Properties of Logarithms

For positive numbers b, m, and n with $b \neq 1$, the following properties hold.

$\log_b mn = \log_b m + \log_b n$ ..... Product Property of Logarithms

The log of the product of two quantities
equals
the sum of the individual logs.

Write as a single logarithm.

1. $\log_3 W + \log_3 8 = \log_3 8W$

2. $\log A + \log 5 + \log C = \log 5AC$

Expand each logarithm.

1. $\log_7 4D = \log_7 4 + \log_7 D$

2. $\log_7 9PQ = \log_7 9 + \log_7 P + \log_7 Q$

Properties of Logarithms

For positive numbers b, m, and n with $b \neq 1$, the following properties hold.

$\log_b \frac{m}{n} = \log_b m - \log_b n$ ..... Quotient Property of Logarithms

The log of the quotient of two quantities
equals
the difference of the individual logs.

Write as a single logarithm.

1. $\ln 5 - \ln B = \ln \frac{5}{B}$

2. $\log_2 24 - \log_2 8 - \log_2 G = \log_2 \frac{24}{8G}$ or $\log_2 \frac{3}{G}$

Expand each logarithm.

$\log \frac{C}{7} = \log C - \log 7$

$\log \frac{5}{2M} = \log 5 - \log 2 - \log M$

or

$= \log 5 - (\log 2 + \log M)$

Write as a single logarithm.

log8 - logH + log3 - log6

$= \log \frac{8 \cdot 3}{H6} = \log \frac{24}{6H}$ or $\log \frac{4}{H}$

Properties of Logarithms

For positive numbers b, m, and n with $b \neq 1$, the following properties hold.

$\log_b m^n = n \log_b m$ ..... Power Property of Logarithms

The exponent inside a logarithm
becomes
the coefficient of the log.

Expand this logarithm. $\log_4 A^2B^5$

$= \log_4 A^2 + \log_4 B^5$

$= 2\log_4 A + 5\log_4 B$

Example 3

What is the expression written as a single logarithm?

A. $4\log_4 m + 3\log_4 n - \log_4 p$

$= \log_4 m^4 + \log_4 n^3 - \log_4 p$

$= \log_4 m^4 n^3 - \log_4 p$

$= \log_4 \frac{m^4 n^3}{p}$

What is the expression written as a single logarithm?

B. $3\ln 2 - 2\ln 5$

$= \ln 2^3 - \ln 5^2$

$= \ln 8 - \ln 25$

$= \ln \frac{8}{25}$

Example 3 **Try It!**

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3. Write each expression as a single logarithm.

a. $5\log_2 c - 7\log_2 n$

$= \log_2 c^5 - \log_2 n^7$

$= \log_2 \frac{c^5}{n^7}$

b. $2\ln 7 + \ln 2$

$= \ln 7^2 + \ln 2$

$= \ln 7^2 \cdot 2 = \ln 98$

Example 2

How can you use the properties of logarithms to expand the expression?

A. $\log_5(a^2b^7)$ $= \log_5 a^2 + \log_5 b^7$

$= 2\log_5 a + 7\log_5 b$

How can you use the properties of logarithms to expand the expression?

B. $\ln\left(\frac{25}{3}\right)$ $= \ln 25 - \ln 3$

Example 2

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Try It!

2. Use the properties of logarithms to expand each expression.

a. $\log_7\left(\frac{r^3t^4}{v}\right) = 3\log_7 r + 4\log_7 t - \log_7 v$

b. $\ln\left(\frac{7}{225}\right) = \ln 7 - \ln 225$

$= \ln 7 - \ln 15^2$

$= \ln 7 - 2\ln 15$

Write as a single logarithm then solve:

$\log_5 x + \log_5 2 = 3$

$\log_5 2x = 3$

$5^3 = 2x$

$\frac{125}{2} = \frac{2x}{2}$

$x = 62.5$