Thursday, March 26, 2020

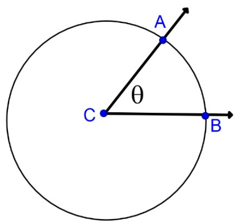
Sec 7-2: Radian Measure of an Angle.

What is the measure of an angle?

The size of an angle
or
The amount of rotation to move from
one side of the angle to the other side.

Units used to measure angles:
- Degrees
- Radians

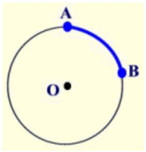
Central Angle: An angle whose vertex is at the center of a circle.



The central angle shown is named:

$\angle ACB$ or $\angle C$
or $\angle\theta$

What is an Arc ?



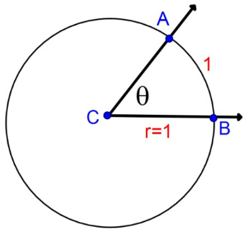
An arc of a circle is a "portion" of the circumference.

The blue hightlighted arc shown is named $\overset{\frown}{AB}$ "arc AB"

What is Arc Length ?

The length of an arc is the distance from one end of the arc to the other end along the outside of the circle. It's the length of its part of the circumference.

In a circle with radius=1 unit ("Unit Circle") Central Angle
θ = 1 radian when the length of the intercepted arc = 1 unit.

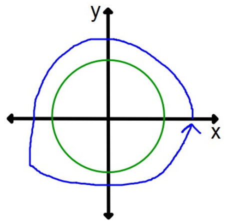


If intercepted $\overset{\frown}{AB}$ = 1

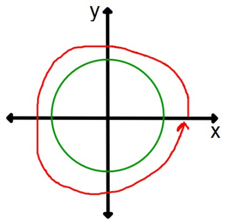
Then θ = 1 radian

In general, the Radian Measure of a central angle θ is equal to the length of the intercepted arc when the radius of the circle is 1 unit.

One full turn around a circle measured in degrees = 360°



One full turn around a circle measured in radians = 2π radians



Therefore, the relationship between degrees and radians is:

$$2\pi = 360^\circ$$

This can be simplified into: $\pi = 180^\circ$

This relationship: $\pi = 180^\circ$

can be written as the following two conversion factors:

$$\frac{\pi}{180^\circ} \quad or \quad \frac{180^\circ}{\pi}$$

Conversion Factors: $\frac{\pi}{180°}$ $\frac{180°}{\pi}$

Convert each angle into degrees. Round to the nearest tenth when needed.

1. $\frac{2\pi}{3} \cdot \frac{180°}{\pi}$

$=120°$

2. $\frac{5\pi}{9} \cdot \frac{180°}{\pi}$

$=100°$

Conversion Factors: $\frac{\pi}{180°}$ $\frac{180°}{\pi}$

Convert each angle into radians. Give answer in terms of π and as a simplified fraction.

1. $45° \cdot \frac{\pi}{180°}$

$=\frac{\pi}{4}$

2. $150° \cdot \frac{\pi}{180°}$

$=\frac{5\pi}{6}$

Conversion Factors: $\frac{\pi}{180°}$ $\frac{180°}{\pi}$

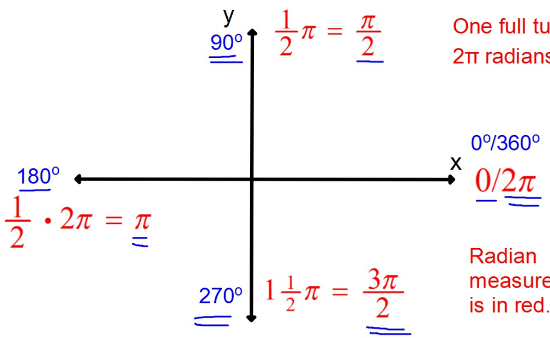
Convert this angle. Round to the nearest hundredth.

$\theta = 5 \cdot \frac{180°}{\pi}$

Even though angles measured in radians usually have **π** in it, this is not always the case. There isn't a symbol for radians, therefore, if you don't see a degree symbol the angle must be measured in radians.

$=286.48°$

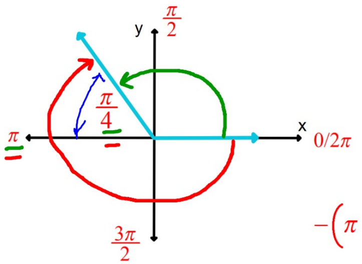
Angle measure of the axes on the coordinate plane:



One full turn is 2π radians.

Radian measure is in red.

State both a positive and a negative measure, in radians, for this angle in Standard Position:



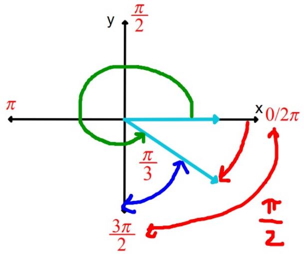
Pos:

$$\pi - \frac{\pi}{4} = \frac{4\pi}{4} - \frac{\pi}{4}$$
$$= \frac{3\pi}{4}$$

Neg:

$$-\left(\pi + \frac{\pi}{4}\right) = -\left(\frac{4\pi}{4} + \frac{\pi}{4}\right)$$
$$= \frac{-5\pi}{4}$$

State both a positive and a negative measure, in radians, for this angle in Standard Position:



Pos:

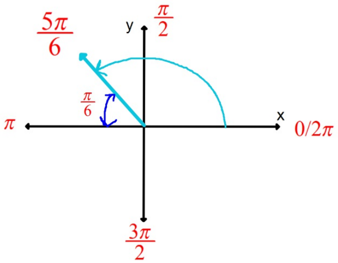
$$\frac{3}{3}\cdot\frac{3\pi}{2} + \frac{\pi}{3}\cdot\frac{2}{2} = \frac{9\pi}{6} + \frac{2\pi}{6}$$
$$= \frac{11\pi}{6}$$

Neg:

$$-\left(\frac{3}{3}\cdot\frac{\pi}{2} - \frac{\pi}{3}\cdot\frac{2}{2}\right) = -\left(\frac{3\pi}{6} - \frac{2\pi}{6}\right)$$
$$= \frac{-\pi}{6}$$

State the Reference Angle, in radians, for this angle in Standard Position.

1. $\theta = \frac{5\pi}{6}$

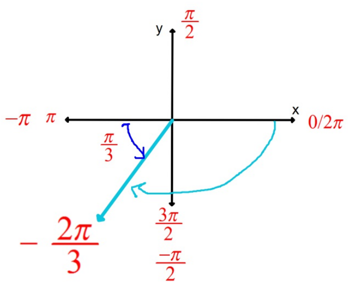


Reference angle:

$$\pi - \frac{5\pi}{6} = \frac{6\pi}{6} - \frac{5\pi}{6}$$
$$= \frac{\pi}{6}$$

State the Reference Angle, in radians, for this angle in Standard Position.

2. $\theta = -\frac{2\pi}{3}$



Reference Angle:

$$\frac{-2\pi}{3} + \pi = \frac{-2\pi}{3} + \frac{3\pi}{3}$$
$$= \frac{\pi}{3}$$

You can now do
Practice #9 which
is posted on my blog.