Write the equations of the Horizontal Asymptotes, if any, for each Rational Function.

1. $y = \dfrac{x^3 + 7x^2 + 11x}{x^2 + 4x - 3}$

NO HA

2. $y = \dfrac{8x^2 - 7x}{x^3 + 4x^2 + 9x - 1}$

$y = 0$

3. $y = \dfrac{2x^2 - 5x + 3}{x^2 - 6x - 10}$

$y = 2$

4. $y = \dfrac{8x^4 + 3x^2 + 11x - 9}{4x^3 - 7x^2 - x^4 - 3}$

NO HA

Horizontal Asymptotes: Depends on the degrees of the Numerator and the Denominator

Case 1: Degree of the Numerator > Degree of the Denominator

HA: None

Case 2: Degree of the Numerator = Degree of the Denominator

HA: y = ratio of the Leading Coefficients

Case 3: Degree of the Denominator > Degree of the Numerator

HA: $y = 0$

x and y intercepts of Rational Functions:

Y-Intercepts: the result of replacing x with zero.

Find the y-intercepts of each function.

$y = \dfrac{x^2 - 9x + 20}{x^2 + 7x + 10} = \dfrac{0^2 - 9(0) + 20}{0^2 + 7(0) + 10} = \dfrac{20}{10}$ y-int: $y = 2$

$y = \dfrac{x^2 - 4}{2x^2 + 6x} = \dfrac{0^2 - 4}{2(0)^2 + 6(0)} = \dfrac{-4}{0}$ y-int: No y-int

undefined!

In general, the y-intercepts of Rational Functions are the:

Ratio of the Constants

A graph can have at most ONE y-intercept.

X-Intercepts: the result of replacing y with zero.

This means you are setting the ratio equal to zero and solving for x.

Find the x-intercepts of this rational function:

$$y = \frac{x^2 + 3x - 10}{x^2 - 1}$$

$$(x^2-1)\,0 = \frac{x^2+3x-10}{x^2-1} \cdot (x^2-1)$$

$$0 = x^2 + 3x - 10$$

$$0 = (x+5)(x-2)$$

$$x = -5, 2$$

$$\boxed{\text{x-int} = -5, 2}$$

The only way a fraction equals zero is if the NUMERATOR equals zero.

Find the x-intercepts of this Rational Function

$$y = \frac{x^2 - 6x + 8}{x^2 + 4x + 3}$$

$$x^2 - 6x + 8 = 0$$

$$(x-4)(x-2) = 0$$

$$x = 2, 4$$

$$\boxed{\text{x-int} = 2, 4}$$

In general, the x-intercepts of a Rational Function are the:

Zeros of the numerator.

Exception to this rule?
Zeros of the numerator as long as they don't match zeros of the denominator, otherwise, they would be HOLES.

A graph <u>can have more than one</u> x-intercept.

find the x-intercepts of each function.

1. $y = \frac{x^2 - 9x + 20}{x^2 + 7x + 10} = \frac{(x-4)(x-5)}{(x+5)(x+2)} \rightarrow x = 4, 5$

x-int: $\boxed{x = 4, 5}$

neither of these values are also zeros of the denominator so they are both x-int.

2. $y = \frac{x^2 - 4}{2x^2 + 6x} = \frac{(x+2)(x-2)}{2x(x+3)} \rightarrow x = \pm 2$

x-int: $\boxed{x = \pm 2}$

neither of these values are also zeros of the denominator so they are both x-int.

3. $y = \frac{3x^2 + 5}{x^2 - 2x - 3} \rightarrow$ No Real zeros

x-int: $\boxed{\text{No x-int}}$

Hwk #42:

Due Tomorrow

Practice Sheet: Horizontal Asymptotes and x & y-intercepts

Topic 9/10 Topic Quiz FRIDAY

Let's put this all together.
Graph the following Rational Function showing:

- All asymptotes as dashed lines
- X & Y-intercepts, if any
- Correct behavior around each asymptote.

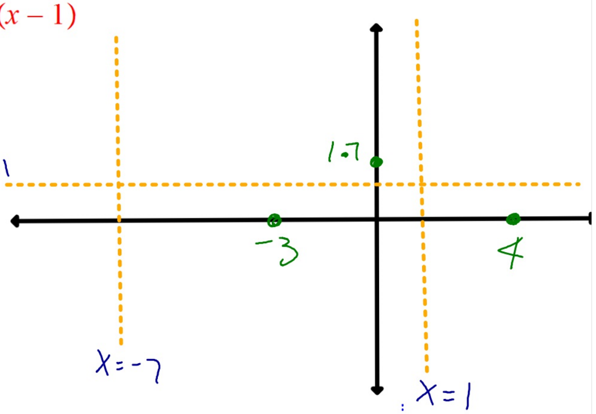
$$y = \frac{x^2 - x - 12}{x^2 + 6x - 7} = \frac{(x-4)(x+3)}{(x+7)(x-1)}$$

x-int: $x = {}^{-}3, 4$

y-int: $y = 12/7 \approx 1.7$

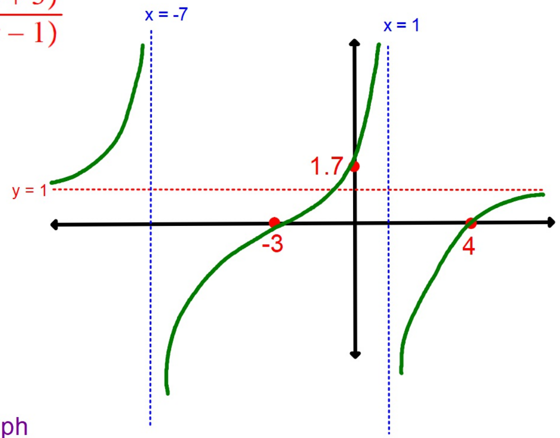
VA: $X = 1, -7$

HA: $y = 1$



$$y = \frac{x^2 - x - 12}{x^2 + 6x - 7} = \frac{(x-4)(x+3)}{(x+7)(x-1)}$$

What do you think this graph will look like?



The graph can never touch VA, must cross the axes only at the interepts, and approaches the HA at the far left and right.

Because there is two VA, the graph has three parts. As a graph approaches a VA it either goes up or down very quickly. And usually, when graph goes up on one side of a VA it does down on the other side.