 x) = 1$ by starting with the left-hand side and rewriting it until the other side appears.

28. Prove that $\sin\left(\theta - \frac{\pi}{4}\right) = -\cos\left(\theta + \frac{\pi}{4}\right)$ for all θ by rewriting each side of the equation independently until equal expressions are obtained.

29. Prove that $\csc^2 x - \cot^2 x = 1$ for all $x \neq n\pi$, where n is an integer, starting with the Pythagorean Identity.

30. Prove: For all θ for which the equation is defined, $\frac{\sin\theta}{\cos\theta \cdot \tan\theta} = 1$.

Objective G: *Describe singularities of functions.* *(Lesson 13-3)*

31. Explain why the restriction $x \neq n\pi$ (n an integer) is necessary for the identity in Question 29.

32. Consider the identity given in Question 30.

a. Determine all the singularities of the functions mentioned in the equation.

b. Give the biggest domain on which the identity holds.

33. a. Determine the singularities (if any) of the functions with the given equations.

i. $f(x) = \frac{x^3 - 27}{x - 3}$ **ii.** $g(x) = \frac{x^3 - 27}{x^2 + 3x + 9}$

b. *True or false.* The proposed identity is true for all real numbers x. Explain your answer.

i. $\frac{x^3 - 27}{x - 3} = x^2 + 3x + 9$

ii. $\frac{x^3 - 27}{x^2 + 3x + 9} = x - 3$

34. *True or false.* The functions $f(x) = \frac{1}{1 - \cos x}$ and $g(x) = \frac{1}{1 - \cos^2 x}$ have the same singularities. Explain your answer.

USES DEAL WITH APPLICATIONS OF MATHEMATICS IN REAL SITUATIONS.

There are no objectives relating to uses in this chapter.

REPRESENTATIONS DEAL WITH PICTURES, GRAPHS, OR OBJECTS THAT ILLUSTRATE CONCEPTS.

Objective H: *Use an automatic grapher to test a proposed identity.* *(Lessons 13-2, 13-3)*

35. *True or false.* Showing that the graphs of the functions on the two sides of a given equality in a single variable coincide when created by an automatic grapher proves that the equation is an identity.

In 36–38, an equation is given.
a. Graph the functions related to the two sides of the proposed identity. **b.** Decide if a person should attempt to prove the identity. Explain your reasoning.

36. $\frac{x^2 - 4x - 5}{x + 1} = x - 5$

37. $\sin^2 x(1 + \tan^2 x) = \tan^2 x$

38. $\frac{\csc x}{\sec x} = \tan x$

Objective I: *Given polar coordinates of a point, determine its rectangular coordinates and vice versa.* *(Lesson 13-4)*

In 39 and 40, convert from polar coordinates to rectangular coordinates.

39. $\left[4, \frac{3\pi}{2}\right]$

40. $[-3, 85°]$

In 41 and 42, give one pair of polar coordinates for each (x, y) pair.

41. $(5, 2)$

42. $(-2, -3)$

43. A point is located at $(-4\sqrt{3}, 4)$ in a rectangular coordinate system. Find three pairs of polar coordinates $[r, \theta]$ that name this same point. Assume θ is in radians.