

Solve each radical equation. CHECK FOR EXTRANEEOUS SOLUTIONS

1. $3\sqrt{x} + 3 = 15$

$3\sqrt{x} = 12$

$\sqrt{x} = 4$

$\boxed{x = 16}$

Check

$3\sqrt{16} + 3 = 15$

$3(4) + 3 = 15$

$15 = 15 \checkmark$

3. $\sqrt{x+3} = 5$

$x+3 = 25$

$\boxed{x = 22}$

Check

$\sqrt{22+3} = 5$

$\sqrt{25} = 5$

$5 = 5 \checkmark$

2. $4\sqrt{x} - 1 = 3$

$4\sqrt{x} = 4$

$\sqrt{x} = 1$

$\boxed{x = 1}$

check

~~$4\sqrt{1} - 1 = 3$~~

$4\sqrt{1} - 1 = 3$

$4(1) - 1 = 3$

$3 = 3 \checkmark$

4. $\sqrt{3x+4} = 4$

$3x+4 = 16$

$3x = 12$

$\boxed{x = 4}$

$\sqrt{3(4)+4} = 4$

$\sqrt{12+4} = 4$

$\sqrt{16} = 4$

$4 = 4 \checkmark$

5. $\sqrt{2x+3} - 7 = 0$

$\sqrt{2x+3} = 7$

$2x+3 = 49$

$2x = 46$

$\boxed{x = 23}$

Check

$\sqrt{2(23)+3} - 7 = 0$

$\sqrt{46+3} - 7 = 0$

$\sqrt{49} - 7 = 0$

$7 - 7 = 0$

$0 = 0 \checkmark$

6. $\sqrt{6-3x} - 2 = 0$

check

$\sqrt{6-3x} = 2$

$6-3x = 4$

$-3x = -2$

$\boxed{x = 2/3}$

$\sqrt{6-3(2/3)} - 2 = 0$

$\sqrt{6-2} - 2 = 0$

$\sqrt{4} - 2 = 0$

$2 - 2 = 0$

$0 = 0 \checkmark$

15. $\sqrt{11x+3} - 2x = 0$

$\sqrt{11x+3} = 2x$

$11x+3 = 4x^2$

$0 = 4x^2 - 11x - 3$

Check

$\sqrt{11(3)+3} - 2(3) = 0$

$\sqrt{33+3} - 6 = 0$

$\sqrt{36} - 6 = 0$

$6 - 6 = 0$

$0 = 0 \checkmark$

$0 = (x-3)(4x+1)$

$\boxed{x = 3}$

~~$\boxed{x = -1/4}$~~

$\sqrt{11(-1/4)+3} - 2(-1/4) = 0$
extraneous $\sqrt{-11/4+3} + 1/2 \neq 0$

16) $\sqrt{5x+4} - 3x = 0$

$\sqrt{5x+4} = 3x$

$5x+4 = 9x^2$

$0 = 9x^2 - 5x - 4$

$0 = (x-9)(9x+4)$

~~$\boxed{x = 9}$~~

~~$\boxed{x = -4/9}$~~

check

$\sqrt{5(9)+4} - 3(9) = 0$

$\sqrt{49} - 27 = 0$

$7 - 27 = 0$

$-20 \neq 0$

~~$\sqrt{5(-4/9)+4} - 3(-4/9) = 0$~~
 $\sqrt{-20/9+4} + 4/3 = 0$
 $\sqrt{16/9} + 4/3 = 0$
 $4/3 + 4/3 = 0$
 $8/3 \neq 0$
NO solutions

17. $\sqrt{3x+13} - 5 = x$

$$\begin{aligned}\sqrt{3x+13} &= x+5 \\ 3x+13 &= (x+5)^2 \\ 3x+13 &= x^2+10x+25 \\ 0 &= x^2+7x+12 \\ 0 &= (x+3)(x+4) \\ \boxed{x=-3} \quad \boxed{x=-4}\end{aligned}$$

Check

$$\begin{aligned}\sqrt{3(-3)+13} - 5 &= -3 \\ \sqrt{4} - 5 &= -3 \\ 2 - 5 &= -3 \\ -3 &= -3 \checkmark \\ \sqrt{3(-4)+13} - 5 &= -4 \\ \sqrt{1} - 5 &= -4 \\ 1 - 5 &= -4 \\ -4 &= -4 \checkmark\end{aligned}$$

18. $\sqrt{x+7} + 5 = x$

Check

$$\begin{aligned}\sqrt{x+7} &= x-5 \\ x+7 &= (x-5)^2 \\ x+7 &= x^2-10x+25 \\ 0 &= x^2-11x+18 \\ 0 &= (x-9)(x-2) \\ \boxed{x=9} \quad \boxed{x=2}\end{aligned}$$

$$\begin{aligned}\sqrt{9+7} + 5 &= 9 \\ \sqrt{16} + 5 &= 9 \\ 4 + 5 &= 9 \\ 9 &= 9 \checkmark \\ \sqrt{2+7} + 5 &= 2 \\ \sqrt{9} + 5 &= 2 \\ 3 + 5 &\neq 2\end{aligned}$$

19) $\sqrt{x+3} - 1 = x$

$$\begin{aligned}\sqrt{x+3} &= x+1 \\ x+3 &= (x+1)^2 \\ x+3 &= x^2+2x+1 \\ 0 &= x^2+x-2 \\ 0 &= (x+2)(x-1) \\ \boxed{x=-2} \quad \boxed{x=1}\end{aligned}$$

$$\begin{aligned}\sqrt{-2+3} - 1 &= -2 \\ \sqrt{1} - 1 &= -2 \\ 1 - 1 &= -2 \\ 0 &\neq -2 \\ \sqrt{1+3} - 1 &= 1 \\ \sqrt{4} - 1 &= 1 \\ 2 - 1 &= 1 \\ 1 &= 1 \checkmark\end{aligned}$$

20) $\sqrt{5-x} = x+1$

Check

$$\begin{aligned}5-x &= (x+1)^2 \\ 5-x &= x^2+2x+1 \\ 0 &= x^2+3x-4 \\ 0 &= (x+4)(x-1) \\ \boxed{x=-4} \quad \boxed{x=1}\end{aligned}$$

$$\begin{aligned}\sqrt{5+4} &= -4+1 \\ \sqrt{9} &= -3 \\ 3 &\neq 3 \\ \sqrt{5-1} &= 1+1 \\ \sqrt{4} &= 2 \\ 2 &= 2 \checkmark\end{aligned}$$

Simplify the following expressions using the properties of exponents.

26) $\frac{2x^4y^{-4}z^{-3}}{3x^2y^{-3}z^4}$

$$\begin{aligned}&\frac{2x^4y^{-4}z^{-3}}{3x^2y^{-3}z^4} \\ &= \frac{2x^4y^{-4}z^{-3}}{3x^2y^{-3}z^4} = \frac{2x^2y^{-1}z^{-7}}{3z^7} \\ &= \frac{2x^2}{3yz^7}\end{aligned}$$

28) $\frac{2h^3j^{-3}k^4}{3jk}$

$$\begin{aligned}&= \frac{2h^3k^4}{3j^4k} = \frac{2h^3k^3}{3j^4}\end{aligned}$$

30) $\frac{3x^3y^{-1}z^{-1}}{x^{-4}y^0z^0}$

$$\begin{aligned}&\frac{3x^3y^{-1}z^{-1}}{x^{-4}y^0z^0} \\ &= \frac{3x^7}{yz}\end{aligned}$$

Convert between rational exponent and radical forms, or vice versa.

19) $(\sqrt[4]{m})^3$

$$\boxed{m^{3/4}}$$

23) $(\sqrt[3]{3a})^4$

$$\boxed{(3a)^{4/3}}$$

4) $7^{4/3}$

$$\boxed{\sqrt[3]{7^4}}$$

14) $(5x)^{-1/2}$

$$\frac{1}{(5x)^{1/2}} = \boxed{\frac{1}{\sqrt{5x}}}$$