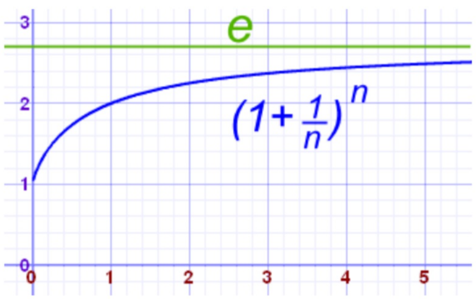
The Number e

You invest $1 at 100% interest for one year.

Frequency of compounding	#times per year compound interest (n)	$1\left(1+\frac{1}{n}\right)^n$	Dollar Value
Annually	$n = 1$	$1\left(1+\frac{1}{1}\right)^1$	2.00
Semiannually	$n = 2$	$1\left(1+\frac{1}{2}\right)^2$	2.25
quarterly	$n = 4$	$1\left(1+\frac{1}{4}\right)^4$	2.441
monthly	$n = 12$		2.613
weekly	$n = 52$		2.693
daily	$n = 365$		2.715
hourly	$n =$ 8760		2.718
every minute	$n =$ 525,600		2.718
every second	$n =$ 31,536,000		2.718

the value of $(1 + 1/n)^n$ approaches **e** as n gets bigger and bigger:



n	$(1 + 1/n)^n$
1	2.00000
2	2.25000
5	2.48832
10	2.59374
100	2.70481
1,000	2.71692
10,000	2.71815
100,000	2.71827

The **natural base e** is defined as the value that the expression $\left(1 + \frac{1}{x}\right)^x$ approaches as $x \to +\infty$. The number e is an irrational number.

$$e = 2.718281828459\ldots$$

Where is e used?

Like π, e is most often found in formulas.

e is called Euler's constant.

Leonhard Euler: Swiss mathematician

Equation of a Catenary: $y = \frac{a}{2}\left(e^{\frac{x}{a}} + e^{\frac{-x}{a}}\right)$

Catenary: A catenary is the shape that a cable assumes when it's supported at its ends and only acted on by its own weight. It is used extensively in construction, especially for suspension bridges

Famous Catenarys:

The more often interest is calculated the more money you will earn.

What is more often than every second?

Compounding Interest **Continuously**

The number e is the base in the **continuously compounded interest formula**.

$$A = Pe^{rt}$$

P = the initial principal invested

e = the natural base

r = annual interest rate, written as a decimal

A = the value of the account after t years

Example 4 **Try It!** **Find Continuously Compounded Interest**

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4. You invest $125,000 in an account that earns 4.75% annual interest, compounded continuously.

a. What is the value of the account after 15 years?

$$A = Pe^{rt}$$

$125{,}000\, e^{.0475 \cdot 15}$

$= \$254{,}885.32$

b. What is the value of the account after 30 years?

$125{,}000\, e^{.0475 \cdot 30}$

$= \$519{,}732.23$

If you invest $20,000 at 6% annual interest for 25 years:

with Simple Interest you'll have $50,000

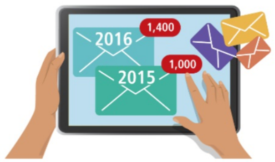
Compounding annually you'll have $85,837.41

Compounding monthly you'll have $89,299.39

Compounding continuously you'll have $89,633.78

Example 5:

Tia knew that the number of e-mails she sent was growing exponentially. She generated a record of the number of e-mails she sent each year since 2009. What is an exponential model that describes the data?



(6,10) & (7,14)

Write an exponential model in the form $y = a \cdot b^x$, with y equal to the number of e-mails in hundreds and x equal to the number of years since 2009. Use the data to find the values of the constants a and b.

When data points have consecutive x-values, the growth factor, b, is the ratio of their y-values. The growth factor for Tia's e-mails in the two consecutive years was $\frac{14}{10}$, or 1.4.

Use the value of b and one of the data points to find the initial value, a.

$y = a \cdot b^x$	Write an exponential growth equation.
$14 = a(1.4)^7$	Substitute 1.4 for b, 7 for x, and 14 for y.
$\frac{14}{(1.4)^7} = a$	Division Property of Equality
$1.33 \approx a$	Simplify.

Another way to do Example 5:

Tia knew that the number of e-mails she sent was growing exponentially. She generated a record of the number of e-mails she sent each year since 2009. What is an exponential model that describes the data?

(6,10) & (7,14)

Write an exponential model in the form $y = a \cdot b^x$, with y equal to the number of e-mails in hundreds and x equal to the number of years since 2009. Use the data to find the values of the constants a and b.

Create a system of equations

use (6,10) → $10 = a \cdot b^6$ use (7,14) → $14 = a \cdot b^7$

$a = \frac{10}{b^6}$ → $14 = \frac{10}{b^6} \cdot b^7$

$14 = 10b$

$\frac{14}{10} = \frac{10b}{10}$

$b = 1.4$

$a = \frac{10}{(1.4)^6}$

$a = 1.33$

$y = 1.33(1.4)^x$

Example 5 Try It! Use Two Points to Find an Exponent

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5. A surveyor determined the value of an area of land over a period of several years since 1950. The land was worth \$31,000 in 1954 and \$35,000 in 1955. Use the data to determine an exponential model that describes the value of the land.

x = # yrs since 1950. $y = a \cdot b^x$

(4, 31,000)   (5, 35,000)

$31{,}000 = a \cdot b^4$ $35{,}000 = a \cdot b^5$

$a = \frac{31{,}000}{b^4}$ → $35000 = \frac{31000}{b^4} b^5$

$35000 = 31000\, b$

$\frac{35000}{31000} = b$

$b \approx 1.13$

$a = \frac{31000}{(1.13)^4}$

$a = 19012.88$

$y = 19012.88\,(1.13)^x$

Find an equation for the exponential function that passes through these two points:

(2,432) and (5,11664)

$y = a \cdot b^x$

$432 = a \cdot b^2$ $11664 = a \cdot b^5$

$a = \frac{432}{b^2}$ → $11664 = \frac{432}{b^2} b^5$

$\frac{11664}{432} = \frac{432\, b^3}{432}$

$\sqrt[3]{27} = \sqrt[3]{b^3}$

$b = 3$

$a = \frac{432}{3^2}$

$a = \frac{432}{9} = 48$

$y = 48 \cdot 3^x$

Hwk #8 Sec 6-2

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Problems 7-9, 12, 19, 29, 32

Find the value of x in each equation:
Round to the nearest hundredth when needed.

1. $\sqrt[3]{x^3} = \sqrt[3]{1728}$

$x = 12$

2. $(47)^2 = \left(\sqrt{x-7}\right)^2$

$2209 = x - 7$

$+7 \quad +7$

$x = 2216$

Every math operation has it's inverse.

Inverse operations "undo" each other.

We solve equations by using inverses to get the variable by itself.

Find the equation of the inverse for this function:

$$y = \frac{\sqrt{x+3}}{7} - 1$$

$$x = \frac{\sqrt{y+3}}{7} - 1$$

$$[7(x+1)]^2 - 3 = y$$

How do you find the equation of the inverse of an exponential equation?

$$y = 10^x$$

To solve for x in an exponential equation: $y = 10^x$
we use the inverse operation called:

Logarithm

Sec 6-3: Logarithms
(the inverse of exponential functions)

Exponential Function

$$y = b^x$$

The base of the Exponential Function

The exponent

Logarithmic Function

$$\log_b y = x$$

The base of the Logarithmic Function

The answer of the logarithm

Changing from one form to the other:

Exponential Function:

$$y = b^x$$

Logarithmic Function:

"Log, base b, of y equals x"

$$\log_b y = x$$

The base is the base

The exponent is the answer

Another way to remember Logarithmic Form:

Exponential Form:		Logarithmic Form:
$x = y^z$	becomes	$z = \text{Log}_y\, x$