When the denominator of a rational function is zero the function is undefined.

Because this value of x can never be used this leads to a break in the graph (it's not continuous)

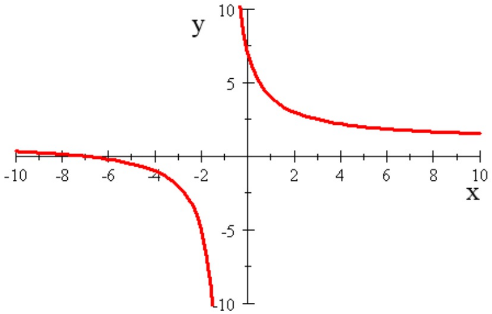
These breaks in the graph are one of two types:

Vertical Asymptotes

Holes

Graph the rational function f(x) in a standard window.

$$f(x) = \frac{x+7}{x+1}$$



There is a break in the graph when x = -1

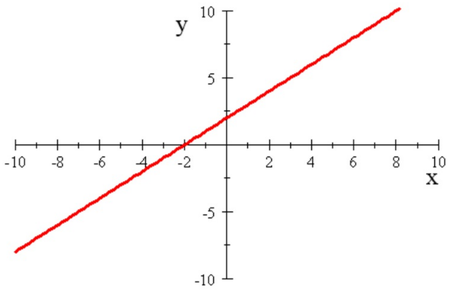
This kind of break in the graph is called a

Vertical Asymptote

Graph the rational function f(x) in a standard window.

$$f(x) = \frac{(x-3)(x+2)}{(x-3)}$$

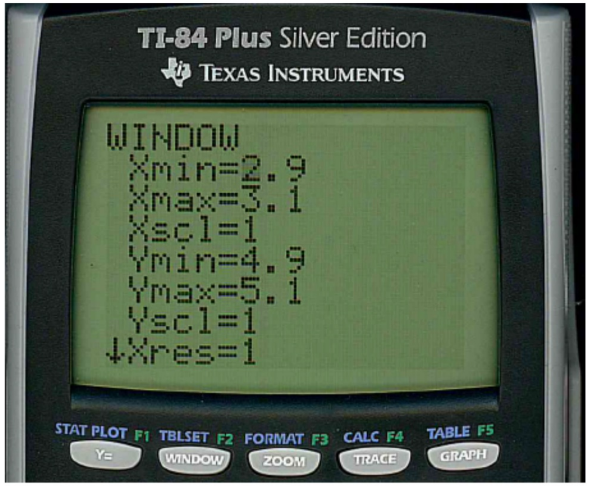
Do you see a vertical asymptote? NO



Why do you think that there isn't a vertical asymptote at x = 3?

except at x=3 the calculator sees this function as y=x+2 since the x-3 "cancel" each other

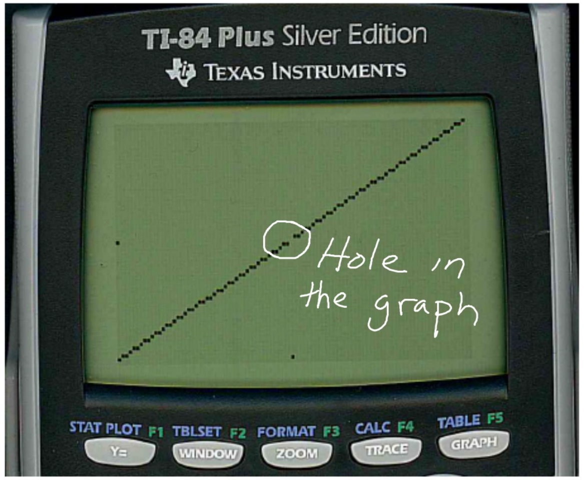
Change the window to the following:



What do you see?

This kind of break in the graph is called a **Hole**

$$f(x) = \frac{(x-3)(x+2)}{(x-3)}$$



Why did this graph have a Vertical Asymptote at x = -1

$$f(x) = \frac{x+7}{x+1}$$

but

this graph had a hole at x = 3?

$$f(x) = \frac{(x-3)(x+2)}{(x-3)}$$

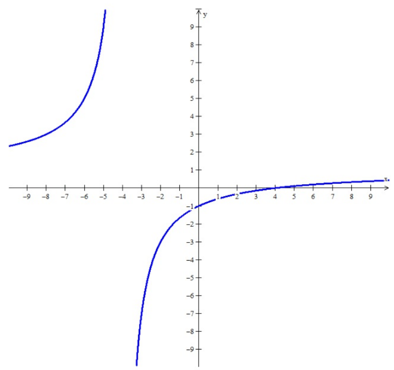
Breaks in the graph are caused by zeros of the denominator and are called:

Points of Discontinuity

Holes	**or**	**Vertical Asymptotes**
Occur at values of x that are zeros of both the denominator AND numerator		Occur at values of x that are zeros of the denominator ONLY.

An exception to this

$$y = \frac{x^2 - 16}{x^2 + 8x + 16} = \frac{\cancel{(x+4)}(x-4)}{\cancel{(x+4)}(x+4)}$$



Why is there a VA at x = -4 and not a hole?

Even though the factors (x+4) are common to the numerator and denominator, when you cancel them there is still (x+4) left in the denominator, thus, creating a VA.

Properties **Vertical Asymptotes**

The rational function $f(x) = \frac{P(x)}{Q(x)}$ has a point of discontinuity for each real zero of $Q(x)$.

If $P(x)$ and $Q(x)$ have no common real zeros, then the graph of $f(x)$ has a vertical asymptote at each real zero of $Q(x)$.

If $P(x)$ and $Q(x)$ have a common real zero a, then there is a hole in the graph or a vertical asymptote at $x = a$.

Find any points of dicontinuity and classify them as Vertical Asymptotes or Holes.

1. $y = \frac{2x^3 - 10x^2 - 12x}{x^2 - 7x + 6}$ $\quad \frac{2x(x-6)(x+1)}{(x-6)(x-1)}$

Pts of Discontinuity: $x = 1, 6$

VA: $x = 1$ $\rightarrow$ zero of denominator ONLY

Holes: $x = 6$ $\rightarrow$ zero of numerator AND denominator

2. $y = \frac{3x^2 - 6}{x^2 - 4} = \frac{3(x^2 - 2)}{(x+2)(x-2)}$ NO Real zeros

Pts of Discontinuity: $x = \pm 2$

VA: $x = \pm 2$ $\rightarrow$ these are zeros of denominator only.

Holes: NONE

3. $y = \frac{x^2 - x - 12}{x^2 - 16}$ $\quad \frac{(x-4)(x+3)}{(x+4)(x-4)}$

Pts of Discontinuity: $x = \pm 4$

VA: $x = -4$

Holes: $x = 4$ $\rightarrow$ since $x=4$ is a zero of both numerator and denominator

4. $y = \dfrac{x^2 + 6x + 9}{x^2 + 5x + 6} = \dfrac{(x+3)(x+3)}{(x+3)(x+2)}$

Pts of Discontinuity: $x = -3, -2$

VA: $x = -2$

Holes: $x = -3$

Since the "extra" $(x+3)$ occurs in the numerator $x=-3$ is only a hole.
If the "extra" $(x+3)$ occured in the denominator $x=-3$ would become a V.A.

5. $y = \dfrac{2x^2}{x^2 + 3}$ → no real # makes this zero. Graph is never undefined.

Pts of Discontinuity: NONE

VA: N.A.

Holes: N.A.

$\underbrace{\text{There are no points of discontinuity because the denominator has no real zeros!}}_{\text{there is no break in the graph}}$

Hwk #41 Due Tomorrow

Practice Sheet: Points of Discontinuity