When simplifying radical expressions your answer should NOT have radicals in the denominator.

Radicals that can't be simplified are irrational numbers. We'd prefer not to have denominators that are irrational.

To eliminate a radical from a denominator is called: Rationalizing the Denominator

$$\sqrt{2} \cdot \sqrt{2} = 2$$

Any square root multiplied by itself equals the radicand.

$$\sqrt[3]{2} \cdot \sqrt[3]{2} \neq 2 \qquad \sqrt[3]{2} \cdot \sqrt[3]{2} = \sqrt[3]{4}$$

A cube root multiplied by itself will not necessarily eliminate the radical as it does with square roots.

Simplify. Rationalize this denominator.

$$\frac{5}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{5\sqrt{3}}{3}}$$

Simplify. Rationalize this denominator.

$$\frac{9}{\sqrt{6a}} \cdot \frac{\sqrt{6a}}{\sqrt{6a}}$$

$$= \frac{9\sqrt{6a}}{6a} = \boxed{\frac{3\sqrt{6a}}{2a}}$$

Simplify. Rationalize this denominator.

One method is to simplify the radical first then rationalize.

$$\frac{2m^3}{\sqrt{18m^5n^3}}$$

$9 \cdot 2$

$$\frac{2m^3}{3m^2n\sqrt{2mn}} \cdot \frac{\sqrt{2mn}}{\sqrt{2mn}} = \frac{2m^3\sqrt{2mn}}{3m^2n \cdot (2mn)}$$

$2mn$

$$= \frac{2m^3\sqrt{2mn}}{6m^3n^2}$$

$$= \boxed{\frac{\sqrt{2mn}}{3n^2}}$$

Simplify. Rationalize this denominator.

Another method is to pick the smallest radicand that will create perfect squares when multiplied by the original radicand.

$$\underbrace{\frac{2m^3}{\sqrt{18m^5n^3}} \cdot \frac{\sqrt{2mn}}{\sqrt{2mn}}}_{\sqrt{36m^6n^4}} = \frac{2m^3\sqrt{2mn}}{6m^3n^2}$$

$$= \boxed{\frac{\sqrt{2mn}}{3n^2}}$$

If both the numerator and denominator are under the same radical you could reduce the fraction first then see what needs to be rationalized.

$$\frac{\sqrt{6cd^2}}{\sqrt{21c^5d^9}} = \frac{\sqrt{2}}{\sqrt{7c^4d^7}}$$

$$= \underbrace{\frac{\sqrt{2}}{\sqrt{7c^4d^7}} \cdot \frac{\sqrt{7d}}{\sqrt{7d}}}_{\sqrt{7^2c^4d^8}} = \boxed{\frac{\sqrt{14d}}{7c^2d^4}}$$

Simplify. Rationalize this denominator.

$$\underbrace{\frac{8y^3}{\sqrt{12x^{17}y^{21}}} \cdot \frac{\sqrt{3xy}}{\sqrt{3xy}}}_{\sqrt{36x^{18}y^{22}}} = \frac{8y^3\sqrt{3xy}}{6x^9y^{11}}$$

$$= \boxed{\frac{4\sqrt{3xy}}{3x^9y^8}}$$

Simplify. Rationalize this denominator.

when rationalizing cube roots we need to create perfect cubes under the radical.

One method is to simplify the radical first then rationalize.

$$\frac{9}{\sqrt[3]{7c^4d^7}} = \frac{9}{cd^2\sqrt[3]{7cd}} \cdot \frac{\sqrt[3]{7^2c^2d^2}}{\sqrt[3]{7^2c^2d^2}} = \frac{9\sqrt[3]{49c^2d^2}}{cd^2\cdot 7cd}$$

$$\sqrt[3]{7^3c^3d^3}$$

$$= \boxed{\frac{9\sqrt[3]{49c^2d^2}}{7c^2d^3}}$$

Another method is to pick the smallest radicand that will create perfect cubes when multiplied by the original radicand.

$$\frac{9}{\sqrt[3]{7c^4d^7}} \cdot \frac{\sqrt[3]{7^2c^2d^2}}{\sqrt[3]{7^2c^2d^2}}$$

$$\sqrt[3]{7^3c^6d^9}$$

$$= \boxed{\frac{9\sqrt[3]{7^2c^2d^2}}{7c^2d^3}}$$

Simplify. Rationalize this denominator.

$$\frac{10b^2}{\sqrt[3]{4a^3b}} \cdot \frac{\sqrt[3]{2b^2}}{\sqrt[3]{2b^2}} = \frac{10b^2\sqrt[3]{2b^2}}{2ab}$$

$$\sqrt[3]{8a^3b^3}$$

$$= \boxed{\frac{5b\sqrt[3]{2b^2}}{a}}$$

Binomial radical expression.

A binomial where one or both terms are radical expressions.

Examples:

$$8 + 3\sqrt{7}$$

$$2\sqrt[3]{5} - \sqrt{6}$$

Find each product.

1. $\sqrt{6}(5-2\sqrt{6})$

$5\sqrt{6} - 2\sqrt{6}\cdot\sqrt{6}$

$= 5\sqrt{6} - 2\cdot 6$

$= \boxed{5\sqrt{6}-12}$

2. $2\sqrt[3]{4}(7+3\sqrt[3]{4})$

$= 2\cdot 7\cdot\sqrt[3]{4} + 2\cdot 3\cdot\sqrt[3]{4}\cdot\sqrt[3]{4}$

$= 14\sqrt[3]{4} + 6\sqrt[3]{\underbrace{16}_{8\cdot 2}}$

$= 14\sqrt[3]{4} + 6\cdot 2\sqrt[3]{2}$

$= \boxed{14\sqrt[3]{4}+12\sqrt[3]{2}}$

$(4-3\sqrt{5})(7+2\sqrt{5})$

	4	$-3\sqrt{5}$
7	28	$-21\sqrt{5}$
$+2\sqrt{5}$	$+8\sqrt{5}$	$-6\cdot 5$ $=-30$

$= \boxed{-2 \quad -13\sqrt{5}}$

What's going to happen here?

$(7+2\sqrt{5})(6-4\sqrt{3})$

Because the radicals are different you won't have any like terms when you expand it and the answer will end up with four terms.

$(4-\sqrt{3})(4+\sqrt{3})$

These two binomials are called CONJUGATES

	4	$-\sqrt{3}$
4	16	$\cancel{-4\sqrt{3}}$
$+\sqrt{3}$	$\cancel{+4\sqrt{3}}$	-3

$= 16-3 = \boxed{13}$

The product of binomial radical conjugates is always a constant.

Rationalizing a Binomial Denominator.

$$\frac{5}{2-\sqrt{3}}$$

multiply the numerator and denominator by the **conjugate** of the denominator.

$$\frac{5}{2-\sqrt{3}}\cdot\frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{5(2+\sqrt{3})}{1} = \boxed{5(2+\sqrt{3}) \text{ or } 10+5\sqrt{3}}$$

	2	$-\sqrt{3}$
2	4	$\cancel{-2\sqrt{3}}$
$+\sqrt{3}$	$\cancel{+2\sqrt{3}}$	-3

$= 4-3=1$

Rationalize the denominator.

$$\frac{24}{7+\sqrt{5}}\cdot\frac{7-\sqrt{5}}{7-\sqrt{5}} = \frac{24(7-\sqrt{5})}{44} = \boxed{\frac{6(7-\sqrt{5})}{11} \text{ or } \frac{42-6\sqrt{5}}{11}}$$

	7	$+\sqrt{5}$
7	49	$\cancel{+7\sqrt{5}}$
$-\sqrt{5}$	$\cancel{-7\sqrt{5}}$	-5

$= 44$

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Problems: 30, 32, 34, 36, 37, 39, 41, 44, 46, 47