Horizontal Asymptotes:

The horizontal line that a graph approaches as you move farther and farther to the left and right. (END BEHAVIOR)

The value of y that the function approaches as x gets larger and larger (neg and pos).

The function should approach the same value of y on both ends or the asymptote wouldn't be horizontal.

One way to find the horizontal asymptote of a rational function is to use the table function on the graphing calculator.

Enter this equation into Y_1 then go to the table. $y = \dfrac{4x^2+3x-7}{7x-2x^2+5}$

To find the Horizontal Asymptote, if there is one, enter the following:

Big positive x-values to represent the far right....what is y getting close to? -2

Big negative x-values to represent the far left....what is y getting close to? -2

What is the Horizontal Asymptote?

$\boxed{y = -2}$

X	Y
100	-2.088
1000	-2.009
10000	-2.001
-100	-1.918
-1000	-1.992
-10000	-1.999

since the y-values are approaching -2 on both the left and right sides, the horizontal asymptote is y = -2

Use this technique to find the Horizontal Asymptote, if there is one, for each of the following:

1 $y = \dfrac{x^2+9x-11}{2x^3-x}$

X	Y
100	0.0054
1000	$5E-4 = 5\times10^{-4} = .0005$
10000	5E-5 = .00005
-100	-0.005
-1000	-5E-4 = -.0005
-10000	-5E-6 = -.00005

These y-values keep getting smaller and smaller (small pos or small neg). This means that the y-values are getting closer to zero on both sides.

The HA is y = 0

2 $y = \dfrac{5x^2+3x-1}{4x+7}$

X	Y
100	123.58
1000	1248.6
10000	12499
-100	-126.5
-100	-1251
-10000	-12501

The y-values are getting bigger positive on the right and bigger negative on the left. Therefore, they aren't approaching the same y-value on both sides. This means that there is NO HA.

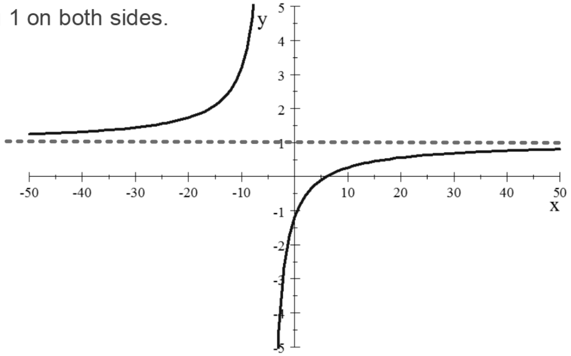
Horizontal Asymptote Exploration:

Students were given an exploration to work on to explore how the HA, if any, is related to what you see in the equation.

Results of the exploration are shown in the following pages.

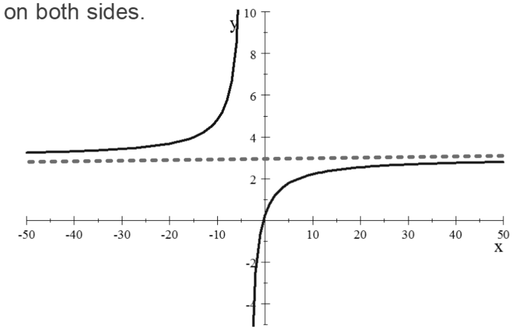
1. $y = \dfrac{x-6}{x+5}$ HA: $y = 1$

y-values in the table were approaching 1 on both sides.



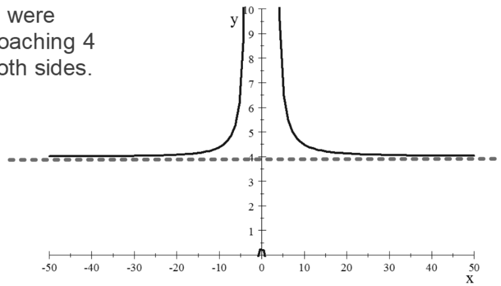
2. $y = \dfrac{3x+1}{x+4}$ HA: $y = 3$

y-values in the table were approaching 3 on both sides.



3. $y = \dfrac{8x^2+x-6}{2x^2-21}$ HA: $y = 4$

y-values in the table were approaching 4 on both sides.



What do you notice in the equations that would give you the HA?

1. $y = \dfrac{x-6}{x+5}$ HA: $y = 1/1$

y = 1

2. $y = \dfrac{3x+1}{x+4}$ HA: $y = 3/1$

y = 3

3. $y = \dfrac{8x^2+x-6}{2x^2-21}$ HA: $y = \dfrac{8}{2}$

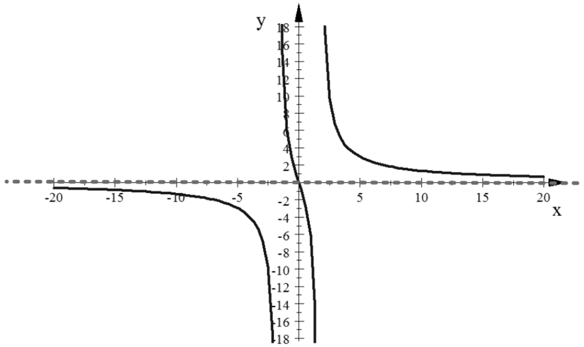
y = 4

What do these three equations have in common?

The degree of the numerator and denominator are the same.

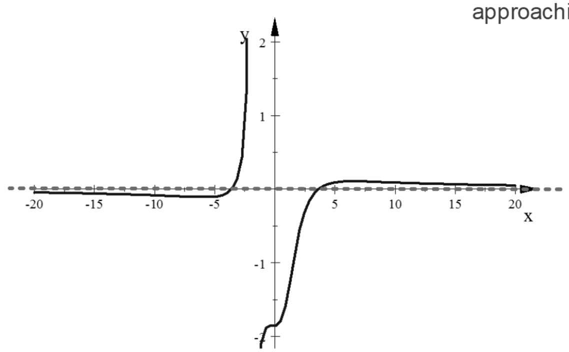
4. $y = \frac{4x + 9x}{x^2 - 3}$ HA: $y=0$

y-values in the table were approaching 0 on both sides.



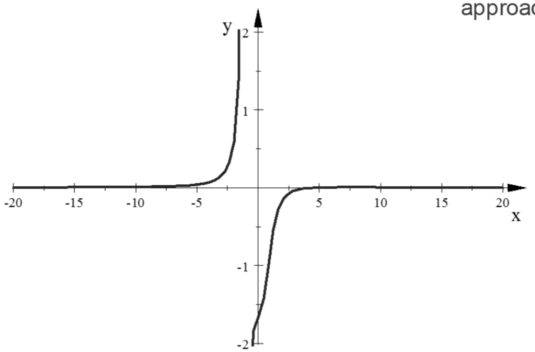
5. $y = \frac{x^2 - 13}{x^3 + 7}$ HA: $y=0$

y-values in the table were approaching 0 on both sides.



6. $y = \frac{x - 5}{2x^3 + 3}$ HA: $y=0$

y-values in the table were approaching 0 on both sides.



What do you notice in the equations that would tell you the HA is y = 0?

4. $y = \frac{4x + 9x}{x^2 - 3}$ HA: y=0

5. $y = \frac{x^2 - 13}{x^3 + 7}$ HA: y=0

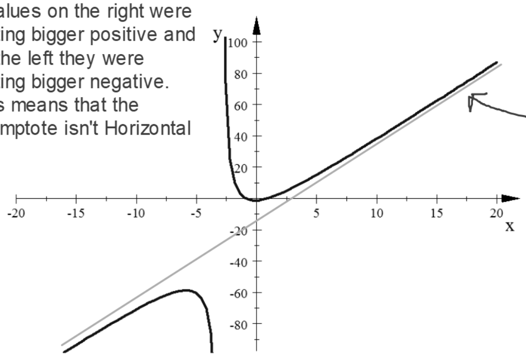
6. $y = \frac{x - 5}{2x^3 + 3}$ HA: y=0

What do these three equations have in common?

The degree of the denominator is greater than the degree of the numerator.

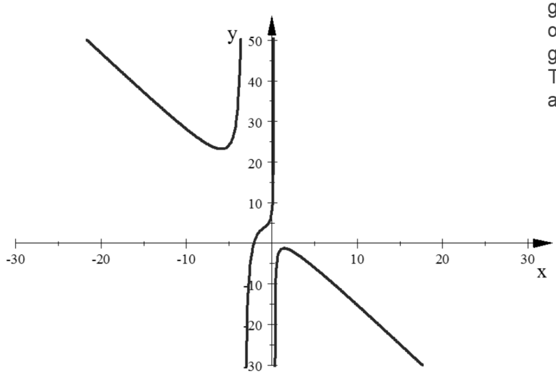
7. $y = \dfrac{5x^2 - 4}{x + 3}$ HA: NO HA

y-values on the right were getting bigger positive and on the left they were getting bigger negative. This means that the asymptote isn't Horizontal



this function
has what is called
a Slant Asymptote.

8. $y = \dfrac{-2x^3 + 5x - 8}{x^2 + 3x - 1}$ HA: NO HA



y-values on the right were getting bigger negative and on the left they were getting bigger positive. This means that the asymptote isn't Horizontal.

What do you notice in the equations that would tell you that there is no HA?

7. $y = \dfrac{5x^2 - 4}{x + 3}$ HA: No HA

8. $y = \dfrac{-2x^3 + 5x - 8}{x^2 + 3x - 1}$ HA: No HA

What do these three equations have in common?

The degree of the numerator is greater than the degree of the denominator.

Without using a table or a graph how could you tell from the equation what the horizontal asymptote is or if it even has one?

Predict the Horizontal Asymptote for each of the rational functions below. if an

a. $y = \dfrac{10x + 7}{5x - 3}$ same degree

HA $y = \dfrac{10}{5} = 2$

b. $y = \dfrac{6x^2 - 5}{2x + 3}$ numerator degree is bigger

HA NO HA

c. $y = \dfrac{12x - 11}{3x^2 - 1}$ denom degree is bigger

HA $y = 0$

Horizontal Asymptotes: Depends on the degrees of the Numerator and the Denominator

Case 1: Degree of the Numerator > Degree of the Denominator

No HA

Case 2: Degree of the Numerator = Degree of the Denominator

HA: y = ratio of the Leading Coefficients

Case 1: Degree of the Denominator > Degree of the Numerator

HA: y = 0