30-60-90 Triange Relationships:

Short Leg = $\frac{1}{2}$ $\cdot$ Hypotenuse

Long Leg = $\sqrt{3}$ $\cdot$ Short Leg

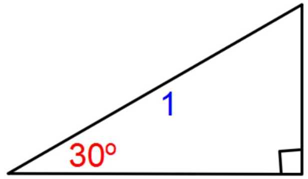
45-45-90 Triange Relationships:

Legs are equal

Hypotenuse = $\sqrt{2}$ $\cdot$ Leg

Use the relationships in Special Right Triangles to find the EXACT values of x and y in each. Simplify fractions and rationalize denominators.

1.



$\frac{1}{2} = y$

$x = \frac{\sqrt{3}}{2}$

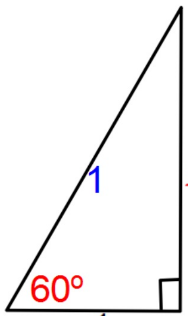
short leg $= \frac{\text{hypot}}{2} = \frac{1}{2}$

long leg = short leg $\cdot \sqrt{3}$

$= \frac{1}{2} \cdot \sqrt{3}$

$= \frac{\sqrt{3}}{2}$

2.



$\frac{\sqrt{3}}{2} = y$

$x = \frac{1}{2}$

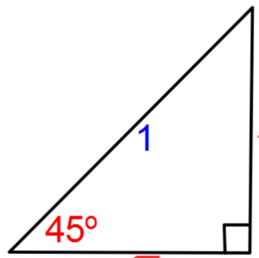
Long leg = short leg $\cdot \sqrt{3}$

$= \frac{1}{2} \cdot \sqrt{3}$

$= \frac{\sqrt{3}}{2}$

short leg $= \frac{\text{hypot}}{2} = \frac{1}{2}$

3.



$\frac{\sqrt{2}}{2} = y$

$x = \frac{\sqrt{2}}{2}$

leg $= \frac{\text{hypot}}{\sqrt{2}}$

$= \frac{1}{\sqrt{2}}$

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{\sqrt{2}}{2}$

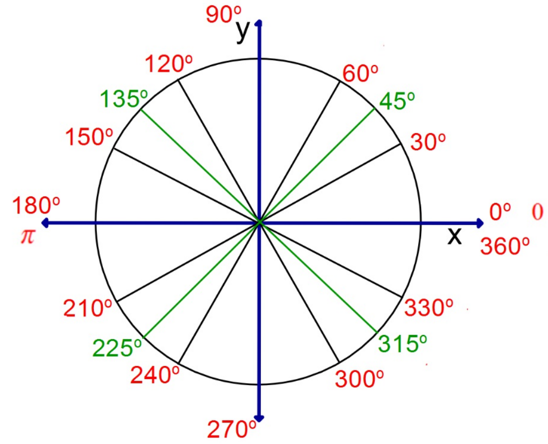
Legs are equal

The Unit Circle:

- Center is the origin
- Radius = 1
- Used to find the EXACT value of Sinθ, Cosθ, and Tanθ without using a calculator.
- Uses the Special Right Triangle relationships.
  This means all the angles on the unit circle are related to either 30°, 60°, or 45°.

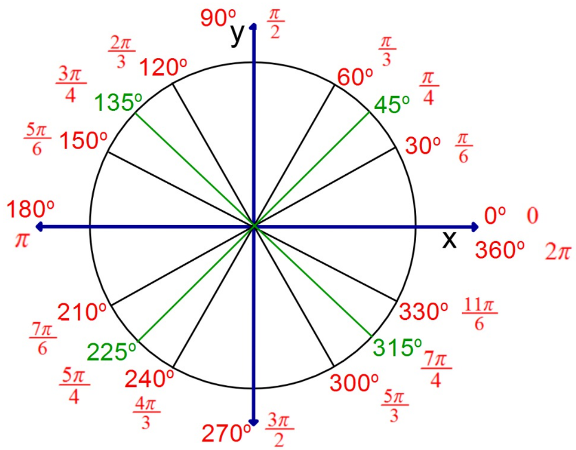
Unit Circle
with degrees

The three angles in each quadrant can be found by starting at an axis then adding 30, 45, and 60 degrees.

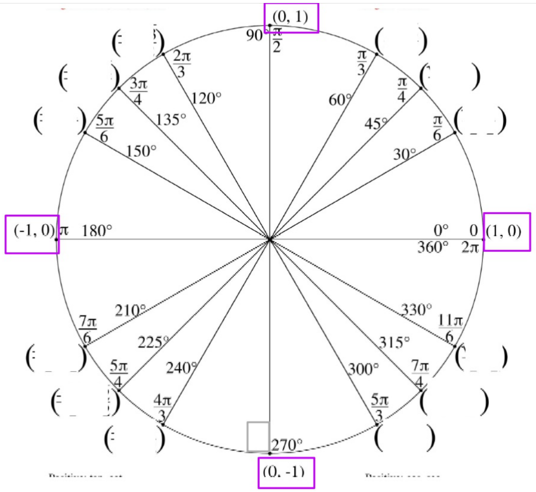


Unit Circle
with degrees
and radians

There are many ways to find the radian measure of all the angles around the Unit Circle.



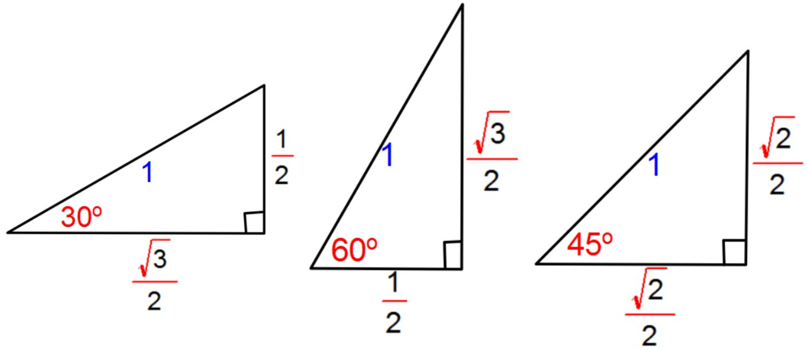
Coordinates of points on the Unit Circle can be used to find the Exact Value of Sin, Cos, and Tan of angles on the Unit Circle.



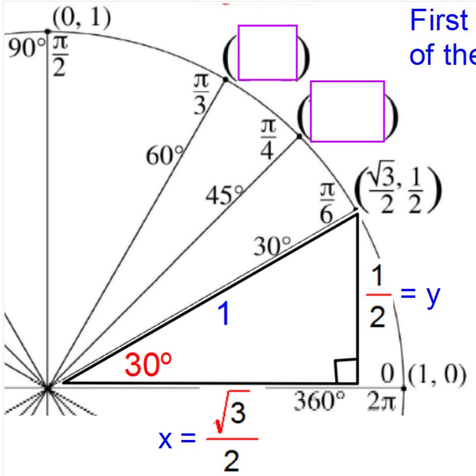
Coordinates on the axes.

Since this is the Unit Circle the radius is 1 and the points on the axes are just 1 unit right, 1 up, 1 left, and 1 down from the origin.

The Unit Circle involves the angles in Special Right Triangles which means it probably involves the sides too!



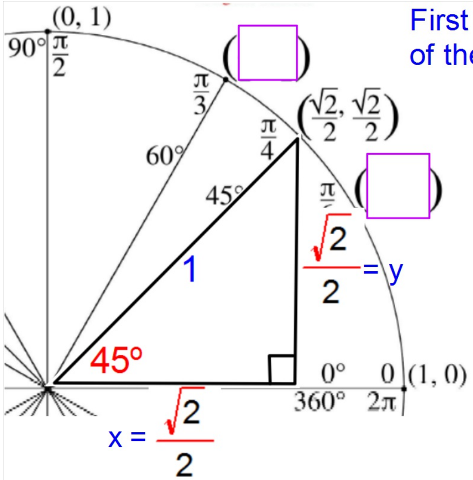
First Quadrant of the Unit Circle



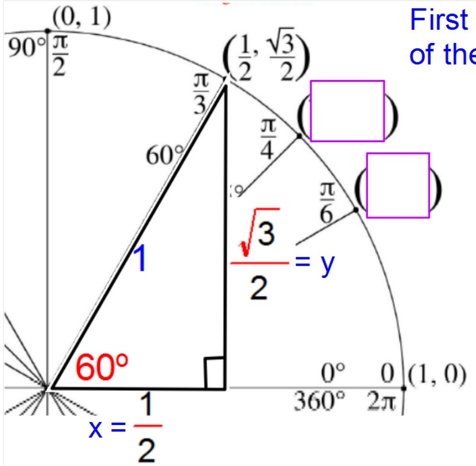
x-coordinates are the horizontal distance from the origin

y-coordinates are the vertical distance from the origin

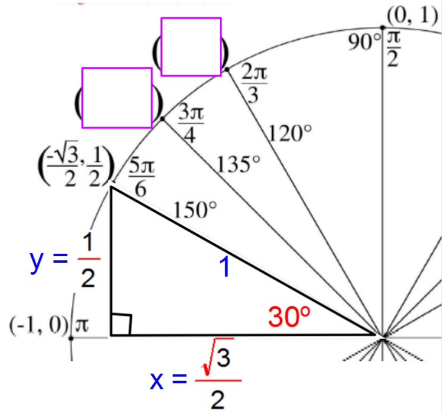
First Quadrant of the Unit Circle



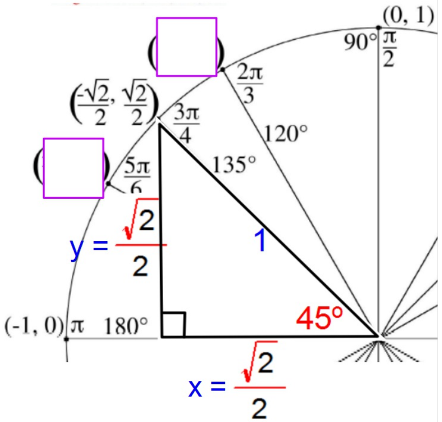
First Quadrant of the Unit Circle



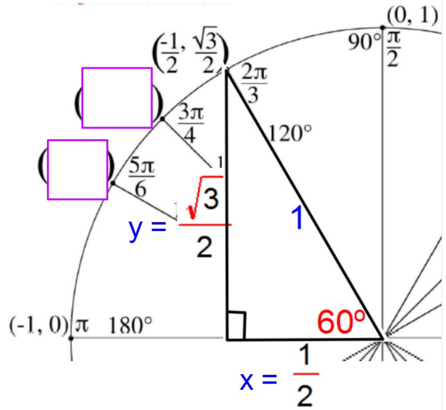
Second Quadrant of the Unit Circle

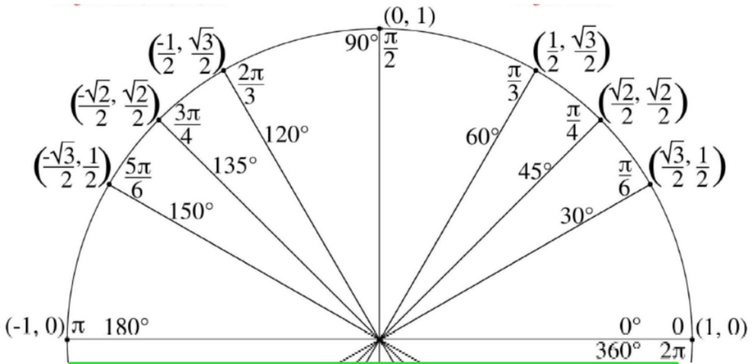


Second Quadrant of the Unit Circle



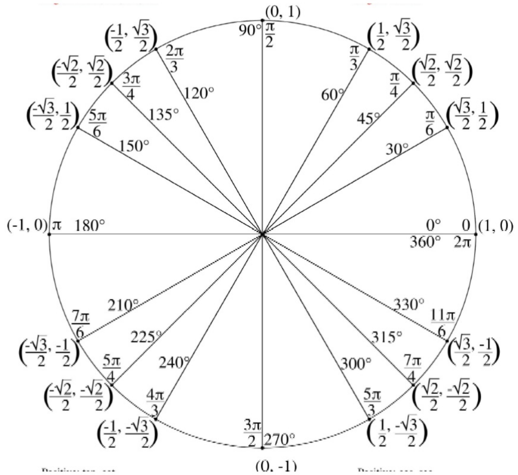
Second Quadrant of the Unit Circle





What do you notice about the relationship amongst the coordinates in the 1st and 2nd Quadrant?

The coordinates of the points in the second quadrant are reflections over the y-axis of the points in the first quadrant but x is negative.

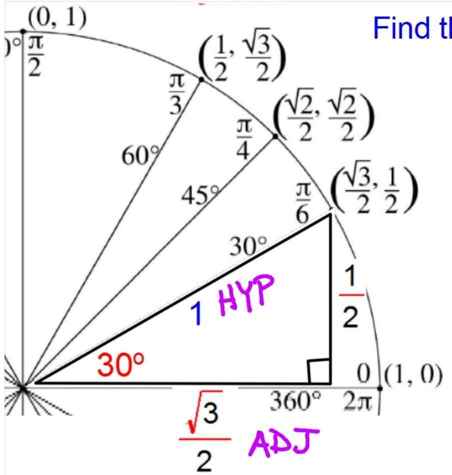


Coordinates in the 3rd and 4th Quadrants.

the coordinates of the points in the third and fourth quadrants are reflections over the x-axis of the points in the first and second quadrants. But in the third quadrant x and y are negative and in the fourth quadrant only y is negative.

You might also notice that if you draw a diameter from any point on the unit circle to the point on the opposite side that they have the same x & y coordinates except the signs will change.

Find the Exact value of each.



$\text{Cos}30^{\circ}$

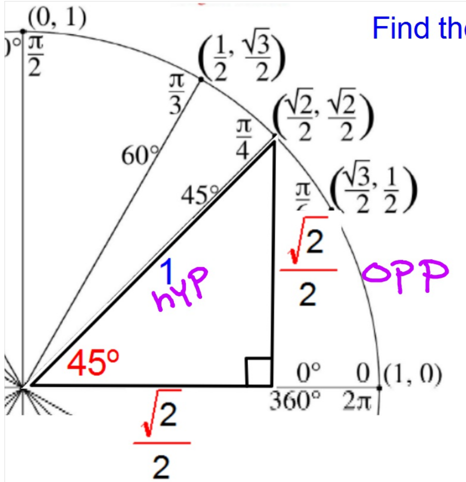
$$\cos 30^{\circ} = \frac{\text{adjacent leg}}{\text{hypotenuse}}$$

$$= \frac{\frac{\sqrt{3}}{2}}{1} = \frac{\sqrt{3}}{2}$$

$$\cos 30^{\circ} = \frac{\sqrt{3}}{2}$$

Notice that this is simply the x-coordinate of the point associated with 30°

Find the Exact value of each.



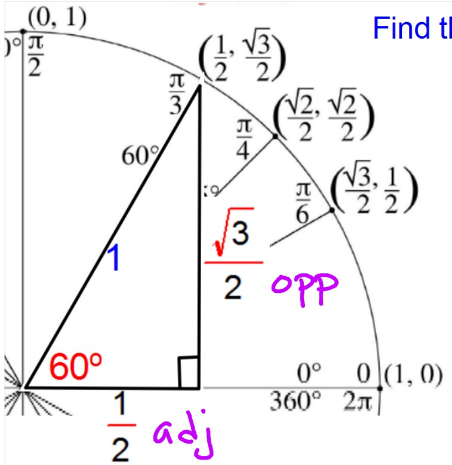
$\text{Sin}45^{\circ}$

$$\sin 45^{\circ} = \frac{\text{opp leg}}{\text{hypot}}$$

$$= \frac{\frac{\sqrt{2}}{2}}{1} = \frac{\sqrt{2}}{2}$$

$$\sin 45^{\circ} = \frac{\sqrt{2}}{2}$$

Notice that this is simply the y-coordinate of the point associated with 45°



Find the Exact value of each.

Tan60°

$$\text{Tan}\,60^\circ = \frac{\text{opp leg}}{\text{adj leg}}$$

$$= \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\sqrt{3}}{2} \cdot \frac{2}{1} = \sqrt{3}$$

$$\text{Tan}\,60^\circ = \sqrt{3}$$

Notice that this is the ratio of
the x & y -coordinates of the
point associated with 60°