Just like in rational equations you can get extraneous solutions when solving radical equations. Therefore, always check your answers.

Answer Example 3 Try It! bottom of page 126
Student Companion 3a and 3b.

a. $x = \sqrt{7x+8}$

a. $(x)^2 = \left(\sqrt{7x+8}\right)^2$

$x^2 = 7x + 8$

$-7x-8 \quad -7x-8$

$x^2 - 7x - 8 = 0$

$(x-8)(x+1) = 0$

$x = -1, 8$

$\boxed{x = 8}$

-1 is an extraneous solution.

b. $x + 2 = \sqrt{x+2}$

b. $(x+2)^2 = \left(\sqrt{x+2}\right)^2$

$x^2 + 4x + 4 = x + 2$

$-x \quad -2 \quad -x \quad -2$

$x^2 + 3x + 2 = 0$

$(x+2)(x+1) = 0$

$\boxed{x = -1, -2}$

Both are solutions (no extraneous sol)

Solving equations with rational exponents.

Solve.

$(x-3)^{\frac{5}{2}} - 11 = 21$

$\qquad +11 \quad +11$

$(x-3)^{5/2} = 32$

$\left(\sqrt{x-3}\right)^5 = 32$

$\sqrt[5]{\left(\sqrt{x-3}\right)^5} = \sqrt[5]{32}$

$\sqrt{x-3} = 2$

$\left(\sqrt{x-3}\right)^2 = (2)^2$

$x-3=4 \longrightarrow \boxed{x=7}$

A second method:

$(x-3)^{\frac{5}{2}} - 11 = 21$

$\qquad +11 \quad +11$

$(x-3)^{5/2} = 32$

$\left[(x-3)^{5/2}\right]^{2/5} = 32^{2/5} = \left(\sqrt[5]{32}\right)^2 = (2)^2$

$x-3=4$

$\boxed{x=7}$

Answer Example 4 Try It! top page 127 Student Companion just do 4a.

a. $(x^2-3x-6)^{\frac{3}{2}} - 14 = -6$

$\qquad +14 \quad +14$

$(x^2-3x-6)^{3/2} = 8$

$\left[(x^2-3x-6)^{3/2}\right]^{2/3} = 8^{2/3} = \left(\sqrt[3]{8}\right)^2 = (2)^2$

$x^2-3x-6=4$

$x^2-3x-10=0$

$(x-5)(x+2)=0$

$\boxed{x=-2,5}$ → no extraneous sol

b. $(x+8)^2 = (x-10)^{\frac{5}{2}}$

b. $\left[(x+8)^2\right]^2 = \left[(x-10)^{\frac{5}{2}}\right]^2$

$(x+8)^4 = (x-10)^5 \longrightarrow$ Very difficult to solve this with algebra!

Some equations are very difficult to solve and in some cases not solvable using algebra. Therefore, you must use other techniques to find solutions.

Another possible way is to solve by graphing

Find **both** solutions to this equation.

$$(x+7)^{\frac{2}{3}} + 2 = 18$$

$$\quad -2 \quad\quad -2$$

$$(x+7)^{2/3} = 16$$

$$\left[(x+7)^{2/3}\right]^{3/2} = (16)^{3/2} = \left(\sqrt{16}\right)^3 = (\pm 4)^3 = \pm 64$$

$$x+7 = \pm 64 \to x+7=64 \to x=57$$

$$ \to x+7=-64 \to x=-71$$

$$\boxed{x=-71, 57}$$

Solve an equation with two radicals.

Solve.

$$\sqrt{x+9} - \sqrt{2x} = 3$$

$$+\sqrt{2x} \quad\quad +\sqrt{2x}$$

$$\sqrt{x+9} = 3+\sqrt{2x}$$

$$\left(\sqrt{x+9}\right)^2 = \left(3+\sqrt{2x}\right)^2$$

	3	$+\sqrt{2x}$
3	9	$+3\sqrt{2x}$
$+\sqrt{2x}$	$+3\sqrt{2x}$	$+2x$

$$x+9 = 2x+9+6\sqrt{2x}$$

$$-2x\ -9 \quad\quad -2x\ -9$$

$$-x = 6\sqrt{2x}$$

$$(-x)^2 = \left(6\sqrt{2x}\right)^2$$

$$x^2 = 36(2x) = 72x$$

$$x^2 - 72x = 0$$

$$x(x-72) = 0$$

$$x = 0, 72$$

extraneous solution

$$\boxed{x=0}$$

Answer Example 5 Try It! Page 127 Student Companion. Answer only 5a.

a. $\sqrt{x+4} - \sqrt{3x} = -2$

$$+\sqrt{3x} \quad\quad +\sqrt{3x}$$

$$\sqrt{x+4} = \sqrt{3x} - 2$$

$$\left(\sqrt{x+4}\right)^2 = \left(\sqrt{3x}-2\right)^2$$

	$\sqrt{3x}$	-2
$\sqrt{3x}$	$3x$	$-2\sqrt{3x}$
-2	$-2\sqrt{3x}$	$+4$

$$x+4 = 3x+4-4\sqrt{3x}$$

$$-3x\ -4 \quad\quad -3x\ -4$$

$$-2x = -4\sqrt{3x}$$

$$(-2x)^2 = \left(-4\sqrt{3x}\right)^2$$

$$4x^2 = 16(3x) = 48x$$

$$4x^2 - 48x = 0$$

$$4x(x-12) = 0$$

$$x = 0, 12$$

extraneous sol

$$\boxed{x=12}$$

Answer Example 5 Try It! Page 127 Student Companion. Now answer 5b.

b. $\sqrt{15-x} - \sqrt{6x} = -3$

$\quad\quad\quad\quad +\sqrt{6x} \quad\quad +\sqrt{6x}$

$\sqrt{15-x} = \sqrt{6x} - 3$

$\left(\sqrt{15-x}\right)^2 = \left(\sqrt{6x} - 3\right)^2$

	$\sqrt{6x}$	-3
$\sqrt{6x}$	$6x$	$-3\sqrt{6x}$
-3	$-3\sqrt{6x}$	$+9$

$15 - x = 6x + 9 - 6\sqrt{6x}$

$-9 \quad -6x \quad\quad -6x \quad -9$

$6 - 7x = -6\sqrt{6x}$

$(6-7x)^2 = \left(-6\sqrt{6x}\right)^2$

$49x^2 - 84x + 36 = 36 \cdot 6x = 216x$

$49x^2 - 300x + 36 = 0$

$(49x - 6)(x - 6) = 0$

$x = 6,\ 6/49$

$\boxed{x = 6}$

extraneous sol.