

Sec 8-3: Logarithms
(the inverse of exponential functions)

Exponential Function

$$y = b^x$$

The base
of the
Exponential
Function

The exponent

Logarithmic Function

$$\log_b y = x$$

The base of the
Logarithmic
Function

Exponential Function:

How do you say this?

$$y = b^x$$

"y equals b to the
power of x"

Logarithmic Function:

How do you say this?

$$\log_b y = x$$

"Log base b of y
equals x"

Exponential Function:

$$y = b^x$$

Logarithmic Function:

$$\log_b y = x$$

"the base is
the base"

The exponent
is the answer

Exponential Equation

Range:
Pos #'s

$$y = b^x$$

$b > 0, b \neq 1$

Domain:
Any real
number

Logarithmic Equation

$$\log_b y = x$$

Range:
Any real
number

Domain:
Pos #'s

$b: b > 0, b \neq 1$

You can get any
real number out
of a logarithm

You can't have a
negative # or zero
as the base. You'll
get an error message
on the calculator.

You can't take
the log of a
negative # or zero.
You'll get an
error message on
the calculator.

Get a whiteboard, marker, and rag

Rewrite each into logarithmic form.

1. $12^x = 100$ $\log_{12} 100 = x$

the exponent is the answer (green arrow from x to 100)
the base is the base (blue arrow from 12 to 12)

2. $6^2 = x$

$\log_6 x = 2$

3. $x^5 = 20$

$\log_x 20 = 5$

Rewrite each into exponential form.

1. $\text{LOG}_5 8 = x$ $5^x = 8$

2. $\text{LOG}_3 x = 12$ $3^{12} = x$

3. $\text{LOG}_x 15 = 30$ $x^{30} = 15$

Write in Logarithmic Form:

$10^x = 125$

$\log_{10} 125 = x$

$\text{LOG}_{10} 125 \rightarrow$ "LOG base 10 of 125" $\rightarrow \text{LOG} 125$

LOG_{10} is called the Common Logarithm and is written without the 10.

The button on the calculator LOG is for Common Logarithms LOG_{10}

Evaluate each without a calculator:

(hint: think of each as an exponential)

1. $\log_4 1$

$4^? = 1$ $? = 1$

2. $\log_3 9$

$3^? = 9$ $? = 2$

3. $\log_7(7)$

$7^? = 7$ $? = 1$

4. $\log_{25} 5$

$25^? = 5$
 $\sqrt{25} = 5$ $? = \frac{1}{2}$
 $25^{\frac{1}{2}} = 5$

5. $\log_6(6^4)$

$6^? = 6^4$
 $? = 4$

6. $\log_2(0.5)$

$2^? = 0.5 \rightarrow \frac{1}{2}$
 $2^? = \frac{1}{2^1}$ $? = -1$

If it's a base other than 10 such as: $\log_3 8$

Method 1:

Press **MATH** then

arrow key down to A \downarrow logBASE(or **MATH** **alpha** **MATH**

Method 2:

Press **alpha** then **window**

choose 5: logBASE(

$\log_{\square} \square$

Using the graphing calculator to do logarithms

If it's \log_{10} just use the **log** button

$\log 75 =$ **log** **7** **5** **=** 1.875

This answer represents the power of 10 that will equal 75. In other words, the logarithm represents the following exponential: $10^x = 75$
 $10^1 = 10$ and $10^2 = 100$. Since 75 is between these two values you should expect the exponent to be between 1 and 2.

Find each to the nearest hundredth.

1. $\log_9 5 = 0.73$ 2. $\log_2 20 = 4.32$

3. $\log 300 = 2.48$

Solve each equation. Round to the nearest tenth.

1. $10^x = 1500$

$$\log 1500 = x$$

$$x = 3.2$$

3. $4^x = 44$

$$\log_4 44 = x$$

$$x = 2.7$$

2. $\frac{4(10)^x}{4} = \frac{570}{4}$

$$10^x = 142.5$$

$$\log 142.5 = x$$

$$x = 2.2$$

4. $12^x = 3$

$$\log_{12} 3 = x$$

$$x = 0.4$$

What if your calculator only has common logarithm button LOG?

Property

Change of Base Formula

For any positive numbers, M , b , and c , with $b \neq 1$ and $c \neq 1$,

$$\log_b M = \frac{\log_c M}{\log_c b}$$

Most of the time
we'll use Common Log
(base=10)

$$\log_5 30 = \frac{\log 30}{\log 5} = 2.11$$

Find each to the nearest hundredth using only common logarithms.

1. $\log_5 16 =$

$$= \frac{\log 16}{\log 5}$$

$$= 1.72$$

2. $\log_8 100 =$

$$= \frac{\log 100}{\log 8}$$

$$= 2.21$$

The value of a house has been decreasing 7.5% each year. The house was worth \$180,000 in 2001.

In how many years will the value fall to \$45,000?
Round to the nearest hundredth.

$$\frac{45,000}{180,000} = \frac{180,000}{180,000} (0.925)^x$$

$$100 - 7.5$$

$$= 92.5\%$$

$$b = 0.925$$

$$0.25 = 0.925^x$$

$$x = \log_{0.925} (0.25) = 17.78 \text{ yrs}$$