Sec 6-6: Exponential and Logarithmic Equations

Solve Exponential Equations Using a Common Base

Symbols	Suppose $b > 0$ and $b \neq 1$, then $b^x = b^y$ if and only if $x = y$.
Words	If two powers of the same base are equal, then their exponents are equal; if two exponents are equal, then the powers with the same base are equal.

Solve. $9^{x-4} = 81^{2x}$

$= (9^2)^{2x}$

$9^{x-4} = 9^{4x}$

$x - 4 = 4x$

$-x \quad -x$

$-4 = 3x$

$\frac{-4}{3} = \frac{3x}{3}$

$\boxed{x = -4/3}$

Property of Equality for Logarithmic Equations

Symbols	If $x > 0$, then $\log_b x = \log_b y$ if and only if $x = y$.
Words	If two logarithms (exponents) of the same base are equal, then the quantities are equal; if two quantities are equal, and the bases are the same, then the logarithms (exponents) are equal.

Solve. $6^{x+3} = 4^{2x}$

Unlike the previous problem you can't make each side have the same base. Therefore, you must use another technique which is to take the Logarithm of both sides. This logarithm can be any base (probably common log or natural log as long as they are the same base on both sides.

$\log 6^{x+3} = \log 4^{2x}$

$(x+3)\log 6 = 2x\log 4$

$x\log 6 + 3\log 6 = 2x\log 4$
$-x\log 6 \qquad -x\log 6$

$3\log 6 = 2x\log 4 - x\log 6$

$3\log 6 = x(2\log 4 - \log 6)$

$\frac{3\log 6}{2\log 4 - \log 6} = \frac{x(2\log 4 - \log 6)}{2\log 4 - \log 6} \Rightarrow x = \frac{3\log 6}{2\log 4 - \log 6}$

$\boxed{x = 5.48}$

Example 3 Try It!

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3. What is the solution to $2^{3x} = 7^{x+1}$?

You can't make the bases the same which means you'll need to take the logarithm of both sides:

$\log 2^{3x} = \log 7^{x+1}$

$3x\log 2 = (x+1)\log 7$

$3x\log 2 = x\log 7 + \log 7$
$-x\log 7 \qquad -x\log 7$

$3x\log 2 - x\log 7 = \log 7$

$\frac{x(\cancel{3\log 2 - \log 7})}{\cancel{3\log 2} - \log 7} = \frac{\log 7}{3\log 2 - \log 7} = \boxed{14.57}$

Example 5

What is the solution to $\ln(x^2 - 16) = \ln(6x)$?

Since these logarithms are equal AND have the same base, their arguments must also be equal

$\boxed{x = 8}, \cancel{-2}$

$\boxed{x^2 - 16} = \boxed{6x}$
$-6x \qquad -6x$

$x^2 - 6x - 16 = 0$
$(x-8)(x+2) = 0$

-16
+2 -8
-6

-2 is an extraneous solution.

Example 5 Try It!

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5. Solve each equation.

b. $\ln(-4x - 1) = \ln(4x^2)$

b. $\ln(-4x - 1) = \ln(4x^2)$

Since the logarithms are equal AND they have the same base their arguments must also be equal.

$$-4x - 1 = 4x^2$$
$$+4x + 1 \qquad +4x + 1$$

$$0 = 4x^2 + 4x + 1 \implies (2x+1)(2x+1) = 0$$

$$\begin{array}{ccc} & +4 & \\ +2 & & +2 \\ & +4 & \end{array}$$

	$2x$	$+1$
$2x$	$4x^2$	$+2x$
$+1$	$+2x$	$+1$

$$\boxed{x = -1/2}$$

even though this is negative it is not an extraneous solution.

Solve. $\log(x^2 - x) = \log 20$

Since the logarithms are equal AND they have the same base their arguments must also be equal.

$$x^2 - x = 20$$
$$-20 \qquad -20$$

$$\begin{array}{ccc} & -20 & \\ -5 & & +4 \\ & -1 & \end{array}$$

$$x^2 - x - 20 = 0$$
$$(x-5)(x+4) = 0$$
$$\boxed{x = -4, 5}$$

Solve. $\log_3(2x + 8) = 2\log_3 x$

Your first step must be to use the Power Property of Log's to move the coefficient on the right side back into the logarithm as an exponent.
Once you've done this you have two logarithms that are equal AND they have the same base, therefore, their arguments must also be equal.

$$\log_3(2x + 8) = 2\log_3 x$$

$$\log_3(2x+8) = \log_3 x^2$$

$$2x + 8 = x^2$$
$$-2x \ -8 \qquad -2x - 8$$

$$0 = x^2 - 2x - 8$$

$$\begin{array}{ccc} & -8 & \\ -4 & & +2 \\ & -2 & \end{array}$$

$$0 = (x-4)(x+2)$$

$$\boxed{x = \cancel{-2}, 4}$$

-2 is an extraneous solution

Solve. $\log_2(x - 6) + \log_2 x = 4$

Because of the extra term (4) you can't solve this the same way as the previous several equations. However, you can use the Product Property of Logarithms to turn the left side of the equation into a single logarithm.

$$\log_2 (x-6)(x) = 4$$

$$2^4 = x(x-6)$$

change to exponential form.

$$16 = x^2 - 6x$$
$$-16 \qquad -16$$

$$0 = x^2 - 6x - 16$$

$$\begin{array}{ccc} & -16 & \\ -8 & & +2 \\ & -6 & \end{array}$$

$$0 = (x-8)(x+2)$$

$$\boxed{x = \cancel{-2}, 8}$$

-2 is an extraneous solution

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Problems 4-6, 26, 30, 35