Monday, April 20, 2020

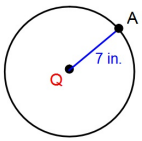
Sec 10-6: Circumference and Arc Length.

Circumference of a Circle:

The distance around the outside of a circle.

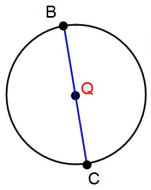
$C = \pi d$ or $C = 2\pi r$

Find the circumference of each circle. Leave your answer in terms of π. Q is the center of each circle.



AQ is a radius so use $C=2\pi r$

$C = 2\pi(7) = 14\pi$ in

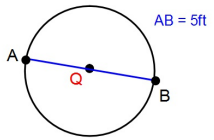


BC = 23 cm

BC is a diameter so use $C=\pi d$

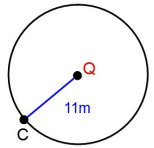
$C = \pi(23) = 23\pi$ cm

Find the circumference of each circle. Round answers to the nearest hundredth. Q is the center of each circle.



AB is a diameter so use $C=\pi d$

$C = \pi(5) = 15.71$ ft



CQ is a radius so use $C=2\pi r$

$C = 2\pi(11) = 69.12$ m

Given the circumference of a circle is 50 cm, find it's radius to the nearest hundredth.

$C = 2\pi r \longrightarrow \frac{50}{2\pi} = \frac{2\pi r}{2\pi}$

$r = \frac{50}{2\pi} = 7.96 \text{ cm}$

$50 \div (2\pi)$

Given the circumference of a circle is 288π in, find it's diameter.

$C = \pi d \longrightarrow \frac{288\pi}{\pi} = \frac{\pi d}{\pi}$

$d = 288 \text{ in}$

The measure of an arc is a number of degrees that represents how much of a whole circle we have, how much out of a possible 360°.

This is different than the LENGTH of an arc.

Arc Length:

The distance between two points on the circle as you trace around the outside of the circle.

It's a portion of the circle's circumference.

Given in units of length such as in., cm., ft., ...

To find the length of an arc you can use a proportion.

Just think of this ratio: $\frac{\text{part of a circle}}{\text{the whole circle}}$

Degrees — Distance

$$\frac{\text{part of a circle in degrees}}{\text{the whole circle in degrees}} = \frac{\text{part of the circumference}}{\text{the whole circumference}}$$

Arc Length proportion:

$$\frac{\text{measure of a Central } \angle}{360^\circ} = \frac{\text{Arc Length } (x)}{\text{circumference } (2\pi r \text{ or } \pi d)}$$

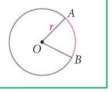
Remember the measure of a Central Angle is the same as the measure of it's corresponding arc.

The textbook gives this formula for Arc Length:

Theorem 10-10 Arc Length

The length of an arc of a circle is the product of the ratio $\frac{\text{measure of the arc}}{360}$ and the circumference of the circle.

$$\text{length of } \overset{\frown}{AB} = \frac{m\overset{\frown}{AB}}{360} \cdot 2\pi r$$

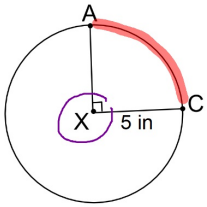


This formula is same as the proportion

$$\frac{\text{measure of a Central } \angle}{360^\circ} = \frac{\text{Arc Length } (x)}{\text{circumference } (2\pi r \text{ or } \pi d)}$$

where both sides were multiplied by the circumference

Find the length of $\overset{\frown}{AC}$ to the nearest tenth.



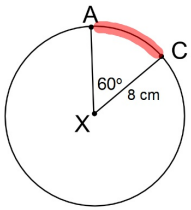
1st: Find the circumference: $C = 2\pi(5) = 10\pi$

2nd: Set up the proportion:

$$\frac{90^\circ}{360^\circ} = \frac{x}{10\pi}$$

3rd: Cross-multiply and round: x = 7.9 in

Find the length of $\overset{\frown}{AC}$ to the nearest tenth.



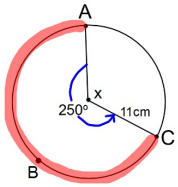
1st: Find the circumference: $C = 2\pi(8) = 16\pi$

2nd: Set up the proportion:

$$\frac{60^\circ}{360^\circ} = \frac{x}{16\pi}$$

3rd: Cross-multiply and round: x = 8.4 cm

Find the length of $\overset{\frown}{ABC}$ to the nearest tenth.



1st: Find the circumference: $C = 2\pi(11) = 22\pi$

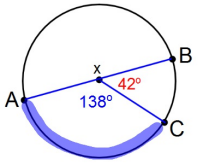
2nd: Set up the proportion:

$$\frac{250^\circ}{360^\circ} = \frac{x}{22\pi}$$

3rd: Cross-multiply and round: x = 48.0 cm

Find the length of $\overset{\frown}{AC}$ to the nearest tenth.

$\overline{AB}$ is a diameter. AB = 6 ft



1st: Find Circumference: $C = \pi d = 6\pi$

2nd: Find the measure of $\angle AXC = \overset{\frown}{AC}$

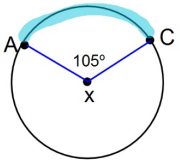
$\overset{\frown}{AC} = 180° - \overset{\frown}{BC} = 180° - 42° = 138°$

3rd: Set up and solve Proportion:

$$\frac{138^\circ}{360^\circ} = \frac{x}{6\pi}$$

x = 7.23 ft

Given the length of $\overset{\frown}{AC}$ is 45 in, find the diameter to nearest tenth.



Set up the proportion with the known information.

$$\frac{105^\circ}{360^\circ} = \frac{45}{\text{circumference}}$$

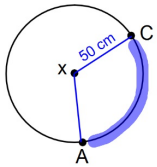
Cross multiply to solve for the circumference:

C= 154.3 in → $\pi d = 154.3$

divide both sides by π to get:

d = 49.1 in

Given the length of $\overset{\frown}{AC}$ is 87 cm, find the measure of $\angle AXC$ to nearest tenth



1st: Find Circumference: $C = 2\pi r = 2\pi(50) = 100\pi$

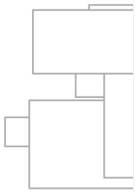
2nd: Set up proportion with known information.

$$\frac{x}{360^\circ} = \frac{87 \text{ cm}}{100\pi}$$

3rd: Cross-multiply to solve for x: x = 99.7

this value of x represents $\angle AXC = \overset{\frown}{AC}$

$\angle AXC = 99.7^\circ$



You can now do Practice #18 which is on my blog.