

Sec 5-5: Function Operations

- Function Addition
- Function Subtraction
- Function Multiplication
- Function Division
- Composite Functions

Function addition and subtraction is simply **combining like terms**.

Function multiplication is simply **expanding**:
Distributive Property, **FOIL**, using the **Box**.

Defining a function includes describing its **DOMAIN**.

The domains of $f + g$, $f - g$, and $f \cdot g$ are the intersections of the domains of the individual functions f and g .

- Where the domains of f and g overlap.
- #'s that are in both domains at the same time.

The domains of $f+g$, $f-g$, and $f \cdot g$ are

In general, if you just add or subtract two functions no new restrictions are going to result that weren't already restrictions on the original functions f and g .

This is also usually true of function multiplication but there may be a rare instance when multiplying that a new restriction is created.

State the domains of each:

$$\begin{aligned} f(x) &= x^2 + 3x - 1 & \text{Domain: } (-\infty, \infty) \\ g(x) &= x + 8 & \text{Domain: } (-\infty, \infty) \end{aligned} \quad \left. \vphantom{\begin{aligned} f(x) &= x^2 + 3x - 1 \\ g(x) &= x + 8 \end{aligned}} \right\} \begin{array}{l} \text{intersection} \\ \text{is:} \\ (-\infty, \infty) \end{array}$$

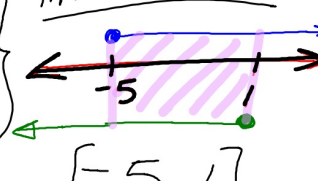
$$\begin{aligned} f+g & \text{ Domain:} \\ f-g & \text{ Domain:} \\ f \cdot g & \text{ Domain:} \end{aligned} \quad \left. \vphantom{\begin{aligned} f+g \\ f-g \\ f \cdot g \end{aligned}} \right\} \begin{array}{l} \text{all domains are} \\ (-\infty, \infty) \end{array}$$

State the domains of each:

$$\begin{aligned} f(x) &= \frac{2}{x+3} & \text{Domain: } x \neq -3 \\ g(x) &= x - 7 & \text{Domain: } (-\infty, \infty) \end{aligned} \quad \left. \vphantom{\begin{aligned} f(x) &= \frac{2}{x+3} \\ g(x) &= x - 7 \end{aligned}} \right\} \begin{array}{l} \text{intersection} \\ \text{is:} \\ x \neq -3 \end{array}$$

$$\begin{aligned} f+g & \text{ Domain:} \\ f-g & \text{ Domain:} \\ f \cdot g & \text{ Domain:} \end{aligned} \quad \left. \vphantom{\begin{aligned} f+g \\ f-g \\ f \cdot g \end{aligned}} \right\} \begin{array}{l} \text{all domains are:} \\ x \neq -3 \end{array}$$

State the domains of each:

$$\begin{aligned} f(x) &= \sqrt{x+5} & \text{Domain: } [-5, \infty) \\ g(x) &= \sqrt{-(x-1)} & \text{Domain: } (-\infty, 1] \end{aligned} \quad \left. \vphantom{\begin{aligned} f(x) &= \sqrt{x+5} \\ g(x) &= \sqrt{-(x-1)} \end{aligned}} \right\} \begin{array}{l} \text{intersection is} \\ [-5, 1] \end{array}$$


$$\begin{aligned} f+g & \text{ Domain:} \\ f-g & \text{ Domain:} \\ f \cdot g & \text{ Domain:} \end{aligned} \quad \left. \vphantom{\begin{aligned} f+g \\ f-g \\ f \cdot g \end{aligned}} \right\} \begin{array}{l} \text{all domains} \\ \text{are} \\ [-5, 1] \end{array}$$

The domain of $\frac{f}{g}$ is the intersection of f , g , and $\frac{f}{g}$

State the domains of each:

$$\begin{aligned} f(x) &= x^2 + 3x - 1 \quad \text{Domain: } (-\infty, \infty) \\ g(x) &= x + 8 \quad \text{Domain: } (-\infty, \infty) \end{aligned} \quad \left. \vphantom{\begin{aligned} f(x) &= x^2 + 3x - 1 \\ g(x) &= x + 8 \end{aligned}} \right\} \begin{array}{l} \text{intersection} \\ \text{is } \\ (-\infty, \infty) \end{array}$$

$\frac{f}{g}$ Domain:

→ since $g(x)$ is now in the denominator there is an additional restriction: $x \neq -8$

final domain is the combination of $(-\infty, \infty)$ and $x \neq -8$

$D: x \neq -8$

State the domains of each:

$$\begin{aligned} f(x) &= \frac{2}{x+3} \quad \text{Domain: } x \neq -3 \\ g(x) &= x - 7 \quad \text{Domain: } (-\infty, \infty) \end{aligned} \quad \left. \vphantom{\begin{aligned} f(x) &= \frac{2}{x+3} \\ g(x) &= x - 7 \end{aligned}} \right\} \begin{array}{l} \text{intersection} \\ \text{is} \\ \text{all real} \\ \text{\#s except} \\ -3: x \neq -3 \end{array}$$

$\frac{f}{g}$ Domain:

→ since $g(x)$ is now in the denominator there is an additional restriction: $x \neq 7$

final domain is the combination of $x \neq -3$ and $x \neq 7$

$D: x \neq -3, 7$

State the domains of each:

$$\begin{aligned} f(x) &= \sqrt{x+5} \quad \text{Domain: } [-5, \infty) \\ g(x) &= \sqrt{-(x-1)} \quad \text{Domain: } (-\infty, 1] \end{aligned} \quad \left. \vphantom{\begin{aligned} f(x) &= \sqrt{x+5} \\ g(x) &= \sqrt{-(x-1)} \end{aligned}} \right\} \begin{array}{l} \text{intersection} \\ \text{is } \\ [-5, 1] \end{array}$$

$\frac{f}{g}$ Domain:

→ since $g(x)$ is in the denominator a new restriction $x \neq 1$

final domain is the combination of $[-5, 1]$ and $x \neq 1$

$D: [-5, 1)$

Finding the rule for $\frac{f}{g}$

- Factor and simplify.
- or
- Perform division.

✓ → we will probably
this most of
the time.

EXAMPLE 3 Try It! Divide Functions

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3. Identify the rule and domain for $\frac{f}{g}$ for each pair of functions.

a. $f(x) = x^2 - 3x - 18$, $g(x) = x + 3$ b. $f(x) = x - 3$, $g(x) = x^2 - x - 6$

$$f(x) = x^2 - 3x - 18 \quad \text{Domain: } (-\infty, \infty)$$

$$= (x - 6)(x + 3)$$

$$g(x) = x + 3 \quad \text{Domain: } (-\infty, \infty)$$

$$\frac{f}{g} = \frac{x^2 - 3x - 18}{x + 3} = \frac{(x - 6)(x + 3)}{x + 3}$$

$$\boxed{\frac{f}{g} = x - 6}$$

Domain: $g(x)$ is now a denominator there is a restriction:

$$\boxed{D: x \neq -3}$$

EXAMPLE 3 Try It! Divide Functions

3. Identify the rule and domain for $\frac{f}{g}$ for each pair of functions.

a. $f(x) = x^2 - 3x - 18$, $g(x) = x + 3$ b. $f(x) = x - 3$, $g(x) = x^2 - x - 6$

$$f(x) = x - 3 \quad \text{Domain: } (-\infty, \infty)$$

$$g(x) = x^2 - x - 6 = (x - 3)(x + 2) \quad \text{Domain: } (-\infty, \infty)$$

$$\frac{f}{g} = \frac{x - 3}{(x - 3)(x + 2)} = \frac{1}{x + 2}$$

$$\boxed{\frac{f}{g} = \frac{1}{x + 2}}$$

Domain: since $g(x)$ is now a denominator there are restrictions

$$\boxed{D: x \neq -2, 3}$$

$$\text{Given: } 5x + 4y = 12 \quad \text{and} \quad y = 2x - 3$$

Use substitution to write a single equation in terms of x . Simplify.

$$\text{Given: } 5x + 4y = 12 \quad \text{and} \quad y = 2x - 3$$

$$5x + 4(2x - 3) = 12$$

$$5x + 4(2x - 3) = 12 \quad \text{DISTRIBUTE}$$

$$5x + 8x - 12 = 12 \quad \text{Combine like-terms}$$

$$\boxed{13x - 12 = 12}$$