

## Sec 13-6: The Tangent Function

Graph the function  $Y_1 = \text{Tan}x$

Use this Window:  $x:[0, 4\pi]$   $y:[-10, 10]$

Use this Rational Function:

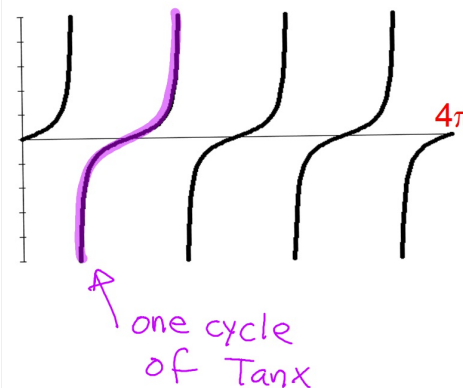
$$y = \frac{x+1}{x^2-9} = \frac{x+1}{(x+3)(x-3)}$$

Find the Vertical Asymptotes  
zeros of the denominator

$$x = \pm 3$$

Find the x-intercepts:  
zeros of the numerator.

$$x = -1$$

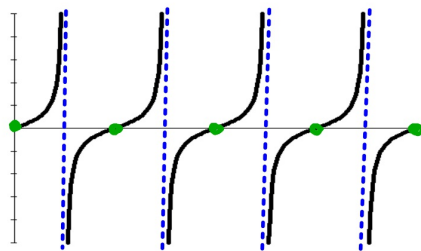


What is the period of  
the Tangent Function?

$$\frac{\text{TOTAL WIDTH}}{\# \text{ cycles}} = \frac{4\pi}{4} = \boxed{\pi}$$

Why does the graph of  
 $y = \tan\theta$  look like this?

$$\tan\theta = \frac{y}{x}$$



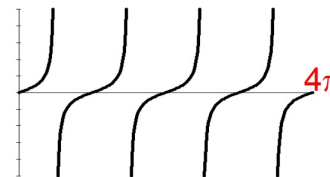
When the numerator = 0  
 $\tan\theta = 0$  and there is an x-intercept

When the denominator = 0  
 $\tan\theta$  is undefined and there is a Vertical Asymptote

The Parent Tangent Function:  $y = \tan x$

Period of  $\tan\theta = \pi$

this means every  $\pi$  units left or right from any point you get the same result.



What are the x-intercepts?

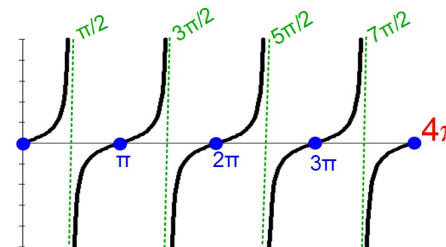
The first x-intercept is  $x=0$  (the origin), therefore, if you keep moving  $\pi$  left or right you'll find more x-intercepts.

$$x\text{-int} = 0, \pm\pi, \pm2\pi, \pm3\pi, \dots$$

What are the Eq's of the Vertical Asymptotes?

Vertical asymptotes (VA) occur exactly halfway between consecutive x-int.

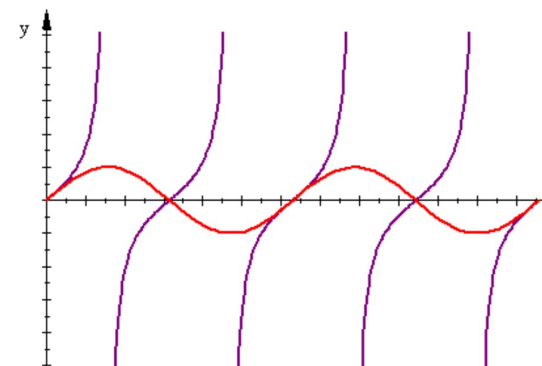
The first VA will be halfway between 0 and  $\pi$  which is at  $\pi/2$ .  
 To find more VA you can add or subtract  $\pi$  in the form  $2\pi/2$   
 $= \pi + 2\pi/2 = 3\pi/2, \dots$



Another definition of Tan:

$$\tan\theta = \frac{y}{x} = \frac{\sin\theta}{\cos\theta}$$

Leave  $Y_1 = \tan x$ . Graph  $Y_2 = \sin x$ .

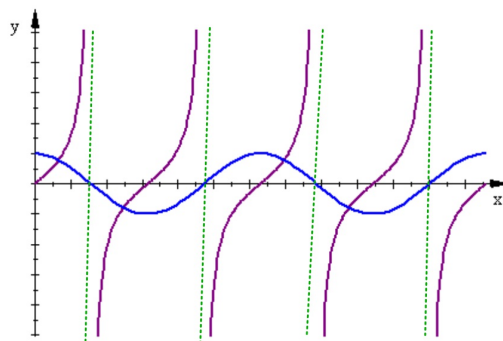


How is the graph of  $\tan x$  related to the graph of  $\sin x$ ?

$\tan x$  is zero whenever  $\sin x$  is zero.

In other words,  $\tan x$  has x-intercepts where ever  $\sin x$  has x-intercepts.

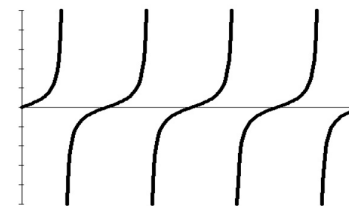
Leave  $Y_1 = \tan x$ . Graph  $Y_2 = \cos x$ .



How is the graph of  $\tan \theta$  related to the graph of  $\cos \theta$ ?

$\tan \theta$  has a VA whenever  $\cos \theta$  is zero.

$y = \tan x$



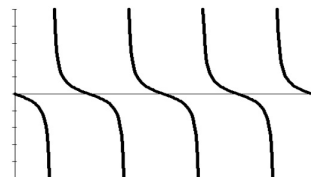
Since  $\tan x = \frac{y}{x}$

you can remember which way  $y = \tan x$  moves if you relate it to slope.

A positive  $\tan x$  moves up and to the right like a positive slope.

$y = a \tan x$      $a$ : If  $a < 0$  there is an x-axis reflection

A negative  $\tan x$ , just like negative slope, moves down and to the right.



$a$  is also a Vertical Stretch or Shrink Factor but..... there are really no points to define how "tall" the Parent Tangent function is so we will only concern ourselves with whether  $a$  is positive or negative.

$y = \tan(bx)$

Just like for  $\sin$  and  $\cos$   $b$  represents a Horizontal Stretch or Shrink.

For  $\sin$  and  $\cos$   $b$  was related to the period in the following ways:

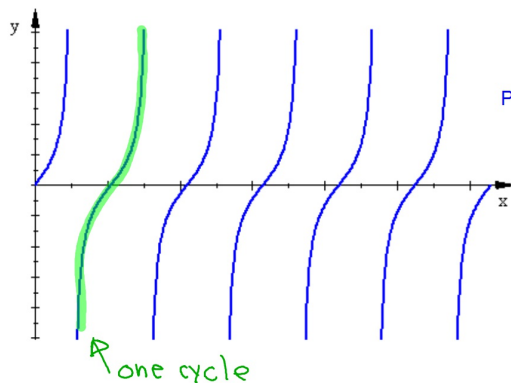
$$\text{Period} = \frac{2\pi}{b} \quad \text{and} \quad b = \frac{2\pi}{\text{period}}$$

There is a similar relationship with Tangent:

$$b: \text{ The period of } \tan bx = \frac{\pi}{b}$$

$$b = \frac{\pi}{\text{period}}$$

The Tangent function is graphed in the window 0 to  $2\pi$ .



1. What is the period?

$$\text{Period} = \frac{\text{total width}}{\# \text{ cycles}} = \frac{2\pi}{6 \text{ cycles}} = \frac{\pi}{3}$$

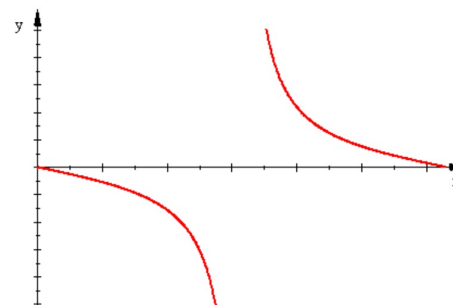
2. What is the equation of this Tangent Function?

$$b = \frac{\pi}{\frac{\pi}{3}} = \pi \cdot \frac{3}{\pi} = 3$$

• The graph moves up & to the right so it's a POS Tangent

$$y = \tan 3x$$

The Tangent function is graphed in the window 0 to  $2\pi$ .



1. What is the period?

$$\text{period} = \frac{2\pi}{1 \text{ cycle}} = 2\pi$$

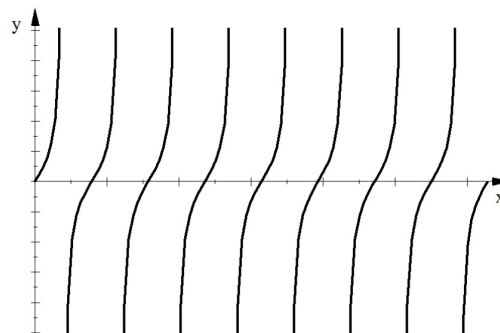
2. What is the equation of this Tangent Function?

$$b = \frac{\pi}{2\pi} = \frac{1}{2}$$

• graph moves down & to the right → This is a Neg Tangent

$$y = -\tan \frac{x}{2}$$

The Tangent function is graphed in the window 0 to  $2\pi$ .



1. What is the period?

$$\text{period} = \frac{2\pi}{8} = \frac{\pi}{4}$$

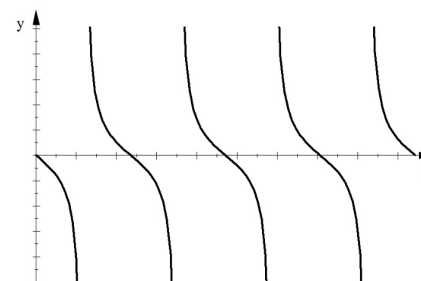
2. What is the equation of this Tangent Function?

$$b = \frac{\pi}{\frac{\pi}{4}} = \pi \cdot \frac{4}{\pi} = 4$$

• graph moves up & to the right → POS tangent.

$$y = \tan 4x$$

The Tangent function is graphed in the window 0 to  $6\pi$ .



1. What is the period?

$$\text{period} = \frac{6\pi}{4} = \frac{3\pi}{2}$$

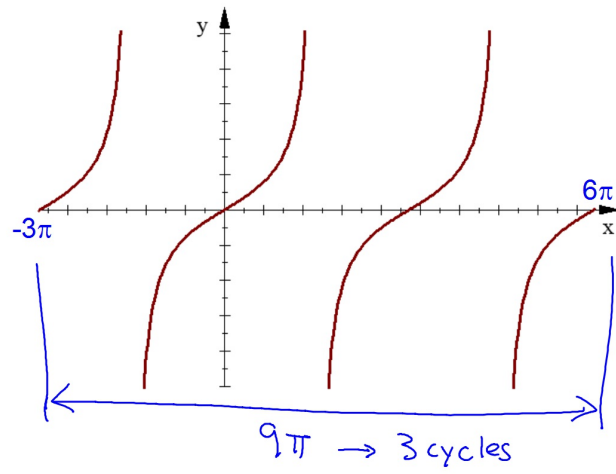
2. What is the equation of this Tangent Function?

$$b = \frac{\pi}{\frac{3\pi}{2}} = \pi \cdot \frac{2}{3\pi} = \frac{2}{3}$$

• graph moves down & to the right → Neg Tangent

$$y = -\tan \frac{2x}{3}$$

Write the equation of this Tangent Function



$$\text{period} = \frac{9\pi}{3} = 3\pi$$

$$b = \frac{\pi}{3\pi} = \frac{1}{3}$$

graph moves  
up & to the  
right  $\rightarrow$  POS  
tangent

$$y = \tan \frac{x}{3}$$