There are five children in a drawing contest. The judges will award five different prizes to these children. How many ways could the judges award these prizes?

$\underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1}$

$= 120$

Factorial: $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

Factorial is usually used if you are arranging ALL of the available items.

There are 12 people on a basketball team and only 12 uniform numbers to pass out.

How many different ways can all 12 uniform numbers be passed out to the players?

$\underline{12} \cdot \underline{11} \cdot \underline{10} \cdot \underline{9} \cdot \underline{8} \cdot \underline{7} \cdot \underline{6} \cdot \underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1}$

OR

$12! = 479{,}001{,}600$

If there were 12 uniforms but only 10 players, how many ways could the uniforms be passed out?

$\underline{12} \cdot \underline{11} \cdot \underline{10} \cdot \underline{9} \cdot \underline{8} \cdot \underline{7} \cdot \underline{6} \cdot \underline{5} \cdot \underline{4} \cdot \underline{3}$

$= 239{,}500{,}800$

There are 5 people running a race.

What if prizes are only awarded to the top three finishers?

In other words, how many ways can 1st, 2nd, and 3rd places be awarded to 5 people running in the race?

Multiplication Counting Principle: $\underline{5} \cdot \underline{4} \cdot \underline{3} = 60$

Permutation: An arrangement of items when order DOES matter.

Permutation Formula: When order DOES matter

Ways to arrange n items r at a time.

$${}_nP_r = \frac{n!}{(n-r)!}$$

n= total # items

r = # arranging at a time

${}_5P_3 = 60$

There are 5 people running a race.

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Multiplication Counting Principle: $5 \cdot 4 \cdot 3 = 60$

OR

Permutation: Arrangment of 5 things 3 at a time

${}_5P_3 = 60$

If there were 12 uniforms but only 10 players, how many ways could the uniforms be passed out?

Instead of using the Multiplication Counting Principle:

Use a Permutation:

${}_{12}P_{10} = 239{,}500{,}800$

You are playing Scrabble. You choose 7 tiles from the pile.

1. How many ways can you arrange all 7 in front of you?

$\underline{7} \cdot \underline{6} \cdot \underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1} = 7! = {}_7P_7$

2. If you can only play 3 at a time, find the number of ways you can arrange 3 of the 7 tiles on the board.

$${}_7P_3 \text{ or } \underline{7} \cdot \underline{6} \cdot \underline{5} = 210$$

The word arrange implies that order IS important, therefore, you can use a Permuation.

Nine students are running for Student Congress offices. How many ways can the positions of President, Vice-President, and Secretary be filled?

Each of these positions is different, therefore, order IS important.

$$\left.\begin{matrix} {}_9P_3 \\ \text{OR} \\ \underline{9} \cdot \underline{8} \cdot \underline{7} \end{matrix}\right\} = 504$$

There are 24 students in a class. If there are 30 seats how many different seating charts are possible?

$${}_{30}P_{24} = 3.68 \times 10^{29}$$

If there are 24 seats available, how many different seating charts are possible?

$${}_{24}P_{24} \text{ or } 24!$$

$$= 6.20 \times 10^{23}$$

A seating chart indicates that order IS important.

How many different four digit sequences can you create using the digits from 0 to 9 if digits can repeat?

$$\underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} = 10{,}000$$

Since #'s can repeat it is not a permuation.

How many different four digit sequences can you create using the digits from 0 to 9 if digits CAN'T repeat?

$$\underline{10} \cdot \underline{9} \cdot \underline{8} \cdot \underline{7} = {}_{10}P_4 = 5040$$

Since #'s can't repeat this becomes a permuation.

You want to order a custom paint that is a mixture of 2 colors. If there are only 4 colors to choose from, how many different custom 2 color mixtures are possible?

Colors:

- Red
- Blue
- Green
- Purple

Does order matter in this situation? No

Create a systematic list to help answer this question.

RB BG GP
RG BP
RP

= 6

6 different 2-color combinations.

Combination:

Selecting a number of items when order DOESN'T matter.

Does this situation represent a Combination or a Permuation?

You order a shake at a shop. There are 7 ingredants to choose from. You buy a shake that contains 4 ingredients. How many different shakes are possible?

This is a combination because when you make a shake you put the ingredients in a blender to mix them up. This means the order in which you pick the ingredients is NOT important.

Combination Formula: When order DOESN'T matter

Ways to choose n items r at a time.

$$_nC_r = \frac{n!}{r!\,(n-r)!}$$

n= total # items

r = # selecting at a time

You want to order a custom paint that is a mixture of 2 colors. If there are only 4 colors to choose from, how many different custom 2 color mixtures are possible?

Colors:

- Red
- Blue
- Green
- Purple

Find this answer using Combinations.

$_4C_2 = 6$

You order a shake at a shop. There are 7 ingrediants to choose from. You buy a shake that contains 4 ingredients. How many different shakes are possible?

This is a combination:

$_7C_4 = 35$

You are taking a test with 10 problems but you only have to choose 5 of them to complete. All problems are worth the same amount of points and of the same degree of difficulty. How may ways can you choose 5 of these problems to do?

Since the problems are worth the same value and are of the same degree of difficulty order is NOT important.

$_{10}C_5 = 252$

There are 24 students in the class.

How many ways could President, Vice-President, and Secretary be assigned?

Assuming the three positions are different jobs then order DOES matter - Permuation.

$_{24}P_3 = 12,144$

How many ways could a committee of 3 students be picked to meet with the principal?

To be on a committee it doesn't matter who was selected first, second, or thire. This means that order DOESN'T matter - Combination.

$_{24}C_3 = 2024$