Properties of Logarithms

For positive numbers b, m, and n with $b \neq 1$, the following properties hold.

$\log_b mn = \log_b m + \log_b n$ ..... Product Property of Logarithms

$\log_b \frac{m}{n} = \log_b m - \log_b n$ ..... Quotient Property of Logarithms

$\log_b m^n = n \log_b m$ ..... Power Property of Logarithms

Use the Properties of Logarithms to write each as a single logarithm:

1. $3\log_4 K - 2\log_4 Q + 5\log_4 W$

$= \log_4 K^3 - \log_4 Q^2 + \log_4 W^5$

$= \log_4 \frac{K^3 W^5}{Q^2}$

Use the Properties of Logarithms to write each as a single logarithm:

2. $3\log_2 A - \frac{1}{3}\log_2 B - 4\log_2 C$

$= \log_2 A^2 - \log_2 B^{1/3} - \log_2 C^4$

$= \log_2 \frac{A^2}{B^{1/3} C^4} = \log_2 \frac{A^2}{C^4 \sqrt[3]{B}}$

Use the Properties of Logarithms to expand each logarithm:

1. $\log_7 \frac{A^4 B^5}{C^3}$

$= \log_7 A^4 + \log_7 B^5 - \log_7 C^3$

$= 4\log_7 A + 5\log_7 B - 3\log_7 C$

Use the Properties of Logarithms to expand each logarithm:

3. $\log_2\left(\frac{m^4}{n^5}\right)^3 = 3\log_2\left(\frac{m^4}{n^5}\right)$

$= 3(\log_2 m^4 - \log_2 n^5)$

$= 3(4\log_2 m - 5\log_2 n)$

or

$12\log_2 m - 15\log_2 n$

Remember, the domain of a logarithm is: Positive Numbers

Solve.

$\log_4(x+6) - \log_4 x = 3$

$\log_4 \frac{x+6}{x} = 3$

$4^3 = \frac{x+6}{x}$

$x \cdot 64 = \frac{x+6}{x} \cdot x$

$64x = x + 6$

$-x \quad -x$

$\frac{63x}{63} = \frac{6}{63}$

$x = \frac{6}{63}$

Solve.

$\log 4x + \log x = 2$

$\log 4x \cdot x = 2$

$10^2 = 4x^2$

$\frac{100}{4} = \frac{4x^2}{4}$

$x^2 = 25$

$x = \sqrt{25}$

$x = \pm 5$

$x = 5$

Solve. $\frac{1}{3}\log_2 x + \log_2 5 = 4$

$3\left(\frac{1}{3}\log_2 x + \log_2 5\right) = (4)\,3$

$\log_2 x + 3\log_2 5 = 12$

$\log_2 x + \log_2 5^3 = 12$

$\log_2 125 \cdot x = 12$

$2^{12} = 125x$

$4096 = 125x$

$x = \frac{4096}{125}$

Solve.

$\log_6(x-5) = 2 - \log_6 x$

$+\log_6 x \qquad +\log_6 x$

Gather all logs on one side

$\log_6(x-5) + \log_6 x = 2$

$\log_6 x(x-5) = 2$

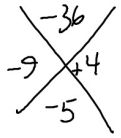
$6^2 = x(x-5)$

$36 = x^2 - 5x$

$-36 \qquad -36$

$0 = x^2 - 5x - 36$

$0 = (x-9)(x+4)$



$x = 9, -4$

$x = 9$