Exponent Properties Notes

Name: KEY

OBJECTIVE: Use patterns to identify integer exponent properties. Represent properties in symbols and words and use them to simplify expressions.

Equivalent Forms	
Use the definition of an exponent to write expressions that are equivalent (equal) to the ones below.	
$x^7 = x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x$	$a \cdot a \cdot a \cdot a = a^4$
All integer exponent properties are an expansion of this definition. We will use this definition as a building block to acquire knowledge about other properties.	

Multiplication Property		
Rewrite each expression using only your knowledge of the definition of an exponent.		
$y^3 \cdot y^2 = y \cdot y \cdot y \cdot y \cdot y = y^5$	$a^3 \cdot a = a \cdot a \cdot a \cdot a = a^4$	
$m \cdot m^3 \cdot m^5 = m \cdot m \cdot m \cdot m \cdot m \cdot m \cdot m \cdot m \cdot m = m^9$	$b^2 \cdot b^6 = b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b = b^8$	
Property: $x^a \cdot x^b = x^{(a+b)}$		
In words: When we multiply two terms with the same base we add the exponents.		
$w^3 \cdot w^{34} = w^{37}$	$a^3 \cdot a \cdot a^{12} = a^{16}$	$x^m \cdot x^n = x^{m+n}$

Power to a Power Property		
Rewrite each expression using only your knowledge of the definition of an exponent and the multiplication property.		
$(m^3)^2 = m^3 \cdot m^3 = m^6$	$((x^2)^2)^3 = x^2 \cdot x^2 \cdot x^2 \cdot x^2 \cdot x^2 \cdot x^2 = x^{12}$	
$(\theta^2)^4 = \theta^2 \cdot \theta^2 \cdot \theta^2 \cdot \theta^2 = \theta^8$	$(y^5)^1 = y^5$	
Property: $(x^a)^b = x^{a \cdot b}$		
In words: when a power is applied to a base with a preexisting power, multiply the exponents		
$(w^4)^{12} = w^{48}$	$((x^2)^4)^3 = x^{24}$	$((w^a)^b)^c = w^{abc}$

Distributive Power Property

Rewrite each expression using only your knowledge of the definition of an exponent and the multiplication property.

$(2y^3)^2 = 2y^3 \cdot 2y^3 = 4y^6$	$(5x^3w^3)^2 = 5x^3w^3 \cdot 5x^3w^3 = 25x^6w^6$
$(4z^2)^3 = 4z^2 \cdot 4z^2 \cdot 4z^2 = 64z^6$	$(4m^{10})^2 = 4m^{10} \cdot 4m^{10} = 16m^{20}$

Property: $(mx^a)^b = m^b \cdot x^{a \cdot b}$

In words: when multiple items inside a set of parenthses are raised to a power, each item is raised to that power.

$(6w^4)^2 = 36w^8$	$(3xy)^3 = 27x^3y^3$	$(5w^{11})^2 = 25w^{22}$

Division Property of Exponents

Rewrite each numerator and denominator using the definition of an exponent. Then simplify by finding quotients equal to 1.

$\frac{z^5}{z^2} = \frac{\cancel{z} \cdot \cancel{z} \cdot z \cdot z \cdot z}{\cancel{z} \cdot \cancel{z}} = z^3$	$\frac{x^5}{x} = \frac{\cancel{x} \cdot x \cdot x \cdot x \cdot x}{\cancel{x}} = x^4$
$\frac{w^3}{w^2} = \frac{\cancel{w} \cdot \cancel{w} \cdot w}{\cancel{w} \cdot \cancel{w}} = w^1$	$\frac{a^5}{a} = \frac{\cancel{a} \cdot a \cdot a \cdot a \cdot a}{\cancel{a}} = a^4$

Property: $\frac{x^a}{x^b} = x^{a-b}$

In words: when dividing two terms with the same base, subtract their exponents

$\frac{(ab)^5}{(ab)^3} = (ab)^2$	$\frac{6x^{105}}{3x^{10}} = 2x^{95}$	$\frac{x^3y^{11}}{xy^6} = x^2y^5$

Negative Power Property

We will use two different methods to simplify the same expression. The results will give us an idea about negative exponents.

Simplify using the division property.	Rewrite each numerator and denominator using the definition of an exponent. Then simplify by finding quotients equal to 1.
$\frac{y^3}{y^7} = y^{-4}$	$\frac{y^3}{y^7} = \frac{\cancel{y}\cdot\cancel{y}\cdot\cancel{y}}{\cancel{y}\cdot\cancel{y}\cdot\cancel{y}\cdot y\cdot y\cdot y\cdot y} = \frac{1}{y^4}$
$\frac{x}{x^7} = x^{-6}$	$\frac{x}{x^7} = \frac{\cancel{x}}{\cancel{x}\cdot x\cdot x\cdot x\cdot x\cdot x\cdot x} = \frac{1}{x^6}$

What About $\frac{1}{x^{-3}}$?

$$\frac{1}{x^{-3}} = x^3$$

Property:

$$x^{-a} = \frac{1}{x^a} \quad \text{and} \quad \frac{1}{x^{-a}} = x^a$$

In words: When you have a negative exponent move it to the numerator or denominator.

$\frac{-2x^{-5}}{4} = \frac{-1}{2x^5}$	$\frac{x^4y^5}{x^6y^{-2}} = \frac{x^4y^5y^2}{x^6} = \frac{y^7}{x^2}$	$\frac{x^{-3}y^{-11}}{x^{-1}y^{-6}} = \frac{x^1y^6}{x^3y^{11}} = \frac{1}{x^2y^5}$

Practice & Review

$\frac{x^4x^5y^{-2}}{x^6y} = \frac{x^9}{x^6yy^2} = \frac{x^3}{y^3}$	$\frac{14x^4y^3}{21x^{12}y^4z^5} = \frac{2}{3x^8yz^5}$

Zero Power Property

Property:	for any $x \neq 0$, $x^0 = 1$
In words:	any non-zero base raised to the power of 0 is equivalent to 1

Justification #1 - Using Multiplication

$x^3 * x^0 = x^{3+0} = x^3$

Justification #2 - Using Division

$\frac{w^5}{w^0} = w^{5-0} = w^5$

Justification #3 – A Comparison

Simplify using the division property.	Simplify by expanding and finding quotients equal to 1.	
$\frac{y^5}{y^5} = y^{5-5} = y^0$	$\frac{y^5}{y^5} = \frac{\cancel{y \cdot y \cdot y \cdot y \cdot y}}{\cancel{y \cdot y \cdot y \cdot y \cdot y}} = 1$	So $y^0 = 1$

Justification #4 – Observing a Pattern

$3^3 = 27$
$3^2 = 9$
$3^1 = 3$
$3^0 = 1$
$3^{-1} = 1/3$

Simplifying Expressions

1	"Distribute" exponents where necessary
2	use multiplication properties w/like bases in numerator/denominator.
3	apply the negative exponent property
4	use division exponent property to make sure only 1 of each base is left.

$\left(\frac{y^{-1}z^5}{y^5}\right)^{-1} = \frac{y^1 \cdot z^{-5}}{y^{-5}} = \frac{y^1 \cdot y^5}{z^5} = \frac{y^6}{z^5}$

$\frac{(3x^2)^3 a^{-12} b^2}{(a^{-2})^{-1} b^3 x^0} = \frac{27x^6 a^{-12} b^2}{a^2 b^3} = \frac{27x^6 b^2}{a^2 a^{12} b^3}$

$= \frac{27x^6}{a^{14} b^1}$