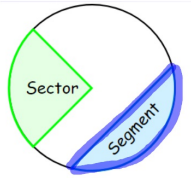
Wednesday, April 22, 2020

Sec 10-7: Area of a Segment of a Circle

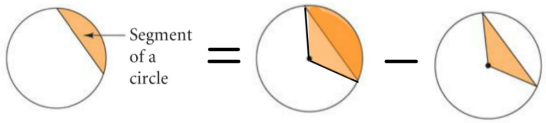
A Sector of a circle is the part of a circles area formed by two radii (slice of pizza)

A Segment of a circle is the part of a circles area formed by an arc and the segment connecting the endpoints of that arc.

The diagram at the right shows examples of a Sector and a Segment.

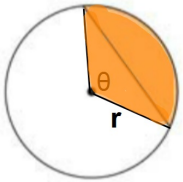


How to find the area of a Segment of a circle.



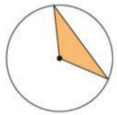
$$A_{segment} = A_{sector} - A_{triangle}$$

Area of a Sector:

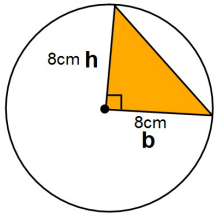


$$\frac{m\text{Central}\angle\theta}{360^\circ} = \frac{\text{area of Sector}}{\pi r^2}$$

Let's focus for a minute on how to find the area of the triangular portion of the figure.



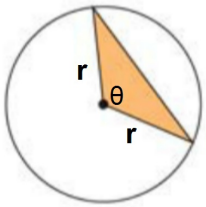
Area of the Δ:

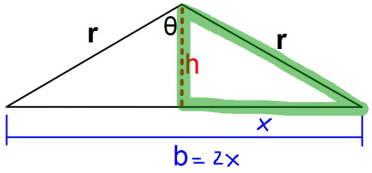


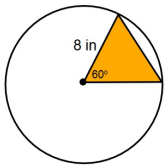
$A = \frac{1}{2}bh$

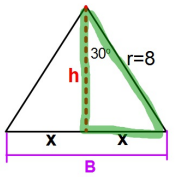
$A = \frac{1}{2}(8)(8) = 32\ \text{cm}^2$

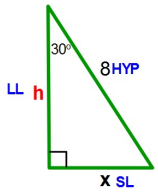
If Central Angle isn't 90°: Area of the Δ











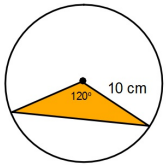
$A = \frac{1}{2}bh$

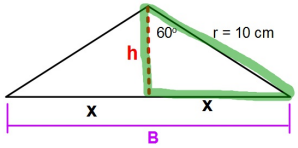
$A = \frac{1}{2}(8)(4\sqrt{3}) = 16\sqrt{3} \approx 27.71\ \text{in}^2$

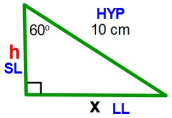
X = SL = HYP÷2 = 4

B = 2x = 2(4) = 8

h = LL = SL√3 = 4√3







h = SL = HYP÷2 = 5

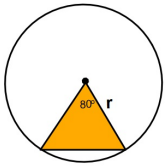
x = LL = SL√3 = 5√3

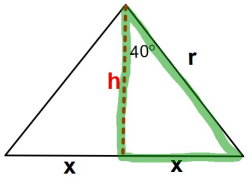
B = 2x = 2(5√3) = 10√3

$A = \frac{1}{2}(10\sqrt{3})(5)$

$= 25\sqrt{3} \approx 43.30 \text{ cm}^2$

What if the Central Angle doesn't lead to a Special Right Δ?



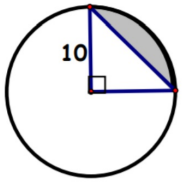


To find **h** and **x** we would need to use

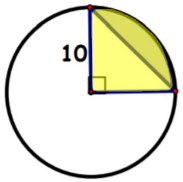
SOHCAHTOA

Now let's put this all together
and find the area of each Segment.

Find the area of this shaded segment to the nearest hundredth.



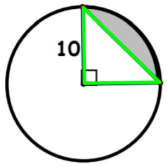
Find the area of this shaded segment to the nearest hundredth.



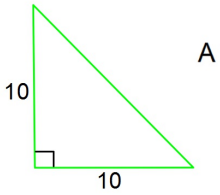
1st: Find the area of the Sector

- Area of the circle = $\pi(10)^2 = 100\pi$
- Set up the Proportion for the Sector:

$$\frac{90°}{360°} = \frac{x}{100\pi} \qquad x = 78.54$$



2nd: Find the area of the Δ:



$$A = \frac{1}{2}(10)(10) = 50$$

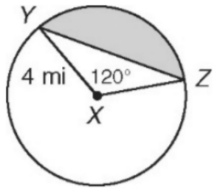
3rd: Put these two together to find the Area of the Segment:

$$A_{segment} = A_{sector} - A_{triangle}$$

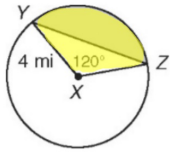
$$= 78.54 - 50$$

$$= 28.54 \text{ units}^2$$

Find the area of this shaded segment to the nearest hundredth.



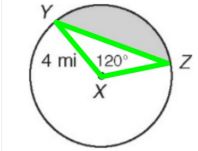
Find the area of this shaded segment to the nearest hundredth.



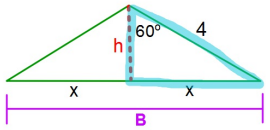
1st: Find the area of the Sector

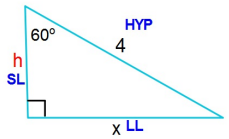
- Area of the circle = $\pi(4)^2 = 16\pi$
- Set up the Proportion for the Sector:

$$\frac{120^\circ}{360^\circ} = \frac{x}{16\pi} \qquad x = 16.76 \text{ mi}^2$$



2nd: Find the area of the Δ:





h = SL = HYP÷2 = 2

x = LL = SL√3 = 2√3

B = 2x = 2(2√3) = 4√3

$$A = \frac{1}{2}(4\sqrt{3})(2)$$

$$= 4\sqrt{3} \approx 6.93 \text{ mi}^2$$

3rd: Put these two together to find the Area of the Segment:

$$A_{segment} = A_{sector} - A_{triangle}$$

$$= 16.76 - 6.93$$

$$= 9.83 \text{ mi}^2$$

You can now do Practice #20.