

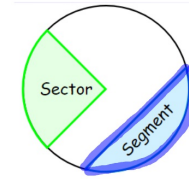
Wednesday, April 22, 2020

Sec 10-7: Area of a Segment of a Circle

A **Sector** of a circle is the part of a circle's area formed by two radii (slice of pizza)

A **Segment** of a circle is the part of a circle's area formed by an arc and the segment connecting the endpoints of that arc.

The diagram at the right shows examples of a **Sector** and a **Segment**.

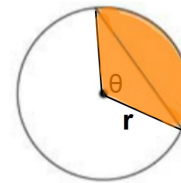


How to find the area of a **Segment** of a circle.

The diagram shows a circle with a small orange segment. An arrow points to it with the label 'Segment of a circle'. This is followed by an equals sign, then a circle with an orange sector, a minus sign, and a circle with an orange triangle. Below this, the formula is written:

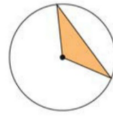
$$A_{\text{segment}} = A_{\text{sector}} - A_{\text{triangle}}$$

Area of a Sector:

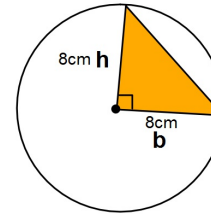


$$\frac{m\text{Central}\angle\theta}{360^\circ} = \frac{\text{area of Sector}}{\pi r^2}$$

Let's focus for a minute on how to find the area of the triangular portion of the figure.



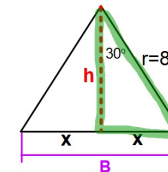
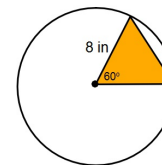
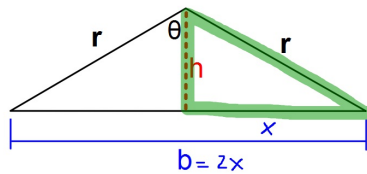
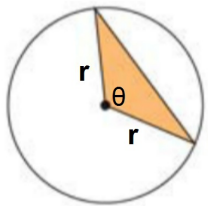
Area of the Δ :



$$A = \frac{1}{2}bh$$

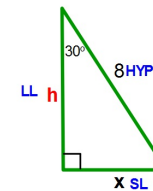
$$A = \frac{1}{2}(8)(8) = 32 \text{ cm}^2$$

If Central Angle isn't 90°: Area of the Δ



$$A = \frac{1}{2}bh$$

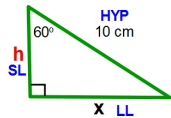
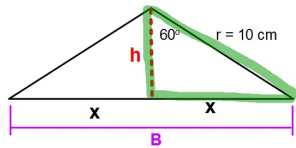
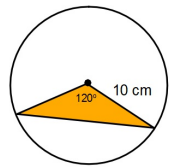
$$A = \frac{1}{2}(8)(4\sqrt{3}) = 16\sqrt{3} \approx 27.71 \text{ in}^2$$



$$x = SL = HYP \div 2 = 4$$

$$B = 2x = 2(4) = 8$$

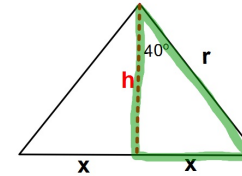
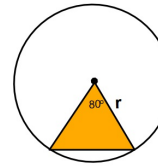
$$h = LL = SL\sqrt{3} = 4\sqrt{3}$$



$$\begin{aligned}
 h &= SL = \text{HYP} \div 2 = 5 \\
 x &= LL = SL \sqrt{3} = 5\sqrt{3} \\
 B &= 2x = 2(5\sqrt{3}) = 10\sqrt{3} \\
 A &= \frac{1}{2}(10\sqrt{3})(5) \\
 &= 25\sqrt{3} \approx 43.30 \text{ cm}^2
 \end{aligned}$$

Now let's put this all together
and find the area of each **Segment**.

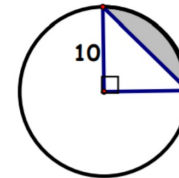
What if the Central Angle doesn't lead to a Special Right Δ ?



To find **h** and **x** we would need to use

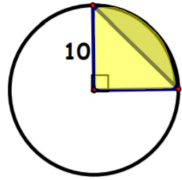
SOHCAHTOA

Find the area of this shaded segment to the nearest hundredth.



Find the area of this shaded segment to the nearest hundredth.

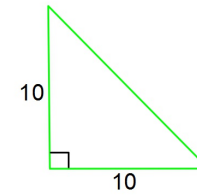
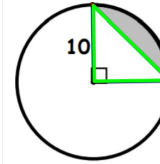
1st: Find the area of the Sector



- Area of the circle = $\pi(10)^2 = 100\pi$
- Set up the Proportion for the Sector:

$$\frac{90^\circ}{360^\circ} = \frac{x}{100\pi} \quad x = 78.54$$

2nd: Find the area of the Δ :



$$A = \frac{1}{2}(10)(10) = 50$$

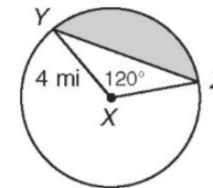
3rd: Put these two together to find the Area of the Segment:

$$A_{\text{segment}} = A_{\text{sector}} - A_{\text{triangle}}$$

$$= 78.54 - 50$$

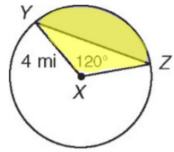
$$= 28.54 \text{ units}^2$$

Find the area of this shaded segment to the nearest hundredth.



Find the area of this shaded segment to the nearest hundredth.

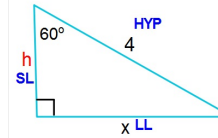
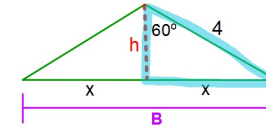
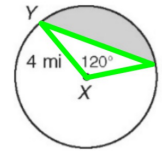
1st: Find the area of the Sector



- Area of the circle = $\pi(4)^2 = 16\pi$
- Set up the Proportion for the Sector:

$$\frac{120^\circ}{360^\circ} = \frac{x}{16\pi} \quad x = 16.76 \text{ mi}^2$$

2nd: Find the area of the Δ :



$$\begin{aligned} h &= SL = HYP \div 2 = 2 \\ x &= LL = SL \sqrt{3} = 2\sqrt{3} \\ B &= 2x = 2(2\sqrt{3}) = 4\sqrt{3} \end{aligned}$$

$$\begin{aligned} A &= \frac{1}{2}(4\sqrt{3})(2) \\ &= 4\sqrt{3} \approx 6.93 \text{ mi}^2 \end{aligned}$$

3rd: Put these two together to find the Area of the Segment:

$$\begin{aligned} A_{\text{segment}} &= A_{\text{sector}} - A_{\text{triangle}} \\ &= 16.76 - 6.93 \\ &= 9.83 \text{ mi}^2 \end{aligned}$$

You can now do Practice #20.