What is true about the inverse relation of the graph of every Quadratic Function?

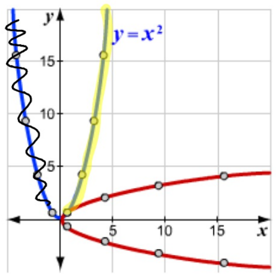
They are sideways parabolas and, therefore, they are NOT functions.

Unless you restrict the Domain!

SAS4 - Topic 2 question #15

15. What must you do to a quadratic function so that the function is a one-to-one function?

One possible way to make $y=x^2$ one-to-one is to erase the left side and only use the right side.



Agilemind - Topic 2 - Exploring

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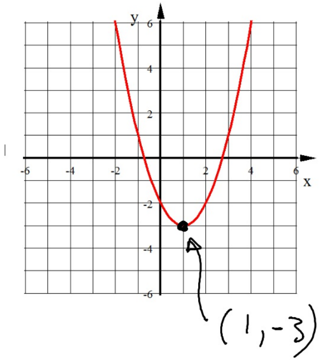
If a function is NOT one-to-one, in order for the inverse to become a function we must restrict the domain of the orginal so that it does become one-to-one.

This may also affect the RANGE.

Below is the graph of $y = (x-1)^2 - 3$



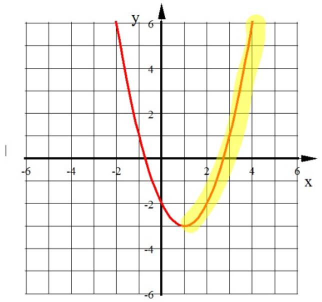
How would you restrict the domain so that the inverse IS a function?

Domain: $x \geq 1$ $[1, \infty)$

(right side of graph)

This restriction is the largest possible set of values such that the inverse is a function.

$y = (x-1)^2 - 3$



To make the inverse a function:

Domain Restriction of the original:

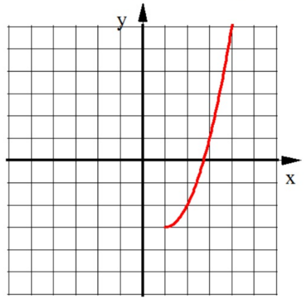
$[1,\infty)$ $x \geq 1$

Resulting Range of the original:

$[-3,\infty)$ $y \geq -3$

$y = (x-1)^2 - 3$

D: $[1,\infty)$ R: $[-3,\infty)$



Write the equation of the inverse:

$x = (y-1)^2 - 3$

$+3$ $+3$

$\sqrt{x+3} = \sqrt{(y-1)^2}$

$\sqrt{x+3} = y - 1$

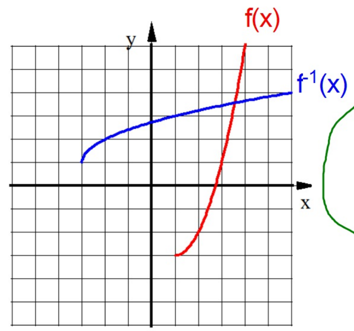
$+1$ $+1$

$$f^{-1}(x) = \sqrt{x+3} + 1$$

Graph the inverse together with the original, restricted, function:

$y = (x-1)^2 - 3$

D: $[1,\infty)$ R: $[-3,\infty)$



$$f^{-1}(x) = \sqrt{x+3} + 1$$

What is the domain and range of the inverse?

D: $[-3,\infty)$ R: $[1,\infty)$

because of this range we DON'T see the bottom half of the sideways parabola.

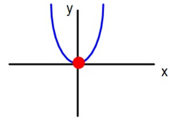
To restict the domain of a quadratic so that the inverse IS a function all you really need to know is.....

The coorinates of the Vertex!

Generally, the restriction will be

$x \geq$ the x-coord of the vertex

The graph of $y = x^2$ is the parent Quadratic Function which is a Parabola whose Vertex is at (0,0).



$y = ax^2$ is a Transformation of the Parent Function.

The value of a determines which way the parabola opens:

$a>0$ parabola opens up.

$a<0$ parabola opens down.

a also represents a vertical stretch or shrink making it taller or shorter than the parent function.

$y = x^2 \pm k$ is a Translation of the Parent Function.

The value of k shifts the parabola k units up (+k) or down (-k)

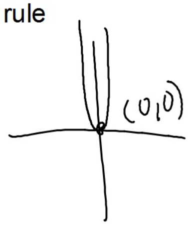
$y = (x \pm h)^2$ is also a Translation of the Parent Function.

The value of h shifts the parabola h units left (x+h) or right (x-h)

Restrict the Domain of f(x) to the largest possible set of values such that the inverse of f is a function. Find an algebraic rule for $f^{-1}(x)$.

$f(x) = 3x^2$

This parabola is three times taller but hasn't shifted in any direction. The Vertex is still (0,0)



Domain Restriction:

restricting the original quadratic to the right side gives us a one-to-one function

$x \geq 0$

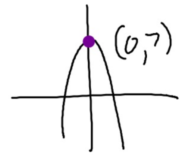
Eq of Inverse:

$f^{-1}(x) = \sqrt{\frac{x}{3}}$

Restrict the Domain of f(x) to the largest possible set of values such that the inverse of f is a function. Find an algebraic rule for $f^{-1}(x)$.

$f(x) = -x^2 + 7$

this parabola opens down and shifted 7 units up. The Vertex is (0,7)



Domain Restriction:

restricting the original quadratic to the right side gives us a one-to-one function

$x \geq 0$

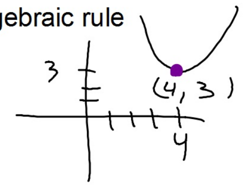
Eq of Inverse:

$f^{-1}(x) = \sqrt{-(x-7)}$

Restrict the Domain of f(x) to the largest possible set of values such that the inverse of f is a function. Find an algebraic rule for $f^{-1}(x)$.

$f(x) = (x-4)^2 + 3$

this parabola has shifted 4 units right and 3 units up. The vertex is (4,3)



Domain Restriction:

restricting the original quadratic to the right side gives us a one-to-one function

$x \geq 4$

Eq of Inverse:

$f^{-1}(x) = \sqrt{x-3} + 4$

Restrict the Domain of f(x) to the largest possible set of values such that the inverse of f is a function. Find an algebraic rule for $f^{-1}(x)$.

$f(x) = x^2 - 6x + 5$

Domain Restriction:

$x \geq 3$

To find the domain restrictions of a quadratic all we need to know is the x-coordinate of the vertex. In Standard Form the x-coordinate of the vertex is the same as the equation for the Line of Symmetry (LOS). Below is the equation of the LOS:

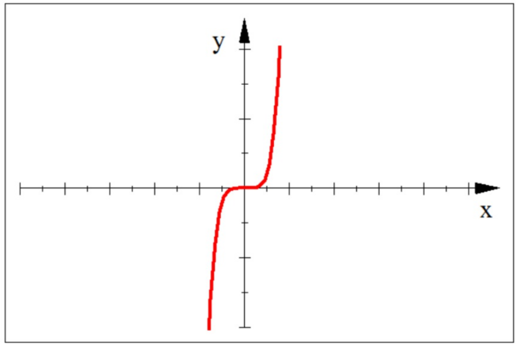
LOS: $x = \frac{-b}{2a}$

In this quadratic a= 1 and b = -6

$x = \frac{6}{2} = 3$

Therefore, the x-coordinate of the vertex is x=3

Is the inverse of $y=x^5$ a function?



Can you get the same y-value by raising two different x-values to the 5th power?

No, when x is negative x^5 is also negative. And when x is positive x^5 is also positive. Therefore, there is no way to get the same y-value from different x-values.

This means $y=x^5$ is a one-to-one function and its inverse IS a function.

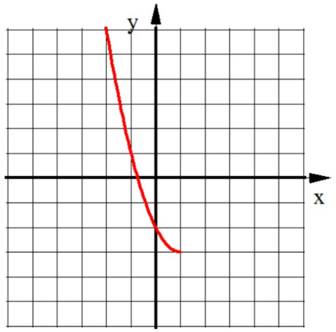
Hwk #11 SAS4 - Topic 2

Problems 16 & 17

What if we restricted the original quadratic to the left side?

$$y = (x-1)^2 - 3$$

D: [-∞,1) R: [-3,∞)



$$f^{-1}(x) = \sqrt{x+3} + 1$$

The domain and range of the inverse would have to be

D: [-3,∞) R: [-∞,1)

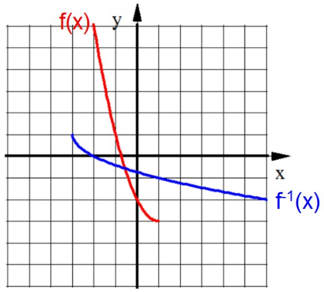
$$f^{-1}(x) = \sqrt{x+3} + 1$$

The only way the range of the inverse could be [-∞,1) is if $\sqrt{x+3}$ were negative!

Doing this gives us the Bottom part of the "sideways parabola".

This would make the equation of the inverse:

$$f^{-1}(x) = -\sqrt{x+3} + 1$$



Normally when we find the square roots of something there are two answers:

The square roots of 16 are ±4

$x^2 = 16$ leads to $x = \pm\sqrt{16} = \pm 4$

In order to make the inverse of y = x^2 a function we must restrict the domain for it to be one-to-one.

When we restrict the domain to the right side of the parabola the inverse becomes the top half of the "sideways parabola".

Instead of the inverse of $y = (x-1)^2 - 3$

being $y = \pm\sqrt{x+3} + 1$

it becomes $y = +\sqrt{x+3} + 1 = \sqrt{x+3} + 1$

When we restrict the domain to the left side of the parabola the inverse becomes the bottom half of the "sideways parabola".

Instead of the inverse of $y = (x-1)^2 - 3$

being $y = \pm\sqrt{x+3} + 1$

it becomes $y = -\sqrt{x+3} + 1$

Agilemind website - Defining Parabolas

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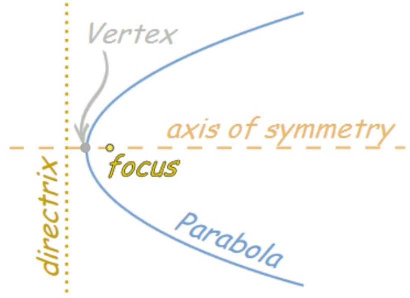
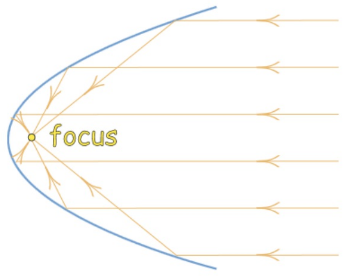
Vocabulary of a Parabola

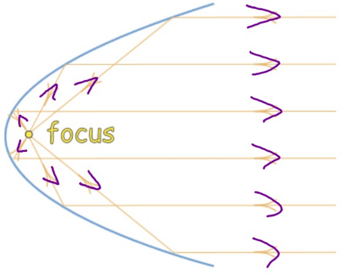


How does a parabola work in Real Life?



as a sattelite dish that collects signals from sattelites in orbit and focuses them on the receiver which must be located at the focus of the parabola.

This idea also works in reverse.



car headlight

photographers lighting

When the focus contains a light source (bulb) the rays emitted bounce off of the parabolic surface and are sent outward in parallel paths, focusing the light into a beam.

We are now done with this Topic

Understanding inverse relationships.