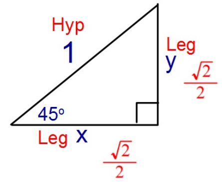
Tuesday, March 31, 2020

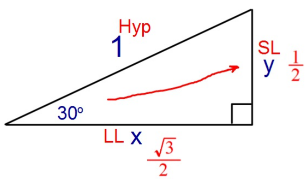
Sec 7-2 & 7-3: The Unit Circle

45°-45°-90° Δ



$$Leg = \frac{hyp}{\sqrt{2}}$$
$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$
$$= \frac{\sqrt{2}}{2}$$

30°-60°-90° Δ

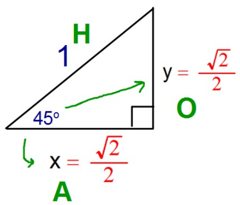


$$SL = \frac{hyp}{2} = \frac{1}{2}$$

$$LL = SL \cdot \sqrt{3}$$
$$= \frac{1}{2} \cdot \frac{\sqrt{3}}{1}$$
$$= \frac{\sqrt{3}}{2}$$

45°-45°-90° Δ

SOHCAHTOA

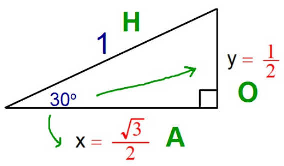


$$\text{Sin}45° = \frac{y}{1} = \frac{\frac{\sqrt{2}}{2}}{1} = \frac{\sqrt{2}}{2}$$

$$\text{Cos}45° = \frac{x}{1} = \frac{\frac{\sqrt{2}}{2}}{1} = \frac{\sqrt{2}}{2}$$

$\boxed{30^\circ\text{-}60^\circ\text{-}90^\circ\ \Delta}$

SOHCAHTOA



$$\text{Sin}30^\circ = \frac{y}{1} = \frac{\frac{1}{2}}{1} = \frac{1}{2}$$

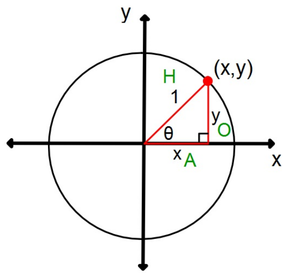
$$\text{Cos}30^\circ = \frac{x}{1} = \frac{\frac{\sqrt{3}}{2}}{1} = \frac{\sqrt{3}}{2}$$

The Unit Circle: A circle that has its center at the origin and has a radius of 1 unit.

1 If you pick any point on the circle, (x,y), you can make a right triangle by drawing a radius to that point.

3 Using the reference angle θ, you can define Sinθ and Cosθ as follows.

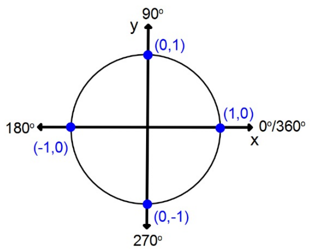
2



4 $\sin\theta = \frac{y}{1} = y$ Sinθ=y-coord of a point on the Unit Circle

5 $\cos\theta = \frac{x}{1} = x$ Cosθ = x-coord of a point on the Unit Circle

Since the radius of the Unit Circle = 1 you can define the coordinates of points on the axes as follows:



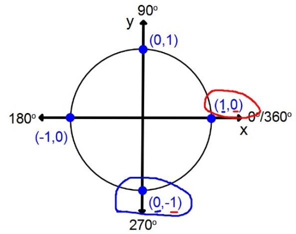
Using the Unit Circle and the following definitions of Sinθ and Cosθ we can find the Sin and Cos of angles that were impossible using right triangle trigonometry.

1

2 $\sin\theta = y$ $\cos\theta = x$

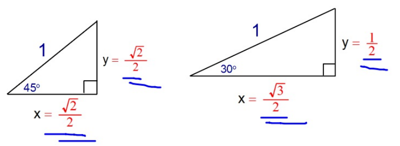
3 Sin0° = 0 Cos0° = 1

4 Sin270° = -1 Cos270° = 0

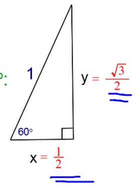


With the exception of the axes the Unit Circle is built on the special right triangles we've been discussing. It allows us to find the EXACT value of Sin and Cos of angles related to 30º, 45º, and 60º.

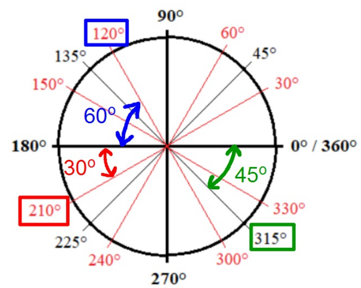
It uses the Special Right Δ's we discussed earlier with a hypot=1:



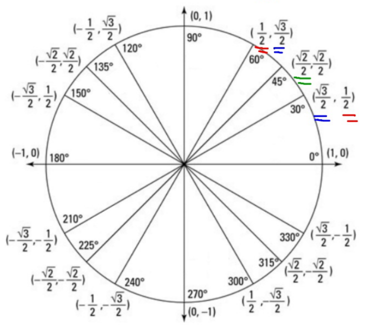
And the related 30º-60º-90º:



The Unit Circle with Degrees:



The Unit Circle with Degrees and coordinates:



Except for the axes, the coordinates of points on the Unit Circle are derived from the legs of the Special Right Triangles when the hypotenuse =1.

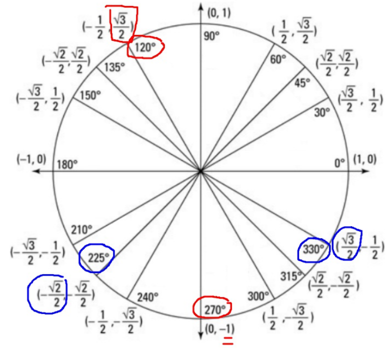
Use the Unit Circle to find the EXACT value of each.

1. Sin 120º = $\frac{\sqrt{3}}{2}$

2. Cos 225º = $\frac{-\sqrt{2}}{2}$

3. Sin 270º = -1

4. Cos 330º = $\frac{\sqrt{3}}{2}$



Use the Unit Circle to find the EXACT value of each.

Hint: First use the concept of coterminal angles to turn each angle into a coterminal between 0° and 360°.

1. Sin(-330°) =

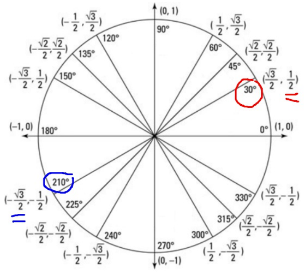
   -330+360 = 30°

   Sin(-330°)=Sin30° $= \boxed{\frac{1}{2}}$

2. Cos 570° =

   570-360 = 210°

   Cos(570°)=Cos210° $= \boxed{\frac{-\sqrt{3}}{2}}$



You can now work on Practice #12 which is posted on my blog.