To solve for x in an exponential equation: $y = b^x$
we use the inverse operation called:

Logarithm

Sec 6-3: Logarithms
(the inverse of exponential functions)

Exponential Function

$y = b^x$

The base of the Exponential Function

The exponent

Logarithmic Function

$\log_b y = x$

The base of the Logarithmic Function

The answer of the logarithm

Changing from one form to the other:

Exponential Function:

$y = b^x$

Logarithmic Function:

"Log, base b, of y equals x"

$\log_b y = x$

The base is the base

The exponent is the answer

Another way to remember Logarithmic Form:

Exponential Form:

$x = y^z$

becomes

Logarithmic Form:

$z = \text{Log}_y x$

Exponential Equation

Range: $y>0$

Domain: Any real number

$y = b^x$

$b>0, b\neq 1$

Logarithmic Equation

$\log_b y = x$

Range: Any real number

Domain: $x > 0$

b: $b>0, b\neq 1$

Example 1 Page 150

Try It!

1. Write the logarithmic form of $y = 8^x$.

$\log_8 y = x$

Example 2 Page 150

Try It!

2. a. What is the logarithmic form of $7^3 = 343$?

$\log_7 343 = 3$

b. What is the exponential form of $\log_4 16 = 2$?

$4^2 = 16$

Rewrite each into logarithmic form.

1. $8^2 = x$ $\quad \log_8 x = 2$
2. $x^3 = 450$ $\quad \log_x 450 = 3$
3. $9^x = 77$ $\quad \log_9 77 = x$

Rewrite each into exponential form.

1. $LOG_5 8 = x$ $\quad 5^x = 8$

2. $LOG_3 x = 12$ $\quad 3^{12} = x$

3. $LOG_x 15 = 30$ $\quad x^{30} = 30$

Write in Logarithmic Form: $10^x = 125$

$LOG_{10}125$ → "LOG base 10 of 125" → LOG125

LOG_{10} is called the Common Logarithm and is written without the 10.

The button on the calculator [LOG] is for Common Logarithms LOG_{10}

Solve by first rewriting in logarithmic form. $10^x = 200$

$\log_{10} 200 = x$

$\log 200 = x$

$x = 2.30$

Log_e is called the Natural Logarithm

and is written as [ln] or [LN]

Solve by first rewriting in logarithmic form. $e^{2x} = 10$

$\frac{\ln 10}{2} = \frac{2x}{2}$

$x = \frac{\ln 10}{2} = 1.15$

Solve by first rewriting in exponential form. $\ln(x-1) = 5$

$e^5 = x - 1$

$+1 \quad +1$

$x = e^5 + 1 = 149.41$

Evaluate .

8. log54 $= 1.73$

Just do this on the calculator.

Evaluate each without a calculator:
(hint: think of each as an exponential)

1. $\log_4 1 = 0$

$4^? = 1 \rightarrow 4^0 = 1$

2. $\log_3 9 = 2$

$3^? = 9 \rightarrow 3^2 = 9$

3. $\log_4(4) = 1$

$4^? = 4 \rightarrow 4^1 = 4$

4. $\log_{25} 5 = 1/2$

$25^? = 5$

$\sqrt{25} = 5$

$25^{1/2} = 5$

5. $\log_6(6^4) = 4$

$6^? = 6^4 \rightarrow 6^4 = 6^4$

6. $\log_2(0.5) = -1$

$2^? = .5$

$2^? = \frac{1}{2} \rightarrow 2^{-1} = \frac{1}{2}$

7. $\log_2 0 \rightarrow$ undefined

$2^? = 0$

THERE IS NO power of 2 that $= 0$

Example 3

Page 150

Try It!

3. What is the value of each logarithmic expression?

a. $\log_3\left(\frac{1}{81}\right) = -4$

$3^? = \frac{1}{81} \Rightarrow 3^{-4} = \frac{1}{81}$

b. $\log_7(-7) \Rightarrow$ undefined

$7^? = -7 \Rightarrow$ there is no power of 7 that equals -7.

c. $\log_5 5^9 = 9$

$5^? = 5^9 \Rightarrow 5^9 = 5^9$

Hwk #9 Sec 6-3

Page 319

Problems 23, 24, 29, 30, 34, 36, 45, 46, 48