30-60-90 Triange Relationships:

Short Leg $= \frac{1}{2} \cdot$ Hypotenuse

Long Leg $= \sqrt{3} \cdot$ Short Leg

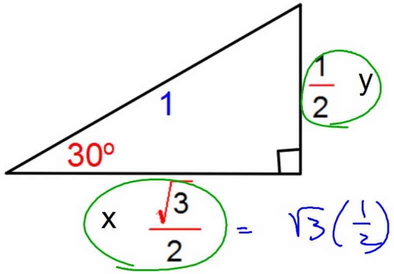
45-45-90 Triange Relationships:

Legs are equal

Hypotenuse $= \sqrt{2} \cdot$ Leg

Use the relationships in Special Right Triangles to find the EXACT values of x and y in each. Simplify fractions and rationalize denominators.

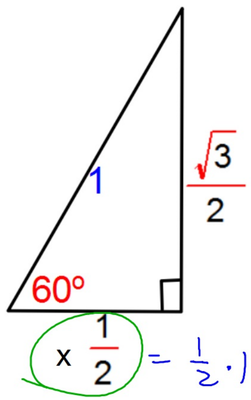
1.



$y = \frac{1}{2}(1)$

$x = \sqrt{3}\left(\frac{1}{2}\right)$

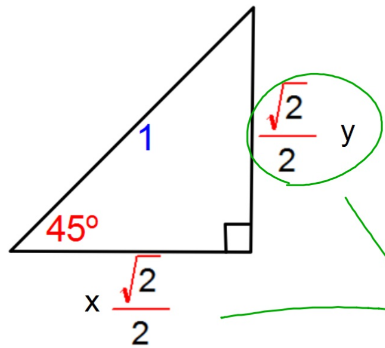
2.



$y = \sqrt{3}\left(\frac{1}{2}\right)$

$x = \frac{1}{2} \cdot 1$

3.

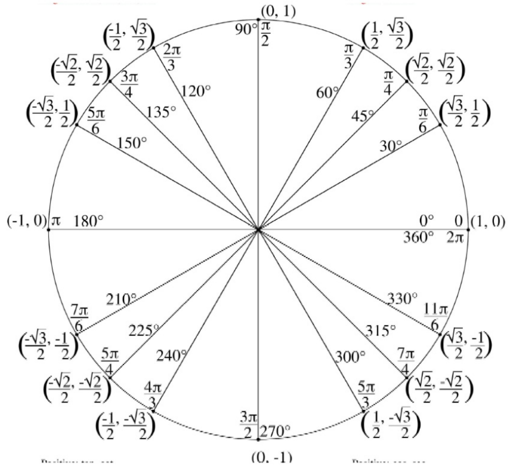


$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

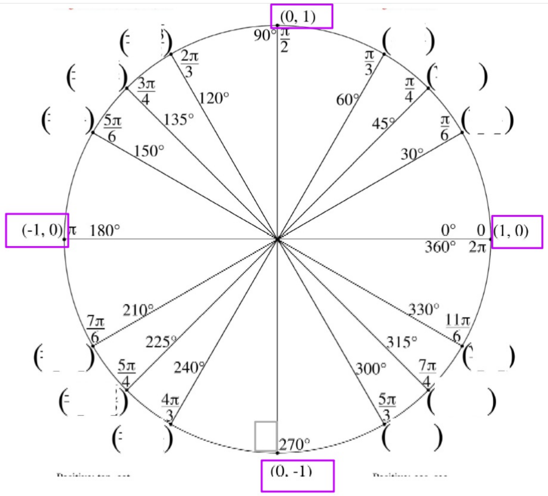
legs are =

The Unit Circle:

- Center is the origin
- Radius = 1
- Used to find the EXACT value of Sinθ, Cosθ, and Tanθ without using a calculator.
- Uses the Special Right Triangle relationships.
  This means all the angles on the unit circle are related to either 30°, 60°, or 45°.



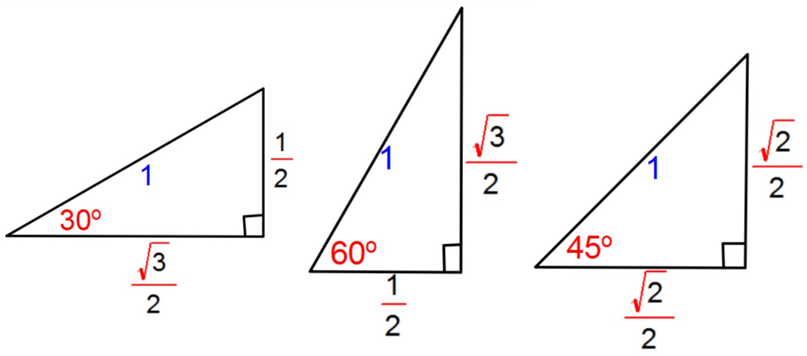
Coordinates of points on the Unit Circle can be used to find the Exact Value of Sin, Cos, and Tan of angles on the Unit Circle.



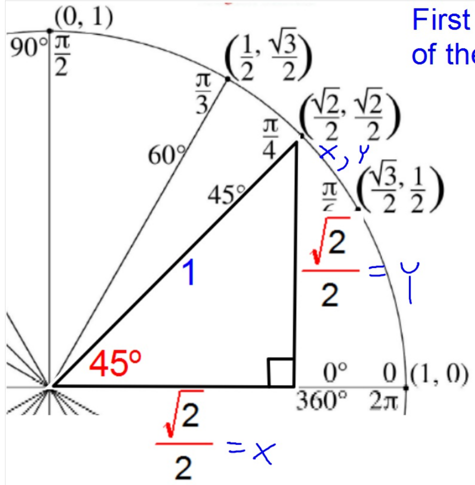
Coordinates on the axes.

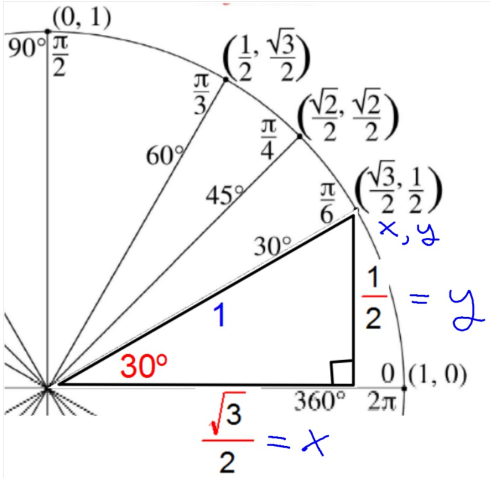
Since this is the Unit Circle the radius is 1 and the points on the axes are just 1 unit right, 1 up, 1 left, and 1 down from the origin.

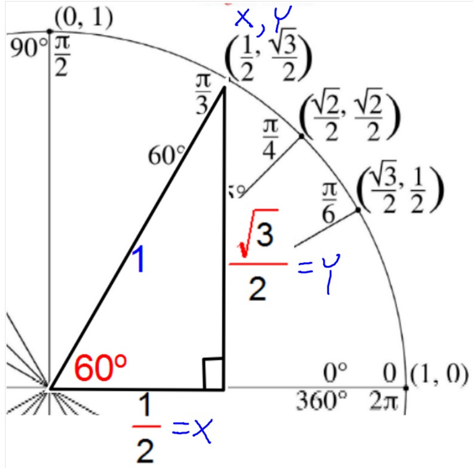
The Unit Circle involves the angles in Special Right Triangles
which means it probably involves the sides too!



First Quadrant
of the Unit Circle

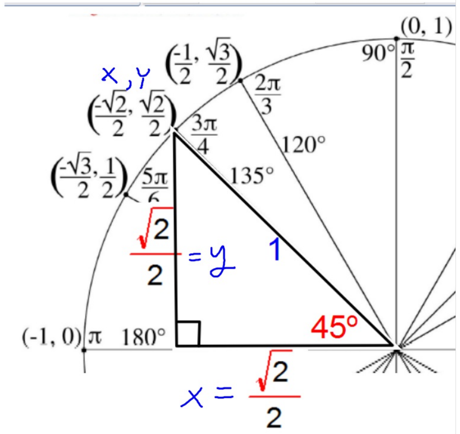




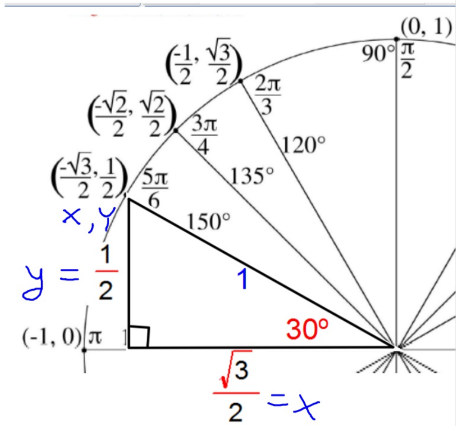


Second Quadrant of the Unit Circle

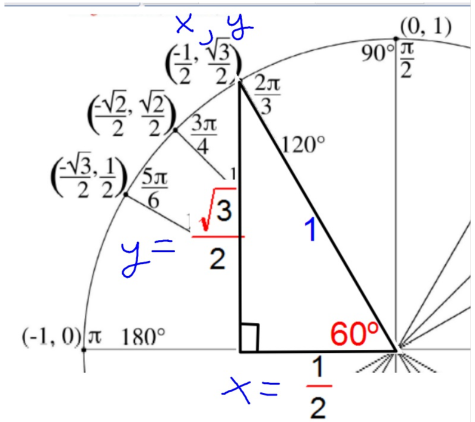
X-coordinates are negative in the second quadrant.

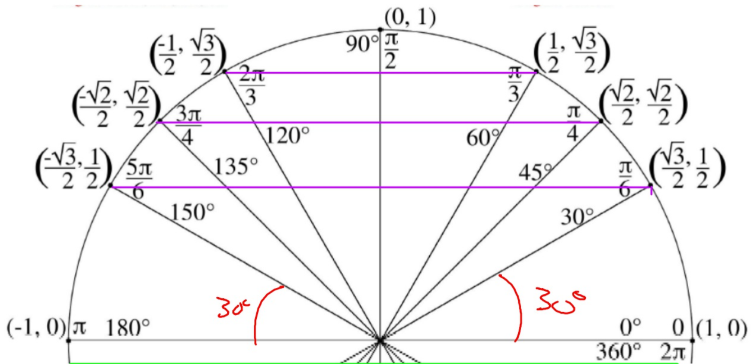


X-coordinates are negative in the second quadrant.



X-coordinates are negative in the second quadrant.

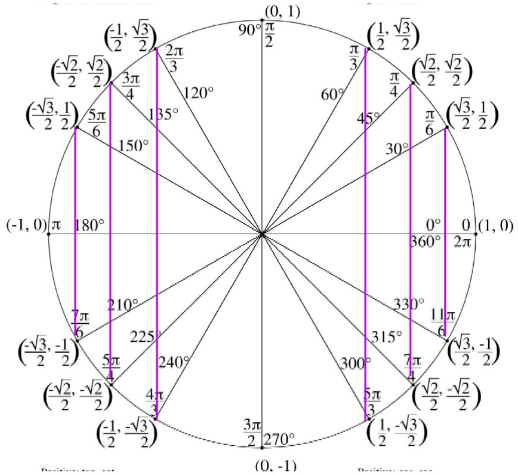




What do you notice about the relationship amongst the coordinates in the 1st and 2nd Quadrant?

The coordinates of the angles in the second quadrant are reflections over the y-axis if the corresponding coordinates from the first quadrant.

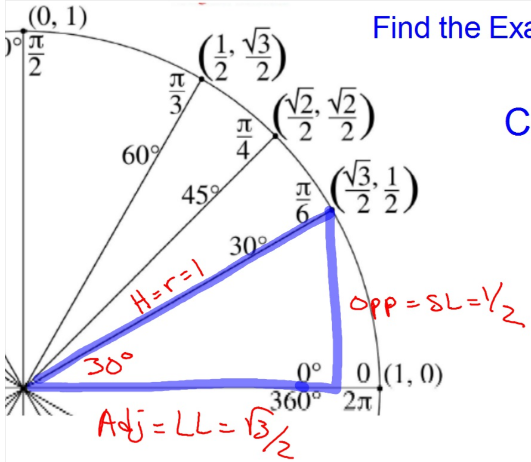
They also use the same reference angles.



Coordinates in the 3rd and 4th Quadrants.

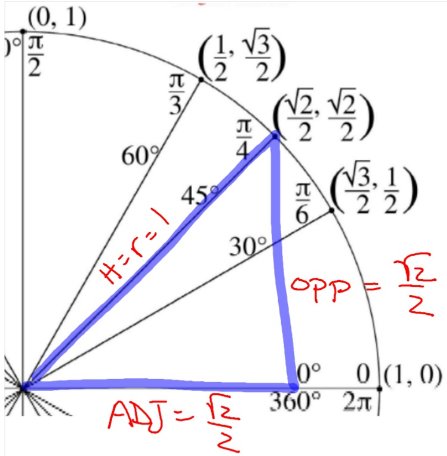
You can reflect quadrants 1 and 2 over the x-axis to create the coordinates of the angles in the third and fourth quadrants. In quadrant 3 all coordinates are negative and in quadrant 4, x-coord are pos and y-coord are neg.

Find the Exact value of each.



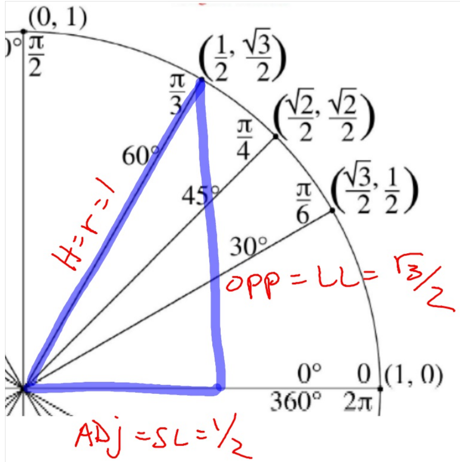
$$\text{Cos}30^\circ = \frac{adj}{hyp} = \frac{\frac{\sqrt{3}}{2}}{1}$$

$$\boxed{\cos 30^\circ = \frac{\sqrt{3}}{2}}$$



$$\text{Sin}45^\circ = \frac{opp}{hyp} = \frac{\frac{\sqrt{2}}{2}}{1}$$

$$\boxed{\sin 45 = \frac{\sqrt{2}}{2}}$$

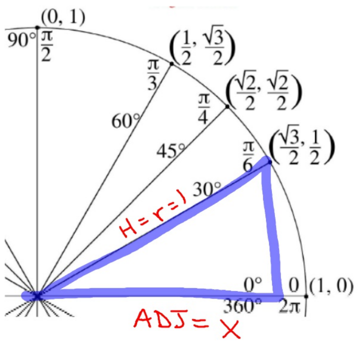


$$\text{Tan}60^\circ = \frac{opp}{adj} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\sqrt{3}}{\cancel{2}} \cdot \frac{\cancel{2}}{1}$$

$$\boxed{\text{Tan}60^\circ = \sqrt{3}}$$

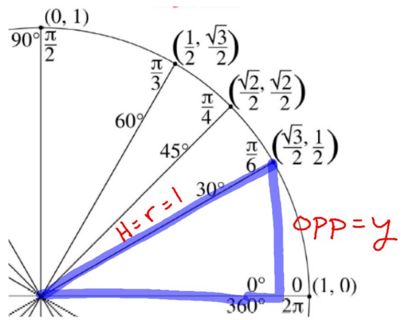
Definitions of Sin,Cos, and Tan using the Unit Circle:

$$\cos\theta = \frac{\text{Adj}}{\text{Hypot}} = \frac{x}{r} = \frac{x}{1} = x$$



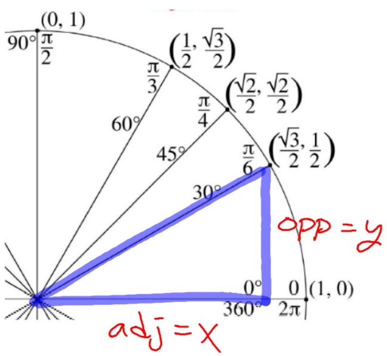
Definitions of Sin,Cos, and Tan using the Unit Circle:

$$\sin\theta = \frac{\text{Opp}}{\text{Hypot}} = \frac{y}{r} = \frac{y}{1} = y$$



Definitions of Sin,Cos, and Tan using the Unit Circle:

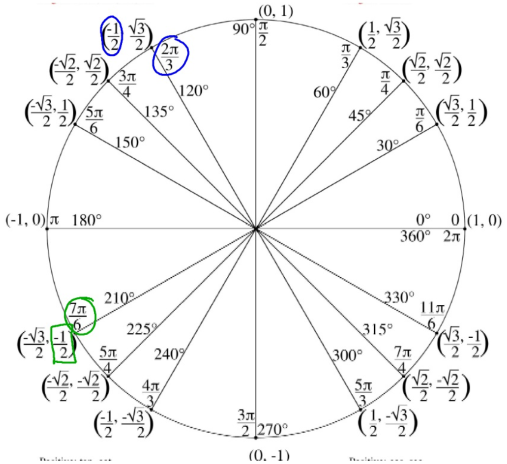
$$\tan\theta = \frac{\text{Opp}}{\text{Adj}} = \frac{y}{x}$$



Coordinates on the Unit Circle:

$$(x, y) \longrightarrow (\cos\theta, \sin\theta)$$

$$\tan\theta = \frac{y}{x} = \frac{\sin\theta}{\cos\theta}$$



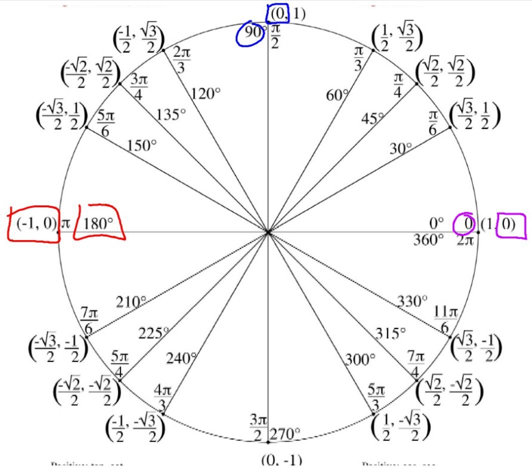
Use the Unit Circle to find the EXACT value of each.

$\text{Cos}\,\frac{2\pi}{3} = -\frac{1}{2}$

x-coord at $\frac{2\pi}{3}$

$\text{Sin}\,\frac{7\pi}{6} = -\frac{1}{2}$

y-coord at $\frac{7\pi}{6}$



Use the Unit Circle to find the EXACT value of each.

Sin0 = 0

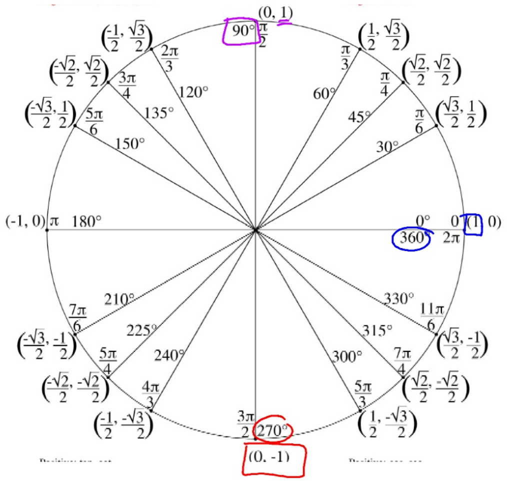
y-coord at 0

Cos90° = 0

x-coord at 90°

$\text{Tan}180° = \frac{0}{-1} = 0$

$\frac{y}{x}$ at 180°



Use the Unit Circle to find the EXACT value of each.

Sin90° = 1

y-coord at 90°

Cos720° = cos 360°

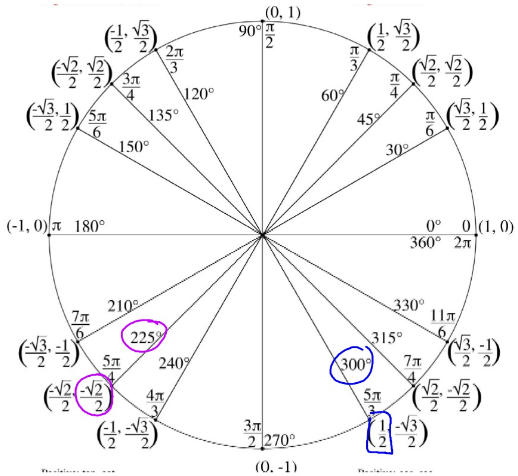
720 − 360 = 360 = 1

x-coord at 360°

$\text{Tan}270° = \frac{-1}{0}$

= undefined

$\frac{y}{x}$ at 270°



$\text{Sin}(-135°) = \sin(225°)$

$$\begin{array}{r} -135 \\ +360 \\ \hline 225 \end{array}$$

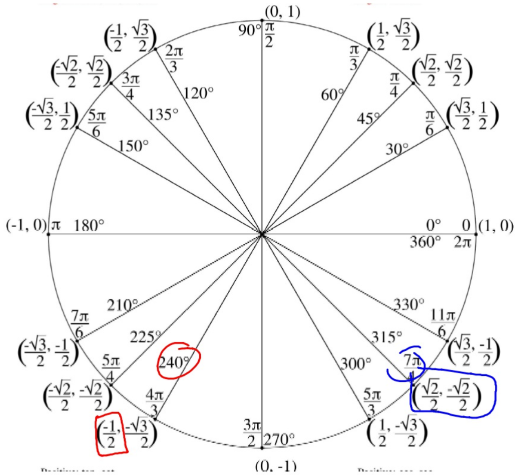
$= -\frac{\sqrt{2}}{2}$

y-coord at 225°

Cos660° = cos300°

$$\begin{array}{r} 660 \\ -360 \\ \hline 300° \end{array}$$

$= \frac{1}{2}$



$\text{Tan}(-\frac{\pi}{4}) = \text{Tan}\,\frac{7\pi}{4} = -1$

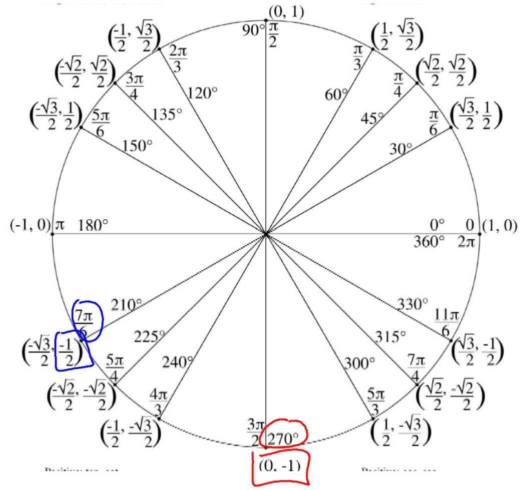
$-\frac{\pi}{4} + 2\pi$

$= -\frac{\pi}{4} + \frac{8\pi}{4} = \frac{7\pi}{4}$

$= \dfrac{-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$

$\text{Cos}(-480°) = \cos 240° = -\frac{1}{2}$

$-480 + 720 = 240°$



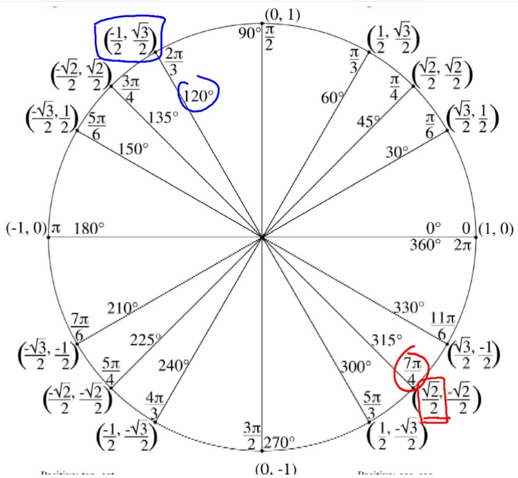
$\text{Sin}(-\frac{17\pi}{6}) = \sin\frac{7\pi}{6} = -\frac{1}{2}$

$-\frac{17\pi}{6} + \frac{12\pi}{6} = -\frac{5\pi}{6}$

$-\frac{5\pi}{6} + \frac{12\pi}{6} = \frac{7\pi}{6}$

2. $\text{Tan}\,630° = \text{Tan}\,270° = -1/0 =$ undefined

$630° - 360° = 270°$



$\text{Tan}\,840° = \text{Tan}\,120° = \dfrac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\sqrt{3}$

$840 - 720 = 120°$

$\text{Cos}(\frac{23\pi}{4}) = \cos\frac{7\pi}{4} = \frac{\sqrt{2}}{2}$

$\frac{23\pi}{4} - \frac{8\pi}{4} = \frac{15\pi}{4}$

$- 8\pi/4 = 7\pi/4$