

You invest \$10,000 in an account that pays 5% interest each year. Find the number of years it will take your investment to reach \$100,000. Round to the nearest tenth.

$$\frac{100,000}{10,000} = \frac{10,000}{10,000} (1.05)^t \quad 100 + 5 = 105\% \\ b = 1.05$$

$$10 = 1.05^t$$

$$\log_{1.05}(10) = t$$

$$t = 47.2 \text{ yrs}$$

The population of a city is decreasing 2.5% each year.

$$100 - 2.5 = 97.5\%$$

The population in 2005 was 160,000. In how many years, to the nearest hundredth, will it take for the population to reach 120,000?

$$\frac{120,000}{160,000} = \frac{160,000}{160,000} (.975)^t$$

$$.75 = (.975)^t$$

$$\log_{.975}(.75) = t$$

$$t = 11.36 \text{ yrs}$$

You invest \$125,000 in an account that pays 8% interest compounded continuously.  $y = Pe^{rt}$

Find the number of years, to the nearest hundredth, it will take to reach \$1,000,000.

$$1,000,000 = 125,000 e^{.08t}$$

$$8 = e^{.08t}$$

$$\ln 8 = .08t$$

$$t = \frac{\ln 8}{.08}$$

$$25.99 \text{ yrs}$$

You invest in an account that pays 6% annual interest compounded monthly.

How many years, to the nearest tenth, will it take to double your investment?

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

$$2 = 1 \left(1 + \frac{.06}{12}\right)^{12t}$$

$$2 = (1.005)^{12t}$$

$$\log_{1.005}(2) = 12t$$

$$t = 11.6 \text{ yrs}$$

## Compound Interest RULE OF 72

- IT'S A -  
MONEY  
THING®

Divide the rule number (72) by the annual interest rate (R) to find out the approximate time (T) required for doubling

$$72 \div \overset{\substack{\text{INTEREST} \\ \text{RATE}}}{R} = \overset{\substack{\text{YEARS TO} \\ \text{DOUBLE}}}{T}$$

## Hwk #10

Practice Sheet:  
Solving exponential and logarithmic equations

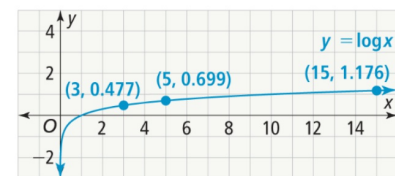
You are now prepared for Quiz #2: Sec 6-1 to 6-3

Page 157 Student Companion

## EXPLORE & REASON

Answer Parts A, B, and C.

Look at the graph of  $y = \log x$  and the ordered pairs shown.



A. Complete the table shown.

$x$	3	5	15
$\log x$	0.477	0.699	1.176

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$\log x$	0.477	0.699	1.176

**B. Look for Relationships** What is the relationship between the numbers 3, 5, and 15? What is the relationship between the logarithms of 3, 5, and 15?

15 is the product of 3 and 5:

$$3 \cdot 5 = 15$$

$$\log 3 + \log 5 = \log 15$$

$$\log 3 \cdot 5 = \log 3 + \log 5$$

The log of the product of 3 and 15 equals the sum of the individual logs.

$x$	3	5	15
$\log x$	0.477	0.699	1.176

**C.** What is your prediction for the value of  $\log 45$ ?  $\log 75$ ? Explain.

$$\begin{aligned}\log 45 &= \log 3 \cdot 15 = \log 3 + \log 15 \\ &= 0.477 + 1.176 \\ &= 1.653\end{aligned}$$

$$\begin{aligned}\log 75 &= \log 5 \cdot 15 = \log 5 + \log 15 \\ &= 0.699 + 1.176 \\ &= 1.875\end{aligned}$$

## HABITS OF MIND Page 157

**Generalize** Do you think that the relationships you found in the Explore & Reason activity would also hold for natural logarithms? Give an example.

Yes,  $\ln 15 = \ln 3 \cdot 5 = \ln 3 + \ln 5$

## Properties of Logarithms

For positive numbers  $b$ ,  $m$ , and  $n$  with  $b \neq 1$ , the following properties hold.

$$\log_b mn = \log_b m + \log_b n \dots \dots \dots \text{Product Property of Logarithms}$$

The log of the **product of two quantities** equals the **sum of the individual logs**.