Sec 6-6: Exponential and Logarithmic Equations

Solve Exponential Equations Using a Common Base

Symbols	Suppose $b > 0$ and $b \neq 1$, then $b^x = b^y$ if and only if $x = y$.
Words	If two powers of the same base are equal, then their exponents are equal; if two exponents are equal, then the powers with the same base are equal.

Example 1

What is the solution to $\left(\frac{1}{2}\right)^{x+7} = 4^{3x}$?

$(2^{-1})^{x+7} = (2^2)^{3x}$

Turn each power into the same base

$2^{-1(x+7)} = 2^{2 \cdot 3x}$

Now set the exponents equal to each other

$-(x+7) = 2 \cdot 3x$

$-x - 7 = 6x$

$+x \quad +x$

$\frac{-7}{7} = \frac{7x}{7}$

$x = -1$

Example 1

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Try It!

1. Solve each equation using a common base.

a. $25^{3x} = 125^{x+2}$

b. $0.001 = 10^{6x}$

a. $25^{3x} = 125^{x+2}$

$(5^2)^{3x} = (5^3)^{x+2}$

$5^{6x} = 5^{3(x+2)}$

$6x = 3(x+2)$

$6x = 3x + 6$

$-3x \quad -3x$

$\frac{3x}{3} = \frac{6}{3}$

$x = 2$

b. $0.001 = 10^{6x}$

$10^{-3} = 10^{6x}$

$\frac{-3}{6} = \frac{6x}{6}$

$x = -\frac{1}{2}$

Property of Equality for Logarithmic Equations

Symbols	If $x > 0$, then $\log_b x = \log_b y$ if and only if $x = y$.
Words	If two logarithms (exponents) of the same base are equal, then the quantities are equal; if two quantities are equal, and the bases are the same, then the logarithms (exponents) are equal.

Example 3 What is the solution to $3^{x+1} = 5^x$?

$\log 3^{x+1} = \log 5^x$ Take the log of both sides and move exponents to coefficients

distribute log3

$(x+1)\log 3 = x \log 5$

$x\log 3 + \log 3 = x \log 5$

$-x\log 3 \quad -x\log 3$ move all x terms to one side

$\log 3 = x \log 5 - x \log 3$

$\log 3 = x(\log 5 - \log 3)$ factor out the x from the two terms on the right side then divide to get x by itself

$\frac{\log 3}{\log 5 - \log 3} = \frac{x(\log 5 - \log 3)}{\log 5 - \log 3}$

$x = \frac{\log 3}{\log 5 - \log 3}$

$x = 2.15$