

Topic 9: Rational expressions and functions

Rational Expression:

The ratio of two polynomials.

Simplify.

$$\frac{8x + 26}{4x}$$

$$= \frac{\overset{\div 2}{\cancel{2}}(4x + 13)}{\overset{\div 2}{\cancel{4}}x}$$

$$= \frac{4x + 13}{2x}$$

$$\frac{\overset{\div 2}{\cancel{8}}x + \overset{\div 2}{\cancel{26}}}{\overset{\div 2}{\cancel{4}}x}$$

$$= \frac{4x + 13}{2x}$$

When a ratio has terms separated by + or - all terms must have a common factor to be able to reduce.

Simplify each without a calculator. Give answer as a single fraction in reduced form.

1. $\frac{14}{15} \cdot \frac{20}{21}$

$$= \frac{\overset{\div 7}{\cancel{14}}}{\overset{\div 5}{\cancel{15}}} \cdot \frac{\overset{\div 5}{\cancel{20}}}{\overset{\div 7}{\cancel{21}}}$$

$$= \frac{2}{3} \cdot \frac{4}{3}$$

$$= \frac{8}{9}$$

2. $\frac{28}{30} \div \frac{44}{18}$

$$= \frac{28}{30} \cdot \frac{18}{44}$$

Instead of dividing by a ratio you can multiply by its reciprocal. This allows you to cross cancel.

$$= \frac{\overset{\div 4}{\cancel{28}}}{\overset{\div 6}{\cancel{30}}} \cdot \frac{\overset{\div 6}{\cancel{18}}}{\overset{\div 4}{\cancel{44}}}$$

$$= \frac{7}{5} \cdot \frac{3}{11} = \frac{21}{55}$$

Simplify.

$$\frac{-6}{-3} \cdot \frac{+2}{-1}$$

$$\frac{4x^3 - 16x}{10x^5 - 10x^4 - 60x^3} \div \frac{2x + 6}{x^3 - 9x}$$

$$= \frac{4x(x^2 - 4)}{10x^3(x^2 - x - 6)} \div \frac{2(x + 3)}{x(x^2 - 9)}$$

$$= \frac{4x(x + 2)(x - 2)}{10x^3(x - 3)(x + 2)} \cdot \frac{x(x + 3)(x - 3)}{2(x + 3)}$$

$$= \frac{4x^2(x - 2)}{20x^3} = \frac{x - 2}{5x}$$

Simplify each without a calculator. Give answer as a single fraction in reduced form.

$$3. \quad \frac{5}{5} \cdot \frac{9}{16} - \frac{4}{5} \cdot \frac{16}{16}$$

Since the original denominators have no common factor the LCD will be their product.

$$= \frac{45 - 64}{5 \cdot 16}$$

$$= \boxed{\frac{-19}{5 \cdot 16}}$$

$$4. \quad \frac{7}{12} + \frac{13}{18}$$

$$= \frac{3}{3} \cdot \frac{7}{6 \cdot 2} + \frac{13}{6 \cdot 3} \cdot \frac{2}{2}$$

Since the original denominators already had a common factor the LCD is not their product.

$$= \frac{21 + 26}{6 \cdot 2 \cdot 3}$$

$$= \boxed{\frac{47}{6 \cdot 2 \cdot 3}}$$

$$\text{Simplify.} \quad \frac{(x+1)}{(x+1)} \cdot \frac{5}{x+3} + \frac{2}{x+1} \cdot \frac{(x+3)}{(x+3)}$$

$$= \frac{5x+5 + 2x+6}{(x+1)(x+3)}$$

$$= \boxed{\frac{7x+11}{(x+1)(x+3)}}$$

Simplify.

$$\frac{9}{x^2 - 16} - \frac{11}{x^2 - x - 12} \rightarrow \begin{array}{c} -12 \\ -4 \times +3 \\ -1 \end{array}$$

$$= \frac{9}{(x+4)(x-4)} - \frac{11}{(x-4)(x+3)}$$

$$= \frac{(x+3)}{(x+3)} \cdot \frac{9}{(x+4)(x-4)} + \frac{-11}{(x-4)(x+3)} \cdot \frac{(x+4)}{(x+4)}$$

Instead of subtracting 11 I've changed this to adding -11. This allows me to distribute the negative and the 11 at the same time.

$$= \frac{9x+27 - 11x - 44}{(x+3)(x+4)} = \boxed{\frac{-2x-17}{(x+3)(x+4)}}$$

Simplify.

$$\begin{array}{c} 25 \\ +5 \times +5 \\ +10 \end{array} \leftarrow \frac{2x}{x^2 + 10x + 25} + \frac{6}{x^2 + 2x - 15} \rightarrow \begin{array}{c} -15 \\ +5 \times -3 \\ +2 \end{array}$$

$$= \frac{2x}{(x+5)(x+5)} + \frac{6}{(x+5)(x-3)}$$

$$\frac{(x-3)}{(x-3)} \cdot \frac{2x}{(x+5)(x+5)} + \frac{6}{(x+5)(x-3)} \cdot \frac{(x+5)}{(x+5)}$$

$$= \frac{2x^2 - 6x + 6x + 30}{(x-3)(x+5)} = \boxed{\frac{2x^2 + 30}{(x-3)(x+5)}}$$

Simplify.

$$\frac{7}{4x^3 - 4x} - \frac{3x}{2x^4 + 10x^3 - 12x^2}$$

$$= \frac{7}{4x(x^2 - 1)} - \frac{3x}{2x^2(x^2 + 5x - 6)} \rightarrow \begin{array}{c} -6 \\ +6 \quad -1 \\ +5 \end{array}$$

$$= \frac{\cancel{x(x+6)} \cdot 7}{\cancel{x(x+6)} 4x(x+1)(x-1)} - \frac{3x}{2x^2(x+6)(x-1)} \cdot \frac{2(x+1)}{2(x+1)}$$

$$= \frac{7x(x+6)}{4x^2(x+6)(x+1)} + \frac{-6x(x+1)}{4x^2(x+6)(x+1)}$$

$$= \frac{7x^2 + 42x - 6x^2 - 6x}{4x^2(x+6)(x+1)} = \boxed{\frac{x^2 + 36x}{4x^2(x+6)(x+1)}}$$

Hwk #36

Due tomorrow

Practice Sheet: Sum, Difference, Product, & Quotient of Rational Expressions