Horizontal Asymptotes:

The horizontal line that a graph approaches as you move farther and farther to the left and right. (END BEHAVIOR)

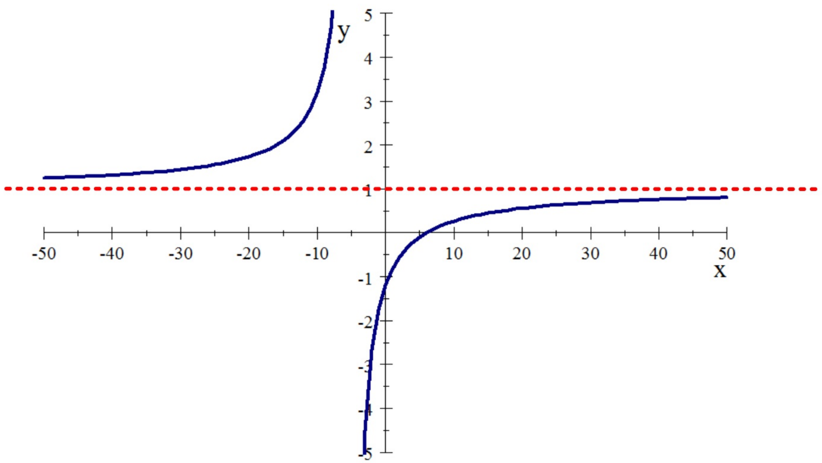
The value of y that the function approaches as x gets larger and larger (pos and neg).

Horizontal Asymptote Exploration:

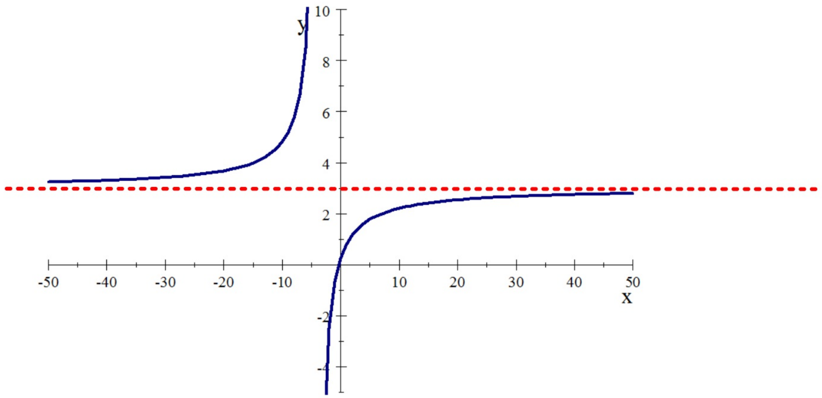
X	Y
100	
1000	
100000	
-100	
-1000	
-100000	

If the function is approaching the same value on both sides then this becomes the HA.

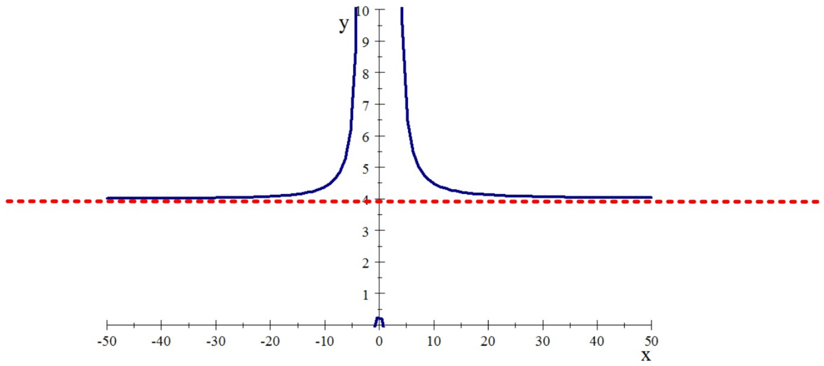
1. $y = \frac{x-6}{x+5}$ HA: $y=1$



2. $y = \frac{3x+1}{x+4}$ HA: $y=3$



3. $y = \dfrac{8x^2 + x - 6}{2x^2 - 21}$ HA:



What do you notice about the equations that would give you the HA without using a table of values?

1. $y = \dfrac{x - 6}{x + 5}$ HA: $y = 1 = \dfrac{1}{1}$

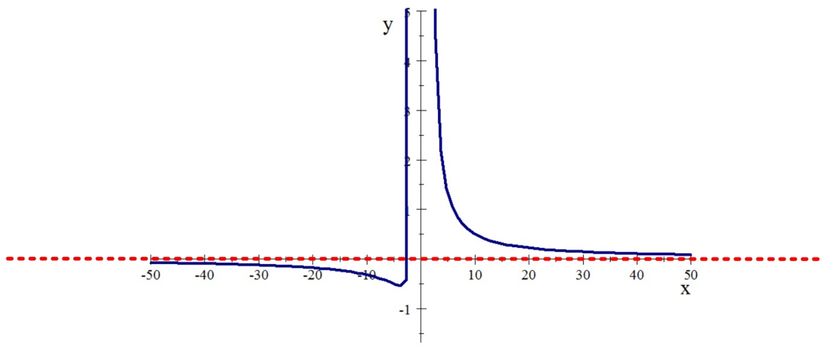
2. $y = \dfrac{3x + 1}{x + 4}$ HA: $y = 3 = \dfrac{3}{1}$

3. $y = \dfrac{8x^2 + x - 6}{2x^2 - 21}$ HA: $y = 4 = \dfrac{8}{2}$

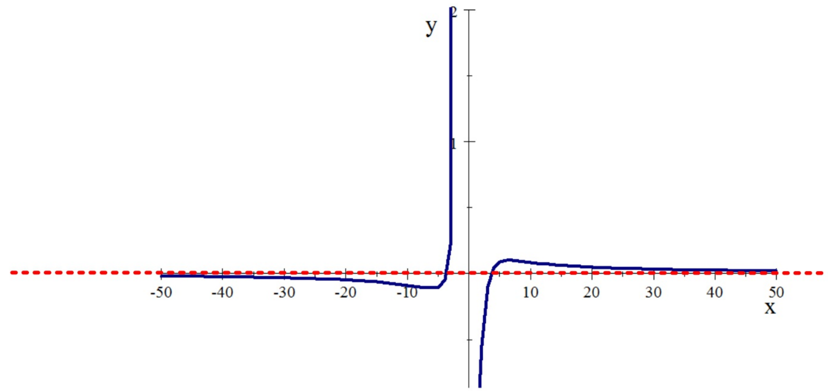
What do these three equations have in common?

The degree of the numerator and denominator are the same.

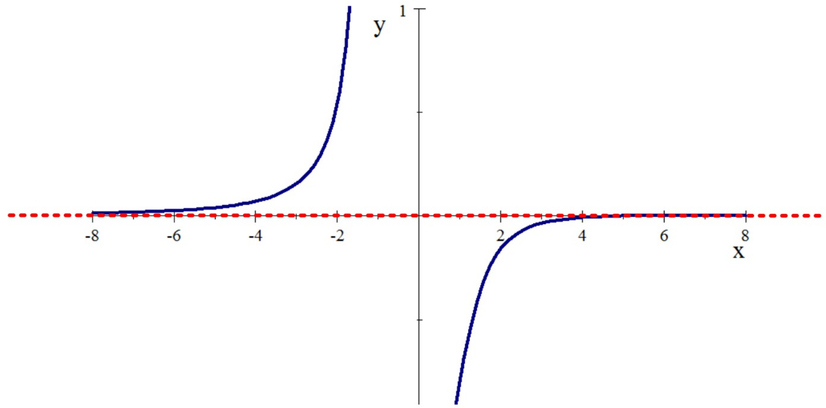
4. $y = \dfrac{4x + 9}{x^2 - 3}$ HA: $y=0$



5. $y = \dfrac{x^2 - 13}{x^3 + 7}$ HA: $y=0$



6. $y = \dfrac{x-5}{2x^3+3}$ HA: $y=0$



What do you notice about the equations that would give you the HA without using a table of values?

4. $y = \dfrac{4x+9x}{x^2-3}$ HA: y=0 $= \dfrac{\#}{\text{much bigger }\#}$

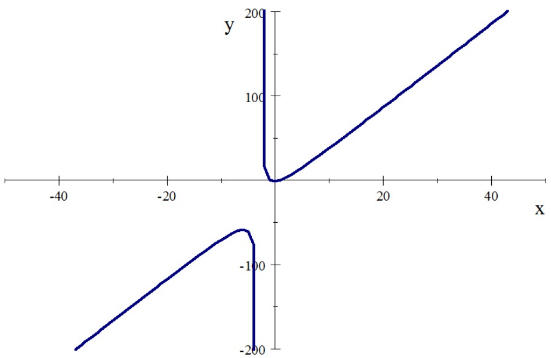
5. $y = \dfrac{x^2-13}{x^3+7}$ HA: y=0 $= \dfrac{\#}{\text{much bigger }\#}$

6. $y = \dfrac{x-5}{2x^3+3}$ HA: y=0 $= \dfrac{\#}{\text{much bigger }\#}$

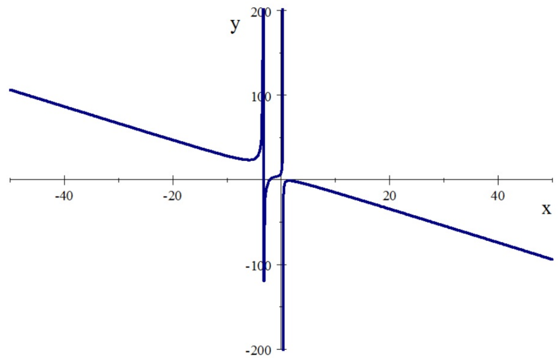
What do these three equations have in common?

The degree of the denominator is greater than the degree of the numerator.

7. $y = \dfrac{5x^2-4}{x+3}$ HA: NONE



8. $y = \dfrac{-2x^3+5x-8}{x^2+3x-1}$ HA: NONE



What do you notice about the equations that would tell you they don't have a HA without using a table of values?

7. $y = \frac{5x^2 - 4}{x + 3}$ HA: No HA

$= \frac{\text{much bigger \#}}{\#}$ as x gets bigger this will keep growing, either bigger pos or neg

8. $y = \frac{-2x^3 + 5x - 8}{x^2 + 3x - 1}$ HA: No HA

$= \frac{\text{much bigger \#}}{\#}$ as x gets bigger this will keep growing, either bigger pos or neg

What do these three equations have in common?

The degree of the numerator is greater than the degree of the denominator.

Without using a table or a graph how could you tell from the equation what the horizontal asymptote is or if it even has one?

HA depend on the degrees of the numerator and denominator.

Predict the Horizontal Asymptote for each of the rational functions below, if any.

a. $y = \frac{10x + 7}{5x - 3}$

Degrees are the same

HA: $y = 10/5$

$y = 2$

b. $y = \frac{6x^2 - 5}{2x + 3}$

bigger degree in numerator

HA: NONE

c. $y = \frac{12x - 11}{3x^2 - 1}$

bigger degree in denominator

HA: $y = 0$

Horizontal Asymptotes: Depends on the degrees of the Numerator and the Denominator

Case 1: Degree of the Numerator > Degree of the Denominator

No HA

Case 2: Degree of the Numerator = Degree of the Denominator

HA: y = ratio of the Leading Coefficients

Case 3: Degree of the Denominator > Degree of the Numerator

HA: y = 0

Determine by the equation the Horizontal Aysmptote for each rational function, if any.

1. $y = \frac{x^3 + 4x^2 - 9}{2x^2 + 6}$

HA: NONE

2. $y = \frac{15x^2 - 2x + 10}{3x^2 + 5}$

HA: $y = 15/3 = 5$

3. $y = \frac{20x + 13}{4x^2 + 9}$

HA: $y = 0$

The last two graphs didn't have horizontal asymptotes, they had slant asymptotes.

The equation of the slant asymptote can be found by actually doing the polynomial division.

The quotient, without the remainder, is the equation of the slant asymptote.

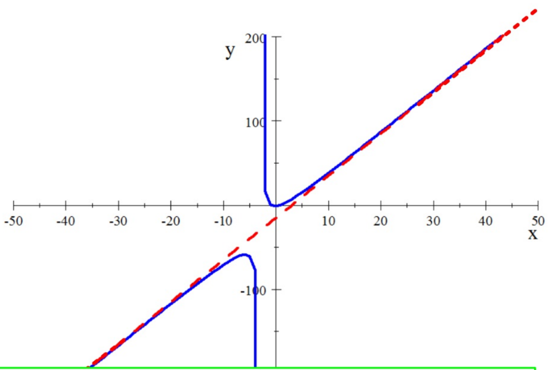
7. $y = \dfrac{5x^2 - 4}{x + 3}$ Find this quotient.

-3	5	0	-4
		-15	45
	5	-15	41

What is the equation of the slant asymptote?

$y = 5x - 15$

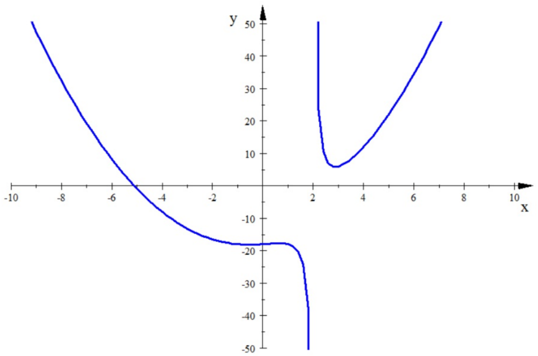
7. $y = \dfrac{5x^2 - 4}{x + 3}$ Graph this equation in Y_1 and the slant asymptote in Y_2.



Graph this rational function in the given window.

$Y_1 = \dfrac{x^3 - 19x + 36}{x - 2}$

X_{min}= -10	Y_{min}= -50
X_{max}= 10	Y_{max}= 50



What kind of end-behavior asymptote does this function seem to have?

Quadratic

Find the equation of the end-behavior asymptote.

$$Y_1 = \frac{x^3 - 19x + 36}{x - 2}$$

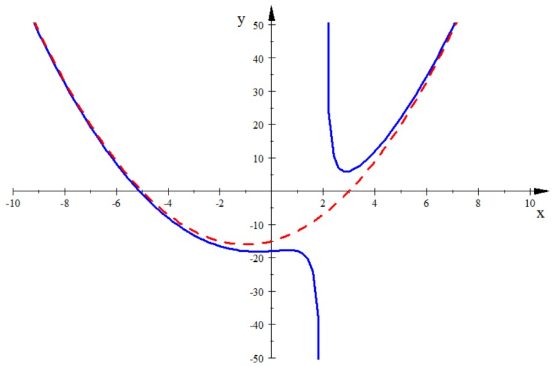
Graph this equation in Y_2.

$$\begin{array}{r|rrrr} 2 & 1 & 0 & -19 & 36 \\ & & 2 & 4 & -30 \\ \hline \end{array}$$

$$\boxed{y = 1x^2 + 2x \quad -15} \quad R=6$$

end. behavior asymptote

End behavior asymptote: $y = x^2 + 2x - 15$



Other information that would be used to graph Rational Functions by hand are:

x-intercepts and y -intercepts

Y-Intercepts: Evaluate the function by replacing x with zero.

Find the y-intercepts of each function.

$$y = \frac{x^2 - 9x + 20}{x^2 + 7x + 10}$$ y-int: $y = \frac{20}{10} = 2$

$$y = \frac{x^2 - 4}{2x^2 + 6x}$$ y-int: $y = \frac{-4}{0} = \text{undefined}$

NO y-int

In general, the y-intercept of a Rational Function is the:

Ratio of the Constants

A graph can have at most ONE y-intercept.