13.4 The Lens Equations

OVERALL EXPECTATIONS

- investigate, through inquiry, the properties of light, and predict its behaviour, particularly with respect to reflection in plane and curved mirrors and refraction in converging lenses
- demonstrate an understanding of various characteristics and properties of light, particularly with respect to reflection in mirrors and reflection and refraction in lenses

SPECIFIC EXPECTATIONS

Developing Skills of Investigation and Communication

- predict, using ray diagrams and algebraic equations, the position and characteristics of an image produced by an converging lens, and test their predictions through inquiry

Understanding Basic Concepts

- explain the conditions required for partial reflection/refraction and for total internal reflection in lenses, and describe the reflection/refraction using labelled ray diagrams
- describe the characteristics and positions of images formed by converging lenses, with the aid of ray diagrams
- identify ways in which the properties of mirrors and lenses determine their use in optical instruments

Time

60–75 min

Vocabulary

- thin lens equation

Assessment Resources

Assessment Rubric 1: Knowledge and Understanding
Assessment Summary 1: Knowledge and Understanding

Other Program Resources

BLM 13.4-1 Lens Equations
Skills Handbook 5. Using Mathematics in Science
Science Perspectives 10 website www.nelson.com /sciencepersectives/10

Related Resources

Gizmos: Refraction; Ray Tracing (Lenses)

Hecht, Eugene. *Theory and Problems of Optics.* McGraw-Hill, 2007.

Science Perspectives 10 ExamView® Test Bank

Science Perspectives 10 Teacher eSource SUITE Upgrade

Science Perspectives 10 website www.nelson.com /sciencepersectives/10

KEY CONCEPTS

- Geometric optics can be used to determine the path of light rays through lenses.

EVIDENCE OF LEARNING

Look for evidence that students can

- identify the terminology used in the lens and magnification equations
- substitute values for image characteristics to use in the lens and magnification equations
- use lens and magnification equations to determine image characteristics

SCIENCE BACKGROUND

The Thin Lens Equation

- Lenses can be classified as either thick or thin. The distinction between them is dependent on the amount of precision one needs to apply to solve a problem. Light entering a lens gets refracted twice—once entering the lens and once exiting it—yet in a thin lens the points of entry and exit are so close as to be considered identical.
- The key to understanding the derivation of the thin lens equation on page 563 of the Student Book is triangle similarity. In geometry, two figures are *similar* if their corresponding angles are congruent and their corresponding sides are in proportion with one another. To establish two triangles as similar, one must show that at least two (of three) corresponding angles of the triangles are congruent.

For triangles *AOF* and *DEF* in Figure 2 on page 563 of the Student Book, angles *DFE* and *AFO* are congruent because they are opposite angles. A second pair of congruent corresponding angles are angles *AOF* and *FDE*—since both are right angles, they are congruent.

- With two pairs of corresponding angles shown to be congruent, the two triangles, *DEF* and *OAF*, can be said to be similar. One can now form proportions between any two corresponding sides or angles in the two triangles because they are known to be similar. The derivation on page 563 of the Student Book begins by stating the proportion *ED*/*DF* = *AO*/*OF* based on the similarity of the two triangles. You can see that side *ED* of triangle *DEF* corresponds to side *AO* of triangle *AOF*. Other statements in the derivation rely on similarity between triangles.
- Why is the thin lens equation stated in terms of the *inverse* of *f*, 1/*f*, rather than *f* itself? Mathematically it is possible to state the relationship in terms of *f*, but the algebra of the situation makes for a "messy" equation. Solved for *f*, the equation is: $f = \frac{(d_o/d_i)}{(d_o + d_i)}$, which is why the inverse form is used.
- The magnification of a lens is the ratio of the height of the image produced to the height of the object (or the negative of the ratio of the image distance to the object distance). These relationships can be stated mathematically in the magnification equation $M = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$.

POSSIBLE MISCONCEPTIONS

Identify

- Students may become confused by negative and positive values when using the magnification equation.

Clarify

- The values of h_i and h_o are positive when measured upward from the principal axis and negative when measured downward.

Ask What They Think Now

- At the end of the section ask, *What does it mean if h_i is calculated to be negative for a real image?* Students should realize that the image is upside down with respect to the object.

TEACHING NOTES

Engage

- Engage students' interest by posing a problem. A film projector uses a converging lens to project an image of a face on a screen. The face measures 1 cm in height on the film. Ask, *Is there any way that you could calculate the size of the face projected on a screen 25 m away?* Have students suggest how they might work backward to solve the problem. Ask, *What information might you need?* Discuss students' responses; then go on to tell them that there is, in fact, a way to make such a calculation that requires knowing the focal length of the lens and a few other variables. They will learn equations in this lesson that will help them compute the size of the face on the screen.

Explore and Explain

- Have students copy Table 1 from page 566 of the Student Book in their notebooks, adding their own notes to help them consolidate the information in the table and adding notes to it as they go through the rest of the lesson. This table appears at the end of the lesson, but it would be helpful to have students begin completing it and organizing the information that it contains earlier in the lesson.
- Go over the terminology and the symbols used in Figure 2 and in the lens equations on page 563 of the Student Book. Identify each feature of the diagram. Make certain students can locate and identify each feature on the diagram. Ask, *Which ray corresponds to* d_o*?* (*BA*) *To* d_i*?* (*OF*) *What does ray* BC *correspond to?* (h_o) *What does ray* DE *correspond to?* (h_i) *How would you identify* F*?* (segment *OF*)
- Review the mathematical proof derivation of the thin lens equation. Walk students through the algebra. Allow students proficient in algebra to explain what is being done or assumed in each step.
- Make sure students are clear on the questions being asked in each of the sample problems. For **Sample Problem 1**, ask, *Which variables do you know the value of?* (f and d_o) *What do you need to find?* (the value of d_i) *How can you find* d_i*?* (Plug the values into the thin lens equation and solve). On the board, go through the steps of solving the equation.
- Have students review **Skills Handbook 5.D. Uncertainty in Measurement** for help with rounding and significant digits. Remind students to round to significant digits for all their calculations. In addition, students may benefit from reviewing section **5.B. Solving Numerical Problems Using the GRASS Method**.
- Work through **Sample Problem 2** with the class. This problem uses the Thin Lens Equation—the same equation that was used for **Sample Problem 1**—but deals with a diverging lens.
- Throughout the lesson, make sure students are careful with positive and negative values. Have students review the rules, as well as the logic behind the rules, for what makes a measurement negative and what that means. Also remind students to be careful when working with negatives in solving and substituting with negatives.
- Write the magnification equation on the board: $M = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$. Explain what each of the variables in the equation represents and explain that the equation is a mathematical statement of the definition of magnification: the ratio of the size of an image to the size of the object.
- Review the sign conventions used previously and add the two following conventions:
  – Object and image heights are positive when measured upward from the principal axis and negative when measured downward.
  – Magnification is positive for an upright image and negative for an inverted image.
- Work through **Sample Problems 3**, **4**, and **5** with the class. Each of the problems involves the magnification equation. **Sample Problems 3** and **5** both deal with determining the magnification based on image and object height; one problem is for a converging lens, whereas the other is for a diverging lens. Sample Problem 4 uses the known magnification to determine the location of the image.
- Review students' notes on Table 1 on page 566 of the Student Book. As a class, discuss where each of the values comes from and answer any questions that students may have.

Learning Tip

Does it Make Sense?

Explain to students that they can always make sure their answer makes sense by substituting it into the original equation and testing it. If it does not work, encourage them to try to see if it is too high or too low, or in the wrong form. Students should then go back and find out where they went wrong.

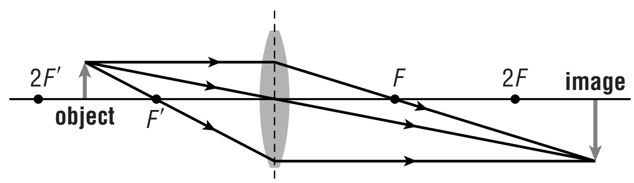
Extend and Assess

- Wrap up the lesson by asking questions based on Table 1. Ask, *If the image distance,* d_i*, is positive, is the image real or virtual?* (real) *On which side of the lens is the image?* (opposite side) *If the image distance,* d_i*, is negative, is the image real or virtual?* (virtual) *On which side of the lens is the image?* (same side) Repeat with similar questions, covering the height of the object and image, focal length, and magnification. It may be helpful to draw diagrams demonstrating each of the scenarios being addressed so that students can better connect the quantities with what they represent.
- You may wish to distribute *BLM 13.4-1 Lens Equations* to students. Thi s BLM contains questions to help students correctly use the equations from this section and contains additional problems for practice in using the equations.
- Have students complete the **Check Your Learning** questions on page 566 of the Student Book.

CHECK YOUR LEARNING

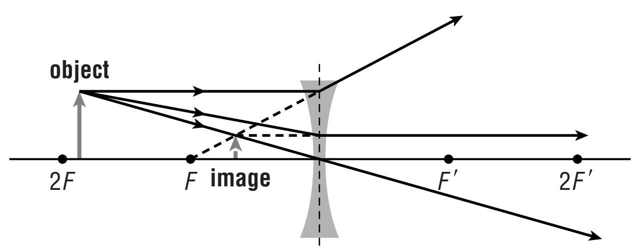
Suggested Answers

1.
$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$
$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}$$
$$\frac{1}{d_i} = \frac{1}{23\text{ cm}} - \frac{1}{32\text{ cm}}$$
$$\frac{1}{d_i} \doteq 0.0122\text{ cm}^{-1}$$
$$d_i \doteq 82\text{ cm}$$

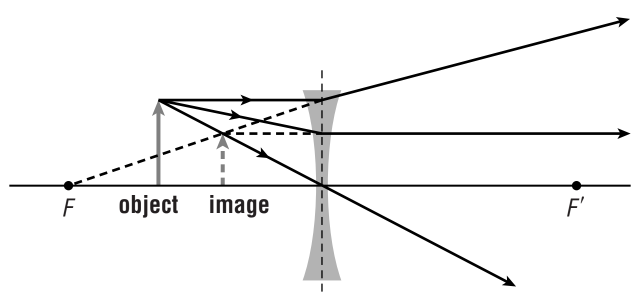


The image of the frog is 82 cm from the centre of the lens on the opposite side of the lens.

2.
$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$
$$\frac{1}{2.53\text{ cm}} + \frac{1}{-18\text{ cm}} = \frac{1}{f}$$
$$-0.0367\text{ cm}^{-1} \doteq \frac{1}{f}$$
$$-27\text{ cm} \doteq f$$



3.
$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$
$$\frac{1}{d_o} = \frac{1}{f} - \frac{1}{d_i}$$
$$\frac{1}{d_o} = \frac{1}{-34\text{ cm}} - \frac{1}{-13\text{ cm}}$$
$$\frac{1}{d_o} \doteq 0.0475\text{ cm}^{-1}$$
$$d_o \doteq 21\text{ cm}$$



The booklet is 21 cm from the centre of the lens on the same side as the image.

4. $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$

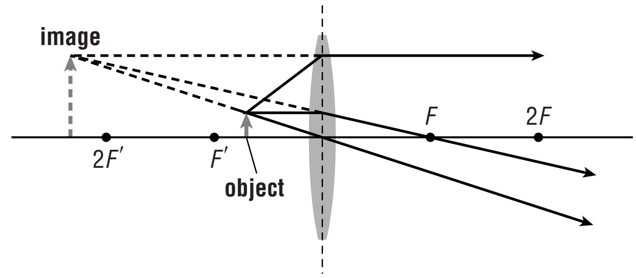
$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}$$

$$\frac{1}{d_i} = \frac{1}{16\text{ cm}} - \frac{1}{11\text{ cm}}$$

$$\frac{1}{d_i} \doteq -0.0284\text{ cm}^{-1}$$

$$d_i \doteq -35\text{ cm}$$

The image will be 35 cm from the lens on the same side as the object.

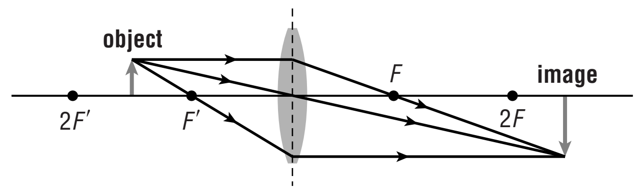


5. **(a)** $M = \frac{h_i}{h_o}$

$$= \frac{-35}{12}$$

$$\doteq -2.9$$

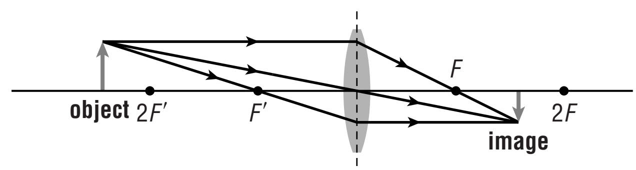
(b) The image is real.



6. $M = \frac{h_i}{h_o}$

$$= \frac{-7.9}{14}$$

$$\doteq -0.56$$

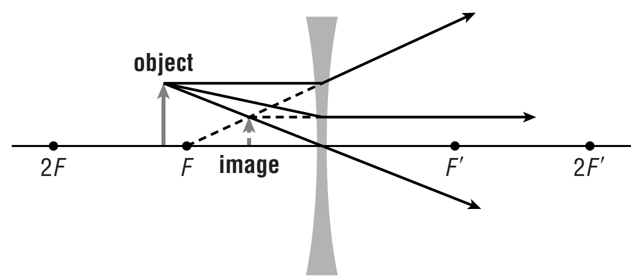


7. **(a)** $M = \frac{h_i}{h_o}$

$$= \frac{1.3}{2.8}$$

$$\doteq 0.46$$

(b) The image is upright.



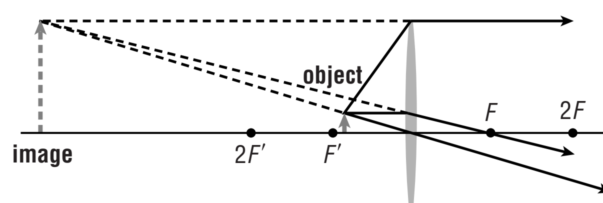
8. **(a)** $M = -\frac{d_i}{d_o}$

$$-Md_o = d_i$$

$$d_i = -Md_o$$

$$= -5.6(9.4\text{ cm})$$

$$\doteq -53\text{ cm}$$



The image is 53 cm from the centre of the lens on the same side as the object.

(b) $\frac{1}{d_o} + \frac{1}{d_1} = \frac{1}{f}$

$$\frac{1}{9.4\text{ cm}} + \frac{1}{-53\text{ cm}} = \frac{1}{f}$$

$$-0.0875\text{ cm}^{-1} \doteq \frac{1}{f}$$

$$11\text{ cm} \doteq f$$

(c) The lens is converging because it has a positive focal length. Also, only a converging lens can produce a virtual image that is larger than the original object. A diverging lens always produces a virtual image that is smaller than the original object.

DIFFERENTIATED INSTRUCTION

- Before going through the derivation of the thin lens equation on page 562 of the Student Book, draw Figure 2 on the board and have students identify the key measurements in Figure 2. Becoming familiar with each term and how it corresponds to the diagram should help students, especially visual/spatial learners, understand the derivation. Make a chart like the one below. Have visual/spatial learners identify and point out each variable and fill in the chart as shown.

f	f′	d_o	d_i	h_o	h_i
OF	*OF′*	*CO*	*OD*	*BC*	*DE*

- Recruit mathematical/logical learners to help explain the steps in the derivation. Visual/spatial learners may be helped by the use of coloured chalk or markers to keep track of equivalent quantities. Verbal/linguistic learners can summarize and restate steps taken so that all students understand.
- You might also suggest that logical/mathematical learners create additional problems (similar to the **Sample Problems**) for the class to solve. Require that each problem be submitted with a fully worked out solution. Have all members of the class solve these problems.
- Interpersonal learners will benefit from having partners take turns reading each of the elements of these illustrations and equations. They can repeat what was read to them or take turns reworking different equations as well as problems. Encourage partners to include the directions of light rays, the angles, and whether images are virtual or real.

ENGLISH LANGUAGE LEARNERS

- Focus on the words *object* and *image* as they are used in the diagrams. Explain that *object* in this context always refers to a real-world, three-dimensional item or thing, whereas *image* refers only to what is seen through the lens or projected on a sheet of paper in two dimensions.
- Review the formation of *magnification* from the verb or adjective *magnifying,* meaning making larger or something that makes a thing appear larger (magnifying glass.)