

Solve each equation. Round to the nearest hundredth.

1. $15^{2x} = 8$

$$\log_{15} 8 = 2x$$

find $\log_{15} 8$ and
divide by 2

$$x = 0.38$$

2. $2^{x+7} = 101$

$$\log_2 101 = x + 7$$

find $\log_2 101$ and
subtract 7

$$x = -0.34$$

3. $5(7)^{4x+1} - 6 = 38$

$$\begin{array}{r} +6 \quad +6 \\ 5(7)^{4x+1} = 44 \\ \hline 5 \end{array}$$

$$7^{4x+1} = 8.8$$

$$\log_7(8.8) = 4x+1$$

find $\log_7(8.8)$ then subtract 1 and
finish by dividing by 4

$$x = 0.03$$

You can now finish Hwk #2.

Practice Sheet: Logarithms

You are also now ready for Quiz #1.

Solve to the nearest hundredth.

$$\log_2(x + 1) = 6$$

$$2^6 = x + 1$$

$$\begin{array}{r} 64 = x + 1 \\ -1 \quad -1 \end{array}$$

$$x = 63$$

Solve to the nearest hundredth.

$$\log_x(x + 6) = 2$$

$$\begin{array}{r} -6 \\ +2 \quad -3 \\ \hline -1 \end{array}$$

$$x^2 = x + 6$$

$$x^2 - x - 6 = 0$$

$$(x-3)(x+2) = 0$$

$$x = 3, -2$$

$$x = 3$$

What is e ?

What is π ?

The number π is a mathematical constant. Originally defined as the ratio of a circle's circumference to its diameter, it now has various equivalent definitions and appears in many formulas in all areas of mathematics and physics

Simple Interest:

You only get paid interest on the initial investment no matter how long you leave your money in the account.

$$I = prt$$

You invest \$5000 at 6% annual interest for 10 years.

How much will you have at the end of 10 years if you get simple interest?

$$I = 5000(0.06)(10) = \$3000$$

$$\text{TOTAL} = 5000 + 3000 = \$8000$$

↑
initial
investment

↑
interest

Compounded Interest:

Interest is added to the initial investment then you get paid interest on that new total, etc...

You invest \$5000 at 6% annual interest for 10 years.
How much will you have at the end of 10 years?

$$100 + 6 = 106\% \quad b = 1.06$$

$$y = 5000(1.06)^{10}$$
$$y = \$8954.24$$

You invest \$5000 at 6% annual interest for 10 years.

How much will you have at the end of 10 years if you get interest monthly?

$$y = 5000(1.005)^{120}$$
$$y = \$9096.98$$

monthly Interest
rate:

$$\frac{6\%}{12} = 0.5\%$$

$$100\% + 0.5\% = 100.5\%$$

$$b = 1.005$$

the exponent will be 120 → 12 times a year for 10 years.

The following formula calculates the total amount of money you will end up with depending on how often you get paid interest.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

P = Principal - original amount

r = Interest rate as a decimal

n = # times per year interest is calculated

t = # years

Suppose you invest \$1 at 100% interest for 1 year.

$$A = P\left(1 + \frac{r}{n}\right)^{nt} \rightarrow A = 1\left(1 + \frac{1}{n}\right)^{n1}$$

$$P = 1$$

$$r = 100\% \rightarrow 1$$

n = # times per year interest is calculated

$$t = 1$$

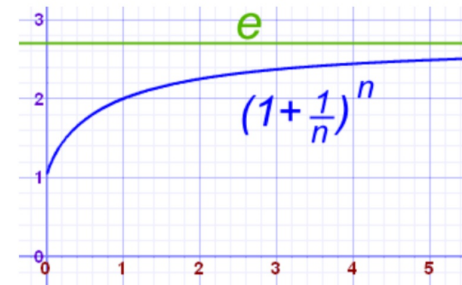
Frequency of compounding	#times per year compound interest (n)	$1\left(1 + \frac{1}{n}\right)^n$	Dollar Value
Annually	$n = 1$	$1\left(1 + \frac{1}{1}\right)^1$	2.00
Semiannually	$n = 2$	$1\left(1 + \frac{1}{2}\right)^2$	2.25
quarterly	$n = 4$	$1\left(1 + \frac{1}{4}\right)^4$	2.441
monthly	$n = 12$		2.613
weekly	$n = 52$		2.693
daily	$n = 365$		2.715
hourly	$n = 8760$		2.718
every minute	$n = 525,600$		2.718
every second	$n = 31,536,000$		2.718

$$A = 1\left(1 + \frac{r}{n}\right)^{nt}$$

After a while there is very little change in the dollar value. This equation is said to have a limit of approximately 2.718

→ e

the value of $\left(1 + \frac{1}{n}\right)^n$ approaches e as n gets bigger and bigger:



n	$\left(1 + \frac{1}{n}\right)^n$
1	2.00000
2	2.25000
5	2.48832
10	2.59374
100	2.70481
1,000	2.71692
10,000	2.71815
100,000	2.71827

$\approx e$

Where is e used?

Like π , e is a mathematical constant most often found in formulas.

e is called Euler's constant.

Leonhard Euler: Swiss mathematician

Equation of a Catenary:
$$y = \frac{a}{2} \left(e^{\frac{x}{a}} + e^{-\frac{x}{a}} \right)$$

Catenary: A catenary is the shape that a cable assumes when it's supported at its ends and only acted on by its own weight. It is used extensively in construction, especially for suspension bridges

A famous Catenary:



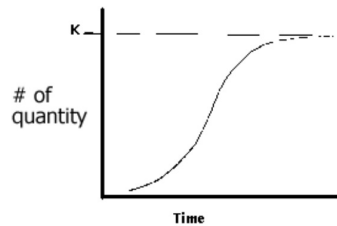
Another famous Catenary.



Logistic Growth:

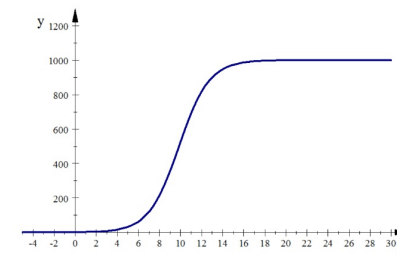
When the rate of growth of a quantity slows down after initially increasing (or decreasing) exponentially.

Graph



The equation below models the spread of flu in a school of 1000 students where y represents the number of students infected after x days.

$$y = \frac{1000}{1 + 990e^{-0.7x}}$$



The more often interest is calculated the more money you will earn.

What is more often than every second?

Continuously

Compounding Interest Continuously

$$y = Pe^{rt}$$

The diagram shows the formula $y = Pe^{rt}$ with four arrows pointing from its components to their respective meanings:

- An arrow from y points to "Amount after t years".
- An arrow from P points to "Principal".
- An arrow from e points to "Annual Interest rate as a decimal".
- An arrow from t points to "# years".