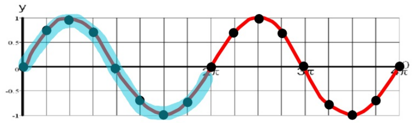
Wednesday, May 13, 2020

Graph of the Cosine Function

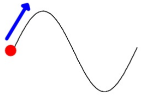
Review: The graph of y=sinx

θ	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π	$\frac{9\pi}{4}$	$\frac{5\pi}{2}$	$\frac{11\pi}{4}$	3π	$\frac{13\pi}{4}$	$\frac{7\pi}{2}$	$\frac{15\pi}{4}$	4π
Sinθ	0	0.71	1	0.71	0	−0.71	−1	−0.71	0	0.71	1	0.71	0	−0.71	−1	−0.71	0

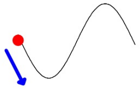


Amplitude = 1 Midline: y=0 Period = 2π

Typical cycles for y = aSinx:

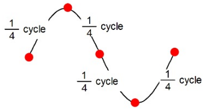


a is Positive



a is Negative

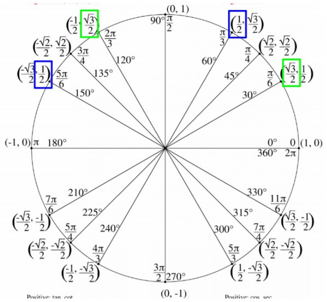
Every cycle of a Sin curve can be broken into fourths:



Below are the values for Cosθ for the same angles that we used for Sinθ

θ	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π	$\frac{9\pi}{4}$	$\frac{5\pi}{2}$	$\frac{11\pi}{4}$	3π	$\frac{13\pi}{4}$	$\frac{7\pi}{2}$	$\frac{15\pi}{4}$	4π
Cosθ	1	0.71	0	-0.71	-1	-0.71	0	0.71	1	0.71	0	-0.71	-1	-0.71	0	0.71	1

Notice that these are the same numbers that are on the table for Sinθ, but in different places.



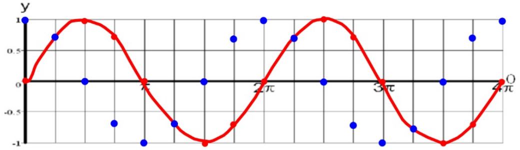
If you look at the coordinates on the Unit Circle you should notice that all of the x-coordinates are the same numbers you see somewhere else as a y-coordinate.

The value of Cosθ will be the same as a value of Sinθ, but at a different spot.

Below are the values for Cosθ for the same angles that we used for Sinθ

θ	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π	$\frac{9\pi}{4}$	$\frac{5\pi}{2}$	$\frac{11\pi}{4}$	3π	$\frac{13\pi}{4}$	$\frac{7\pi}{2}$	$\frac{15\pi}{4}$	4π
Cosθ	1	0.71	0	-0.71	-1	-0.71	0	0.71	1	0.71	0	-0.71	-1	-0.71	0	0.71	1

These points for y=Cosθ are graphed on top of the graph of y=Sinθ



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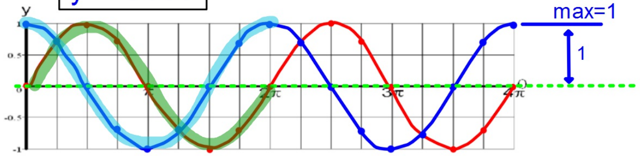
y = Cosθ



min=

y=0

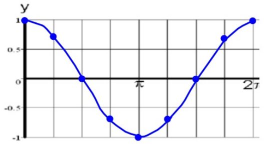
Eq Midline: y=0 Amplitude = 1 Period = 2π

This means that Sinx and Cosx have

- The same Amplitude Amp = 1
- The same Midline y=0
- The same Period period = 2π

The just start in a different spot and what we picture as cycle of each is a different shape

One cycle of the parent Sine function looks like a sideways "S".
What does one cycle of the parent Cosine function look like?



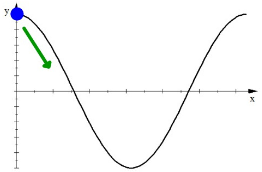
Looks like a "U" or
an upwards opening parabola.

The parent Sine function starts on the midline and goes up.

Where does Cosine "start"? Parent Cosine function starts at
a Maximum.

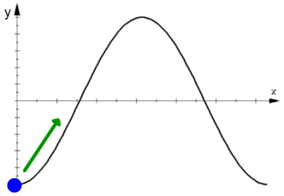
Y = aCosbx

The starting point for the Parent Cosine Function is:
at a Maximum.



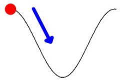
Y = aCosbx

If you start at a Minimum
then the graph is upside down and a is negative in the equation.

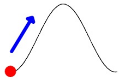


a<0

Typical cycles for y = aCosx:

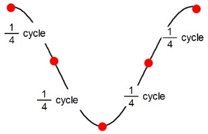


a is Positive



a is Negative

Every cycle of a Cos curve can also be broken into fourths:



Finding the period, amplitude, and midline is the **SAME** for y=cosx as it was for y=sinx.

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y = asinbx

a |a| = Amplitude (vertical Stretch or Shrink factor)

a<0 is an x-axis reflection (upside down)

b Period = $\frac{2\pi}{b}$ $\longrightarrow$ b = $\frac{2\pi}{\text{Period}}$

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y = acosbx

a |a| = Amplitude (vertical Stretch or Shrink factor)

a<0 is an x-axis reflection (upside down)

b Period = $\frac{2\pi}{b}$ $\longrightarrow$ b = $\frac{2\pi}{\text{Period}}$

This is all the **SAME** as for aSinbx

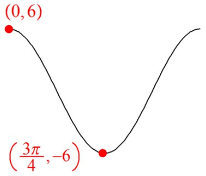
Find the period and amplitude of this Cosine function:

$$y = \overset{a}{-11}\cos\frac{2x}{5} \longrightarrow \overset{b}{\frac{2}{5}}x$$

a = -11 Amplitude = |a| = $\boxed{11}$

b = $\frac{2}{5}$ Period $= \frac{2\pi}{b} = \frac{2\pi}{\frac{2}{5}} = 2\pi \cdot \frac{5}{2} = \boxed{5\pi}$

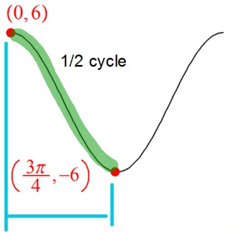
Find the amplitude and period of this cosine function:



Amplitude = 6

$$\frac{\text{max - min}}{2} = \frac{6-(-6)}{2} = \frac{12}{2} = 6$$

Period:
see next page



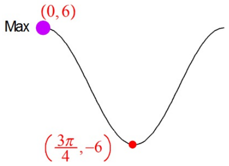
Total width $= \frac{3\pi}{4} - 0 = \frac{3\pi}{4}$

$$\text{Period} = \frac{\text{total width}}{\text{\#cycles}} = \frac{\frac{3\pi}{4}}{\frac{1}{2}}$$

$$= \frac{3\pi}{4} \cdot \frac{2}{1} = \frac{3\pi}{2}$$

$$\text{Period} = \boxed{\frac{3\pi}{2}}$$

Now write the equation of this Cosine Function in y=aCosbx form.



Amplitude = 6 $\longrightarrow \boxed{a = 6}$

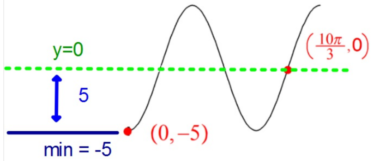
Since this Cos function starts at a max it is NOT upside down so a is positive.

$$\text{Period} = \frac{3\pi}{2} \longrightarrow b = \frac{2\pi}{\text{period}} = \frac{2\pi}{\frac{3\pi}{2}}$$

$$= 2\pi \cdot \frac{2}{3\pi} = \boxed{\frac{4}{3}}$$

EQ: $y = 6\cos\frac{4}{3}x$ or $6\cos\frac{4x}{3}$

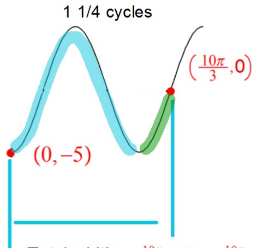
Find the amplitude and period of this cosine function:



Amplitude = 5

distance from
midline of 0 to -5

Period:
see next page



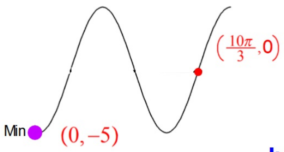
Total width = $\frac{10\pi}{3} - 0 = \frac{10\pi}{3}$

$$\text{Period} = \frac{\text{total width}}{\text{\#cycles}} = \frac{\frac{10\pi}{3}}{1\frac{1}{4}}$$

$$= \frac{\frac{10\pi}{3}}{\frac{5}{4}} = \frac{10\pi}{3} \cdot \frac{4}{5} = \boxed{\frac{8\pi}{3}}$$

Write the equation of this Cosine Function in y=aCosbx form.

Amplitude = 5 $\longrightarrow$ $\boxed{a = -5}$



This graph starts at a minimum, therefore, it's upsidedown and a is negative.

Period = $\frac{8\pi}{3}$

$$\mathbf{b} = \frac{2\pi}{\text{period}} = \frac{2\pi}{\frac{8\pi}{3}} = 2\pi \cdot \frac{3}{8\pi} = \boxed{\frac{3}{4}}$$

Eq: $\boxed{y = -5\cos\frac{3}{4}x \text{ or } -5\cos\frac{3x}{4}}$

You can now do the first part of Practice #27.

We'll finish the remainder of the material tomorrow.

Practice #27 will be due on Saturday, May 16 by 10:00pm