

Thursday, May 28, 2020

Geometric Probability

$$\text{Probability} = \frac{\text{Favorable Outcomes}}{\text{Total Possible Outcomes}}$$

$$\begin{aligned}\text{Geometric Probability} &= \frac{\text{Area of Favorable Region}}{\text{Total Area}} \\ &= \frac{\text{Shaded area}}{\text{Area of Entire "Target" }}\end{aligned}$$

We will be using geometric shapes as "Targets".

For all of the following questions we will be throwing "darts" at these targets. We will assume that the dart always lands somewhere on the target.

All probabilities will be about the dart landing in the shaded area of each target.

Find all probabilities as a percent rounded to the nearest hundredth.

You will need to know some Area formulas:

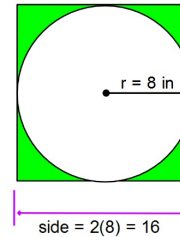
Triangle:  $A = \frac{1}{2}bh$

Rectangle:  $A = bh$

Square:  $A = (\text{side})^2$

Circle:  $A = \pi r^2$

This is a target where a circle is inscribed in a square.



$$P(\text{shaded area}) = \frac{256 - 64\pi}{256} = 21.46\%$$

Area of the Target = Area of the square

The shaded area = (area of square) - (area circle)

$$\text{Area of the circle} = \pi r^2 = \pi(8\text{in})^2 = 64\pi \text{ in}^2$$

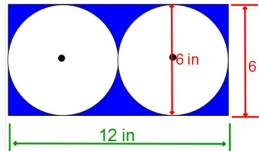
$$\text{Area of the square} = (\text{side})^2$$

The length of each side is = the diameter of the circle.

Diameter of a circle = 2 x radius = 2(8) = 16 in

$$\text{Area of the square} = (16\text{in})^2 = 256 \text{ in}^2$$

The dimensions of the Rectangle are: base = 12in & height = 6 in.



$$P(\text{shaded area}) = \frac{72 - 18\pi}{72} = 21.46\%$$

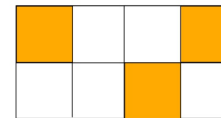
$$\text{Area of the Target} = \text{Area of the rectangle} = (12\text{in})(6\text{in}) = 72 \text{ in}^2$$

$$\text{Shaded area} = (\text{area of Rect}) - (\text{area of 2 circles}) = (72 - 18\pi) \text{ in}^2$$

Height of Rectangle = Diameter of Circles

Radius of Circles = Diameter / 2 = 6/2 = 3 in

$$\text{Area of 2 circles} = 2\pi r^2 = 2\pi(3\text{in})^2 = 18\pi \text{ in}^2$$



$$P(\text{shaded area}) = \frac{3}{8} = 37.5\%$$

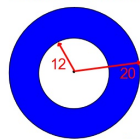
We have no measurements to work with so we can't answer this question by actually calculating areas.

However, the "Target" is a rectangle split into squares of the same size. When this is the case you can find probability by simply counting.

$$P(\text{shaded area}) = \frac{\text{Shaded squares}}{\text{Total \# of squares}}$$

- 3 shaded squares.
- 8 squares total.

The radius of the smaller circle is 12 cm and the radius of the larger circle is 20 cm.

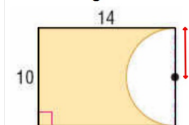


$$P(\text{shaded area}) = \frac{256\pi}{400\pi} = 64\%$$

$$\begin{aligned} \text{Area of Target} &= \text{area of large circle} \\ &= \pi r^2 = \pi(20\text{cm})^2 = 400\pi \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Shaded area} &= \text{blue ring which is found by subtracting the 2 circles.} \\ &= (\text{large circle}) - (\text{small circle}) \\ &= 400\pi - \pi(12)^2 = 400\pi - 144\pi = 256\pi \text{ cm}^2 \end{aligned}$$

This target is created with a rectangle and a semicircle.



$$P(\text{shaded area}) = \frac{140 - 12.5\pi}{140} = 71.95\%$$

$$\text{Area of Target} = \text{area of rectangle} = (10)(14) = 140$$

$$\text{Shaded area} = (\text{rectangle}) - (\text{semicircle})$$

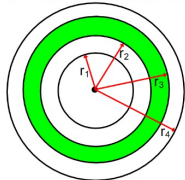
The height of the rectangle, 10, is also the diameter of the semicircle, therefore, the radius of the semicircle =  $10/2 = 5$

$$\text{Shaded area} = 140 - 12.5\pi$$

$$\text{Area of semicircle} = \frac{\pi r^2}{2} = \frac{\pi(5)^2}{2} = \frac{25\pi}{2} = 12.5\pi$$

The target shown has four concentric circles. The measures of the radii are:

$$r_1 = 4\text{in}, r_2 = 6\text{in}, r_3 = 8\text{in}, r_4 = 10\text{in}$$



$$P(\text{shaded area}) = \frac{28\pi}{100\pi} = 28\%$$

$$\begin{aligned} \text{Area of target} &= \text{area of Circle 4} = \pi(r_4)^2 \\ &= \pi(10)^2 = 100\pi \text{ in}^2 \end{aligned}$$

$$\begin{aligned} \text{Shaded area} &= (\text{area of circle 3}) - (\text{area of circle 2}) \\ &= \pi(r_3)^2 - \pi(r_2)^2 = \pi(8)^2 - \pi(6)^2 = 64\pi - 36\pi = 28\pi \end{aligned}$$

You can now finish Practice #29

This Practice is due on Saturday, May 30 by 10:00 pm