

Solving Radical Equations Notes

Definition:

Extraneous Solution: A solution of a simplified version of an equation that does not satisfy the original equation.

Example: Solving with Radicals on One Side of the Equation
BE SURE TO ALWAYS CHECK FOR EXTRANEIOUS SOLUTIONS

1) $\sqrt{x} + 3 = 10$

$$\begin{array}{r} -3 \quad -3 \\ \hline (\sqrt{x})^2 = (7)^2 \\ \hline \boxed{x = 49} \end{array}$$

Check

$$\begin{aligned} \sqrt{49} + 3 &= 10 \\ 7 + 3 &= 10 \\ 10 &= 10 \quad \checkmark \end{aligned}$$

2) $-3\sqrt{x} = -18$

$$\begin{array}{r} -3 \quad -3 \\ \hline (\sqrt{x})^2 = (6)^2 \\ \hline \boxed{x = 36} \end{array}$$

Check

$$\begin{aligned} -3\sqrt{36} &= -18 \\ -3(6) &= -18 \\ -18 &= -18 \quad \checkmark \end{aligned}$$

3) $2\sqrt{x} - 5 = -1$

$$\begin{array}{r} +5 \quad +5 \\ \hline 2\sqrt{x} = 4 \\ \hline \frac{2}{2} \quad \frac{4}{2} \\ (\sqrt{x})^2 = (2)^2 \\ \hline \boxed{x = 4} \end{array}$$

Check

$$\begin{aligned} 2\sqrt{4} - 5 &= -1 \\ 2(2) - 5 &= -1 \\ 4 - 5 &= -1 \\ -1 &= -1 \quad \checkmark \end{aligned}$$

Example: Solving with Radicals on Both Sides of the Equation
BE SURE TO ALWAYS CHECK FOR EXTRANEEOUS SOLUTIONS

$$1) (\sqrt{x+4})^2 = (\sqrt{2x-1})^2$$

$$\begin{array}{r} x+4 = 2x-1 \\ -x \quad -x \\ \hline 4 = x-1 \\ +1 \quad +1 \\ \hline \boxed{5 = x} \end{array}$$

Check

$$\begin{aligned} \sqrt{5+4} &= \sqrt{2(5)-1} \\ \sqrt{9} &= \sqrt{10-1} \\ \sqrt{9} &= \sqrt{9} \\ 3 &= 3 \checkmark \end{aligned}$$

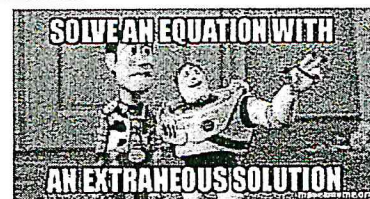
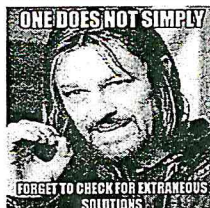
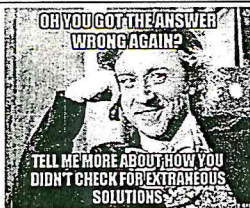
$$2) (\sqrt{3x+8})^2 = (\sqrt{x+4})^2$$

$$\begin{array}{r} 3x+8 = x+4 \\ -x \quad -x \\ \hline 2x+8 = 4 \\ -8 \quad -8 \\ \hline 2x = -4 \\ \frac{2x}{2} = \frac{-4}{2} \\ \boxed{x = -2} \end{array}$$

Check

$$\begin{aligned} \sqrt{3(-2)+8} &= \sqrt{-2+4} \\ \sqrt{-6+8} &= \sqrt{2} \\ \sqrt{2} &= \sqrt{2} \checkmark \end{aligned}$$

BRACE YOURSELF



$$3) (x)^2 = (\sqrt{42-x})^2$$

$$\begin{array}{r} x^2 = 42 - x \\ -42 + x - 42 + x \\ \hline x^2 + x - 42 = 0 \end{array}$$

FACTOR

$$\begin{array}{r} x^2 + x - 42 = 0 \\ 7 \times -6 \\ \hline 1 \end{array}$$

$$\begin{aligned} x^2 + x - 42 &= 0 \\ (x+7)(x-6) &= 0 \end{aligned}$$

$$\begin{aligned} x+7 &= 0 & x-6 &= 0 \\ \boxed{x = -7} & \leftarrow \boxed{x = 6} \end{aligned}$$

extraneous solution

Check

$$\begin{aligned} -7 &= \sqrt{42-(-7)} \\ -7 &= \sqrt{49} \\ -7 &= 7 \text{ FALSE} \end{aligned}$$

$$\begin{aligned} 6 &= \sqrt{42-6} \\ 6 &= \sqrt{36} \\ 6 &= 6 \checkmark \end{aligned}$$

$$4) (\sqrt{2-x})^2 = (x+4)^2$$

$$\begin{aligned} 2-x &= (x+4)^2 \\ 2-x &= (x+4)(x+4) \\ 2-x &= x^2 + 4x + 4x + 16 \\ 2-x &= x^2 + 8x + 16 \\ -2+x & \quad +x \quad -2 \\ \hline 0 &= x^2 + 9x + 14 \end{aligned}$$

$$\begin{array}{r} 14 \times 2 \\ 7 \times 9 \\ \hline 1 \end{array}$$

$$\begin{aligned} 0 &= (x+7)(x+2) \\ x+7 &= 0 & x+2 &= 0 \\ \boxed{x = -7} & \quad \boxed{x = -2} \end{aligned}$$

Extraneous solution

Check

$$\begin{aligned} \sqrt{2-(-7)} &= -7+4 \\ \sqrt{9} &= -3 \\ 3 &\neq -3 \end{aligned}$$

$$\begin{aligned} \sqrt{2-(-2)} &= -2+4 \\ \sqrt{4} &= 2 \\ 2 &= 2 \checkmark \end{aligned}$$