Tuesday, March 31, 2020

Sec 10-3: Angles in Regular Polygons

Regular Polygon:

- All sides are congruent (equilateral)
- All interior angles are congruent (equiangular)

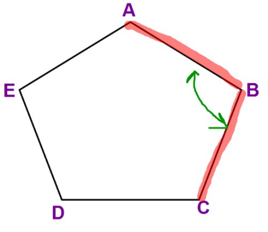
Two Regular Polygons you already know are:

- Equilateral Triangle
- Square

Interior Angle of a Regular Polygon:

Angle formed by consecutive sides.

In Pentagon ABCDE, ∠ABC is an interior angle because it is formed by sides $\overline{AB}$ and $\overline{BC}$.



From Sec 3-5

Polygon Angle-Sum Theorem

The sum of the measures of the interior angles of an n-gon is $(n-2)180$.

Find the sum of the interior angles of the given polygon.

1. A hexagon.
   6-sided polygon

$$Sum = (6 - 2)180 = 720°$$

2. A decagon.
   10-sided polygon

$$Sum = (10 - 2)180 = 1440°$$

3. A 15-gon.
   15-sided polygon

$$Sum = (15 - 2)180 = 2340°$$

In a regular polygon all interior angles are congruent.

Find the measure of one interior angle of each regular polygon.

1. A pentagon.
   5-sided polygon

$$Sum = (5 - 2)180 = 540°$$

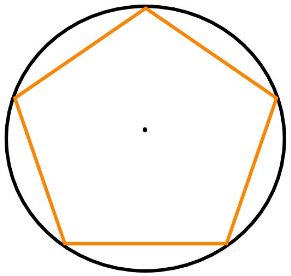
each interior angle $= \frac{540°}{5} = \boxed{108°}$

2. A dodecagon.
   12-sided polygon

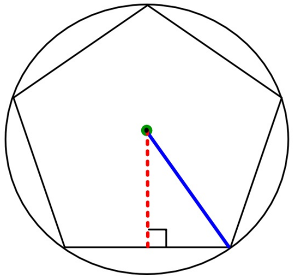
each interior angle $= \frac{(n-2)180}{n}$

$= \frac{(12-2)180}{12} = \boxed{150°}$

The circle is circumscribed about the pentagon.



The circle just touches all the vertices of the Pentagon.



Center of a Regular Polygon

The center of the circumscribed circle

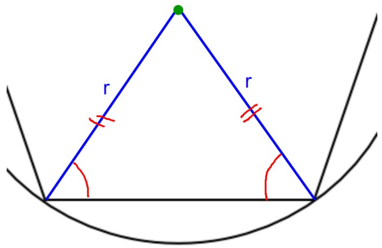
Radius of a Regular Polygon

Segment connecting the center to to a vertex.

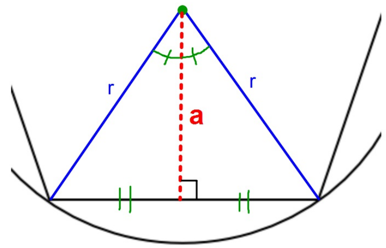
Apothem

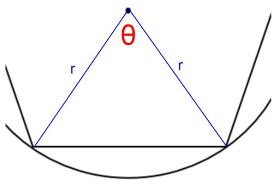
Segment that represents the perpendicular distance from the center to one of the sides.

If you draw two consecutive radii you can create an isosceles triangle with the included side.



If you draw the apothem in this triangle you bisect the base and the angle at the center of the circle.

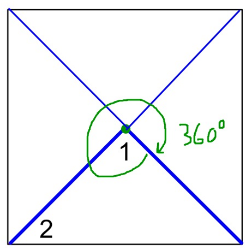




Central Angle of a Regular Polygon:
Angle whose vertex is at the center of the circumscribed circle.

Angle θ is a Central Angle in this figure.

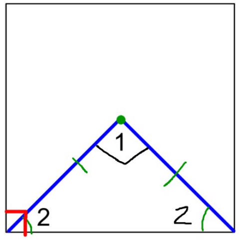
What is the measure of each numbered angle of this Square?



For $\angle 1$:

If you draw all 4 radii you create
4 ≅ Central Angles

$\angle 1 = \frac{360°}{4} = \boxed{90°}$



For ∠2:

The triangle shown is Isosceles which means ∠2 and the third angle are ≅.

$\angle 2 = \frac{180° - \angle 1}{2}$

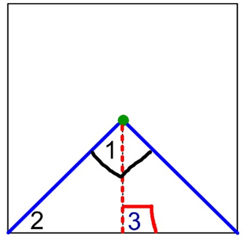
$= \frac{180° - 90°}{2} = \boxed{45°}$

OR

Each radius bisects an interior angle. So if you know the interior angle of the polygon you can just ÷2.

$\angle 2 = \frac{90°}{2} = \boxed{45°}$

What is the measure of each numbered angle after you draw the apothem?



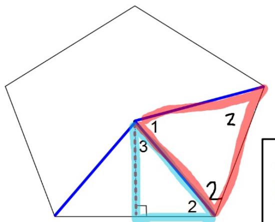
∠2 is still = $\boxed{45°}$

∠3 = $\boxed{90°}$ because the apothem is ⊥ to the side.

∠1 is half of what it used to be (the apothem bisects it)

∠1=90°÷2 = $\boxed{45°}$

Find the measure of each numbered angle in this regular pentagon.



$\angle 1 = \frac{360°}{5} = \boxed{72°}$

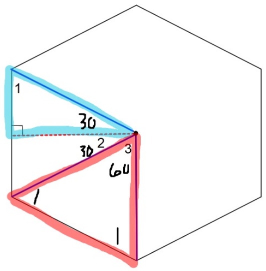
$\angle 3 = \frac{\angle 1}{2} = \frac{72°}{2} = \boxed{36°}$

$\angle 2 = 180 - 90 - 36 = \boxed{54°}$

or

$\angle 2 = \frac{180 - \angle 1}{2} = \frac{180 - 72}{2} = \boxed{54°}$

Find the measure of each numbered angle in this regular hexagon.



$\angle 3 = \frac{360°}{6} = \boxed{60°}$

$\angle 2 = \frac{\angle 3}{2} = \frac{60°}{2} = \boxed{30°}$

$\angle 1 = 180 - 90 - 30 = \boxed{60°}$

or

$\angle 1 = \frac{180 - \angle 3}{2} = \frac{180 - 60}{2} = \boxed{60°}$

You can now do Practice #12 which is posted on my blog.