N.

a) Find the remainder $\frac{2x^3 + 6x^2 + 2x + 5}{x + 3}$

$$\begin{array}{r|rrrr} -3 & 2 & 6 & 2 & 5 \\ & & -6 & 0 & -6 \\ \hline & 2 & 0 & 2 & -1 \end{array}$$

1.

a) Given $f(x) = 2x^3 + 6x^2 + 2x + 5$

find $f(-3)$

$2(-3)^3 + 6(-3)^2 + 2(-3) + 5$

$= -1$

b) Given $f(x) = 3x^3 - 7x^2 - 5x + 1$

find $f(4)$

$3(4)^3 - 7(4)^2 - 5(4) + 1$

$= 61$

b) Find the remainder $\frac{3x^3 - 7x^2 - 5x + 1}{x - 4}$

$$\begin{array}{r|rrrr} 4 & 3 & -7 & -5 & 1 \\ & & 12 & 20 & 60 \\ \hline & 3 & 5 & 15 & \boxed{61} \end{array}$$

These examples show a connection between finding the remainder of a quotient and evaluating the dividend using the zero of the divisor.

The Remainder Theorem

For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$.

From the theorem, you can see that $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

To find the remainder of a polynomial division problem you can:

1. Actually divide the dividend by the divisor.

or

2. Evaluate the dividend using the zero of the divisor.

Given $f(x) = 2x^3 + 7x^2 - 4x + 5$

Find the remainder when $f(x)$ is divided by $x + 3$

1. You could do the division. OR 2. You could find $f(-3)$.

$$\begin{array}{r|rrrr} -3 & 2 & 7 & -4 & 5 \\ & & -6 & -3 & 21 \\ \hline & 2 & 1 & -7 & 26 \end{array}$$

$f(-3) = 26$

Remainder = 26

Find the remainder. $\frac{x^3 - 3x^2 + 5x - 1}{x - 2}$

Do the division OR Use the Remainder Theorem:

$$\begin{array}{r|rrrr} 2 & 1 & -3 & 5 & -1 \\ & & 2 & -2 & 6 \\ \hline & 1 & -1 & 3 & 5 \end{array}$$

$f(2) = 5$

Remainder = 5

Find the remainder. $\frac{x^3 - 2x + 7}{x + 5}$

Do the division or Use the remainder theorem

$$\begin{array}{r|rrrr} -5 & 1 & 0 & -2 & 7 \\ & & -5 & 25 & -115 \\ \hline & 1 & -5 & 23 & -108 \end{array}$$

$f(-5) = -108$

$\boxed{\text{Remainder} = -108}$

Given $f(x) = 2x^3 + 4x^2 - 6x + 10$

Find $f(-5)$ without substituting -5 for x.

$$\begin{array}{r|rrrr} -5 & 2 & 4 & -6 & 10 \\ & & -10 & 30 & -120 \\ \hline & 2 & -6 & 24 & -110 \end{array}$$

$\boxed{f(-5) = -110}$

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In fact, synthetic division is often referred to as **synthetic substitution**.

The previous page and the above statement indicates that instead of evaluating a function for a given the value you can arrive at the same result by doing synthetic division with the same value and the remainder will be the answer.

Is $x - 1$ a factor of $2x^3 - 4x^2 + 9x - 12$?

$$f(1) = -4(1)^2 + 9(1) - 12$$
$$= -4 + 9 - 12$$
$$= -5$$

No, x-1 is not a factor because the remainder isn't Zero.

The Fundamental Theorem of Algebra

The Fundamental Theorem of Algebra

Every non-constant single-variable polynomial of degree ≥ 1 with complex coefficients has at least one complex root.

Every non-constant polynomial has at least one zero.

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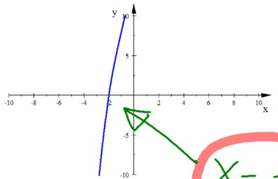
One consequence of the Fundamental Theorem of Algebra is that every nonzero single-variable polynomial with real coefficients has exactly as many nondistinct complex roots as its degree. Do you see why?

This statement says that the number of zeros of a polynomial equals the degree of the polynomial. "Nondistinct" indicates that they don't all have to be different. In other words, zeros may repeat but would still count towards the total number.

Find ALL solutions.

$$y = x^3 + 3x^2 + 10x + 16$$



$$\begin{array}{r|rrrr} -2 & 1 & 3 & 10 & 16 \\ & & -2 & -2 & -16 \\ \hline & 1 & 1 & 8 & 0 \end{array}$$

$x^2 + x + 8$

This doesn't factor therefore, use Quad. Form.

$b^2 - 4ac = -31$

$x = \frac{-1 \pm \sqrt{-31}}{2} = \boxed{\frac{-1 \pm i\sqrt{31}}{2}}$

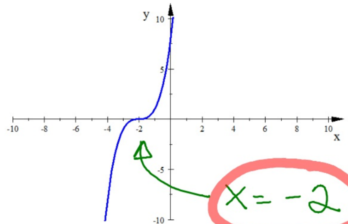
$x = -2$

$x = -2, \frac{-1 \pm i\sqrt{31}}{2}$

These represent all 3 zeros of this cubic.

Find ALL solutions.

$$y = x^3 + 6x^2 + 12x + 8$$



$$\begin{array}{r|rrrr} -2 & 1 & 6 & 12 & 8 \\ & & -2 & -8 & -8 \\ \hline & 1 & 4 & 4 & 0 \end{array}$$

$x^2 + 4x + 4$

$(x+2)(x+2)$

$x = -2, -2$

$x = -2$

$x = -2$

this represents all three zeros of this cubic, they all happen to be the same number.

Hwk #34

Due tomorrow

Practice Sheet - Theorems of Algebra