Composite Functions: When two functions are combined into one function.

$f(g(x))$ is read as "f of g of x"

You are substituting the function $g(x)$ into the function $f(x)$.

Given these two functions: $f(x) = 4x - 1$ $g(x) = 3x - 5$

substitution turns these two functions into one composite function:

$$f(g(x)) \longrightarrow 4(3x - 5) - 1$$

$$= 12x - 20 - 1$$

$$\boxed{f(g(x)) = 12x - 21}$$

The symbol for composite functions is: $\circ$

For example: $(f \circ g)$ means $f(g(x))$

you are substituting the function $g(x)$ into $f(x)$.

$f \circ g$ means $(f \circ g)(x)$ which means $f(g(x))$

Given these two functions: $f(x) = 3x^2 - 7$ and $g(x) = 2x+4$

1. Find f(3)

$$f(3) = 3(3)^2 - 7 = 3(9) - 7 = 27 - 7 = \boxed{20}$$

2. Find g(3)

$$g(3) = 2(3) + 4 = 6 + 4 = \boxed{10}$$

3. Find f(g(3))

$$f(g(3)) = f(g(3)) = f(10) = 3(10)^2 - 7 = 3(100) - 7 = 300 - 7 = \boxed{293}$$

4. Find g(f(3))

$$g(f(3)) = g(20) = 2(20) + 4 = 40 + 4 = \boxed{44}$$

EXAMPLE 4 Try It! Compose Functions

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4. Let $f(x) = 2x - 1$ and $g(x) = 3x$. Identify the rules for the following functions.

a. $f(g(2))$ b. $f(g(x))$

1st: $g(2) = 3(2) = 6$

2nd: $f(g(2)) = f(6) = 2(6) - 1 = 12 - 1 = \boxed{11}$

$$= 2(3x) - 1 = \boxed{6x - 1}$$

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EXAMPLE 5 **Try It!** Write a Rule for a Composite Function

5. Identify the rules for $f \circ g$ and $g \circ f$.

a. $f(x) = x^3$, $g(x) = x + 1$ b. $f(x) = x^2 + 1$, $g(x) = x - 5$

$$f \circ g = (x+1)^3 = \underbrace{(x+1)(x+1)}(x+1)$$
$$= (x^2+2x+1)(x+1)$$
$$= \boxed{x^3+3x^2+3x+1}$$

x	x^2	$+2x$	$+1$
x	x^3	$+2x^2$	$+x$
$+1$	$+x^2$	$+2x$	$+1$

$$g \circ f = (x^3)+1$$
$$= \boxed{x^3+1}$$

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EXAMPLE 5 **Try It!** Write a Rule for a Composite Function

5. Identify the rules for $f \circ g$ and $g \circ f$.

a. $f(x) = x^3$, $g(x) = x + 1$ b. $f(x) = x^2 + 1$, $g(x) = x - 5$

$$f \circ g = (x-5)^2 + 1$$
$$= x^2 - 10x + 25 + 1 = \boxed{x^2-10x+26}$$

$$g \circ f = (x^2+1) - 5 = \boxed{x^2-4}$$

Use these two functions:

$f(x) = x + 4$ $g(x) = 3x^2 - 2$

1. Find $f(g(x))$

$$= (3x^2-2) + 4$$
$$= 3x^2+2$$

2. Find $(g \circ f)(x)$

$$3(x+4)^2 - 2$$
$$3(x^2+8x+16) - 2$$
$$3x^2+24x+48-2$$
$$3x^2+24x+46$$

3. Find $g(f(-8))$

first $f(-8) = -8+4 = -4$

Then $g(-4) = 3(-4)^2 - 2$

$3(16) - 2$

$48 - 2 = 46$

Use these two functions:

$f(x) = 2x + 1$ $g(x) = \dfrac{5x}{6x-8}$

1. Find f(g(x)). Simplify.

$$f(g(x)) = 2\left(\frac{5x}{6x-8}\right) + 1$$
$$= \frac{10x}{6x-8} + 1$$
$$= \frac{10x}{6x-8} + \frac{6x-8}{6x-8} = \frac{16x-8}{6x-8} = \boxed{\frac{8x-4}{3x-4}}$$

Use these two functions:

$f(x) = 2x + 1 \qquad g(x) = \frac{5x}{6x - 8}$

2. Find g(f(x)). Simplify.

$$g(f(x)) = \frac{5(2x+1)}{6(2x+1) - 8} = \frac{10x+5}{12x+6-8} = \boxed{\frac{10x+5}{12x-2}}$$

Hwk #6 Sec 5-5

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Due: Thursday

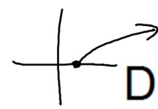
Problems: 14, 21, 23-26, 28

Domain of a composite function:

The domain for $(f \circ g)$ is:
The set of numbers x, that are in the domain $g(x)$,
as long as what comes out of $g(x)$
is in the domain of $f(x)$.

Given these functions:

$g(x) = \sqrt{x - 1} \qquad f(x) = \frac{1}{x - 3}$



$x - 1 \geq 0$

Domain of $f(g(x))$ is

The domain of g(x) is [1,∞)

You can't substitute the value of 3 into f(x). This means g(x)≠3 so you're not allowed to use the x value of 10 either since it will make g(x)=3

domain of g(x): [1,∞) ≠ 10

The combination of these two gives us the domain for f(g(x)):

$\boxed{[1,\infty) \text{ but } x \neq 10 \longrightarrow [1,10) \cup (10,\infty)}$

You have two coupons to use at a store, one gives you 10% off and the other gives you $20 off. You are allowed to use both coupons for the same purchase.

- Does is matter which one you use first?
- If yes, which one should you use first?

<u>10% off</u>

$P(x)=0.9x$

<u>$20 off</u>

$T(x)=x - 20$

x = Original Price

10% off first: $T(P(x))=$ $\boxed{.90x - 20}$ this is a lower price

$20 off first: $P(T(x))=$ $.9(x-20) = .90x - 18$

Therefore, it is better to use the 10% off coupon first followed by the $20 off coupon.