Section 7-7: Inverse Relations

The concept of an Inverse Relation is all about...

switching X and Y

To draw the inverse relation of Y_1:

1. Press 2nd
2. Press PRGM (DRAW)
3. Choose option 8: DrawInv
4. Press ALPHA then TRACE
5. Choose Y_1
6. Press ENTER

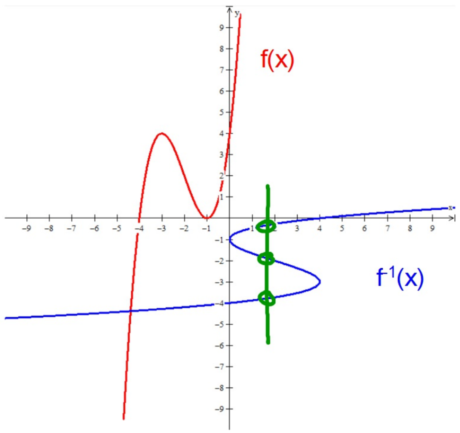
Is the inverse relation to

$$y = x^3 + 6x^2 + 9x + 4$$

a function?

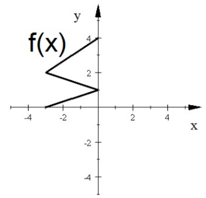
No. The inverse fails the Vertical line test

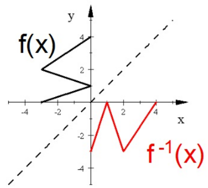
a vertical line intersects the inverse more than once.



Will the inverse relation be a function?

A



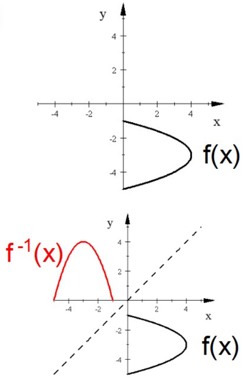


Yes, the inverse passes the Vertical Line Test

No vertical line intersects the inverse more than once.

Will the inverse relation be a function?

B

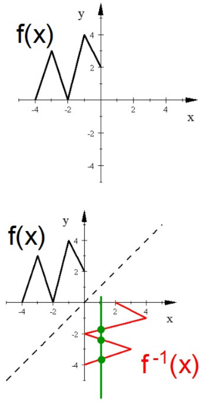


Yes, the inverse passes the Vertical Line Test

No vertical line intersects the inverse more than once.

Will the inverse relation be a function?

C

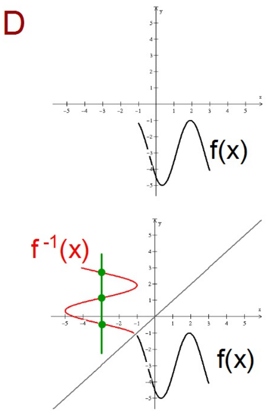


No, the inverse fails the Vertical Line Test

A vertical line intersects the inverse more than once.

Will the inverse relation be a function?

D



No, the inverse fails the Vertical Line Test

A vertical line intersects the inverse more than once.

If you have a graph of the original function or simply know what it looks like you can determine if the inverse is a function or not without actually seeing a graph of the inverse.

It's called: The Horizontal Line Test

Horizontal Line Test: a visual test to determine if the inverse relation will be a function.

If any horizontal line can intersect a graph more than once then the graph of the inverse is NOT a function

this is because:

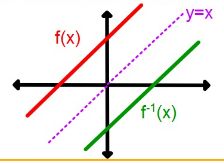
Horizontal lines on f(x) become vertical Lines on $f^{-1}(x)$.

If a Horizontal line touches the original graph of f(x) more than once the corresponding vertical line on $f^{-1}(x)$ will touch the inverse more than once.

Are the following statements true?

If the original relation IS a function, then the inverse is NOT a function?

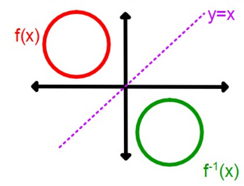
NO. A counterexample is - a linear function.



The inverse would be the green line which is still a function.

If the original relation is NOT a function, then the inverse IS a function?

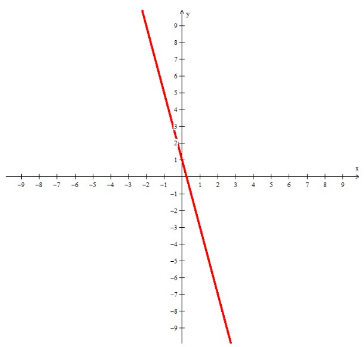
NO. A counterexample is - a circle.



The inverse would be the green circle which is still NOT a function.

Use what you may know about the graph of each or graph them using the graphing calculator to determine if the inverse relation of each is a function or not.

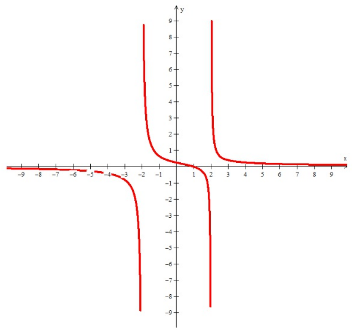
$$f(x) = -4x + 1$$



Yes, the inverse is a function because no Horizontal Line will touch the orginal graph more than once so no Vertical Line will touch the inverse more than once.

Is the inverse a function?

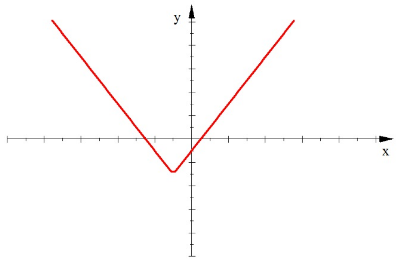
$$y = \frac{x-1}{x^2-4}$$



No, the inverse is not a function because there is a Horizontal Line that will touch the orginal graph more than once which meant that there is Vertical Line that will touch the inverse more than once.

Is the inverse a function?

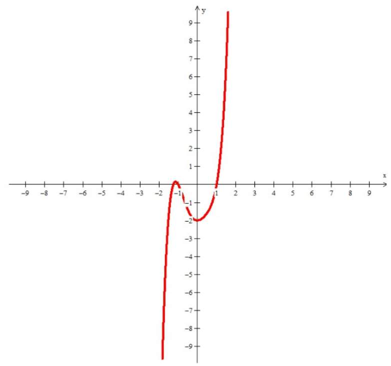
$$y = 2|x+1| - 3$$



No, the inverse is not a function because there is a Horizontal Line that will touch the orginal graph more than once which meant that there is Vertical Line that will touch the inverse more than once.

Is the inverse a function?

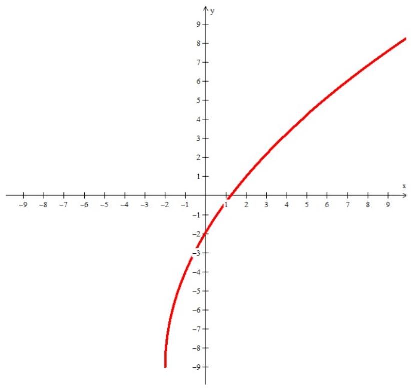
$$y = x^5 - x^3 + 2x^2 - 2$$



No, the inverse is not a function because there is a Horizontal Line that will touch the orginal graph more than once which meant that there is Vertical Line that will touch the inverse more than once.

Is the inverse a function?

$$f(x) = 5\sqrt{x+2} - 9$$

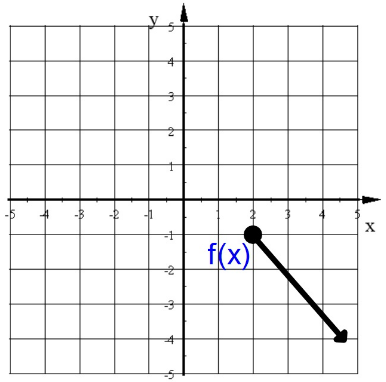


Yes, the inverse is a function because no Horizontal Line will touch the orginal graph more than once so no Vertical Line will touch the inverse more than once.

d

Original Relation		Inverse Relation
f(x)	Becomes	$f^{-1}(x)$
The point (a,b)	Becomes	The point (b,a)
Domain of f(x)	Becomes	Range of $f^{-1}(x)$
Range of f(x)	Becomes	Domain of $f^{-1}(x)$
Graph of f(x)	Reflect over y=x Becomes	Graph of $f^{-1}(x)$

Given the graph of f(x) below, find the domain and range of $f^{-1}(x)$



Start by finding domain and range of the original f(x):

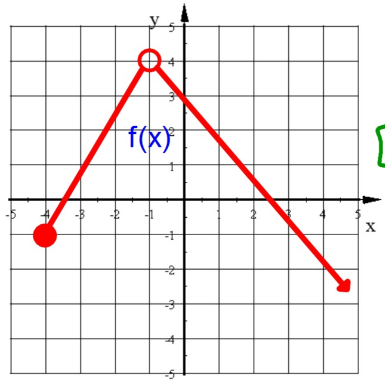
Domain of f(x): $[2, \infty)$

Range of f(x): $(-\infty, -1]$

Domain of $f^{-1}(x)$: $(-\infty, -1]$

Range of $f^{-1}(x)$: $[2, \infty)$

Given the graph of f(x) below, find the domain and range of $f^{-1}(x)$



Start by finding domain and range of the original f(x):

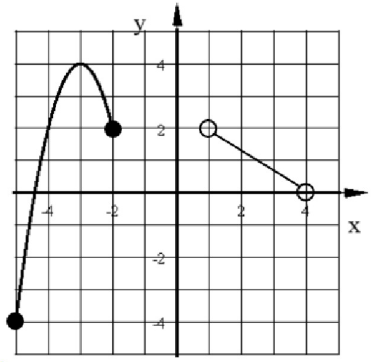
Domain of f(x): $[-4, -1) \cup (1, \infty)$

Range of f(x): $(-\infty, 4)$

Domain of $f^{-1}(x)$: $(-\infty, 4)$

Range of $f^{-1}(x)$: $[-4, -1) \cup (1, \infty)$

Given the graph of f(x) below, find the domain and range of $f^{-1}(x)$



Start by finding domain and range of the original f(x):

Domain of f(x): $[-5, -2] \cup (1, 4)$

Range of f(x): $[-4, 4]$

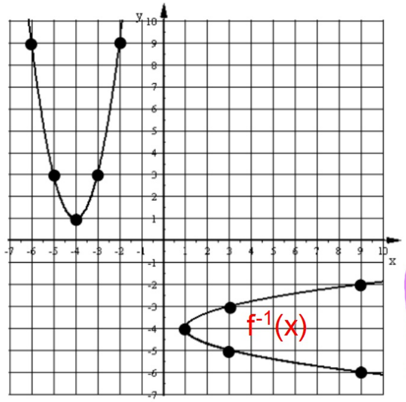
Domain of $f^{-1}(x)$: $[-4, 4]$

Range of $f^{-1}(x)$: $[-5, -2] \cup (1, 4)$

What you should know from Sec 7-7:

1. Given an original relation be able to tell if the inverse is a function or not.
2. Be able to find the domain and range of the inverse relation using the domain and range of the original relation.
3. Be able to write the equation of the inverse relation.

$f(x) = 2(x+4)^2 + 1$



Equations of Inverses

1. Switch the variables x and y

$$x = 2(y+4)^2 + 1$$

2. Solve equation for y

$$y = \pm\sqrt{\frac{x-1}{2}} - 4$$

$\pm$ is required because you are taking an EVEN root.

Find the equation of the inverse for each function.

$f(x) = 2x - 3$

First switch x and y:

$$x = 2y - 3$$

Solve for y:

move -3: $x + 3 = 2y$

move 2: $\frac{x+3}{2} = y$

$$f^{-1}(x) = \frac{x+3}{2}$$

Find the equation of the inverse for each function.

$f(x) = (x+5)^3 - 7$

First switch x and y:

$$x = (y+5)^3 - 7$$

Solve for y:

move -7: $x + 7 = (y+5)^3$

undo power: $\sqrt[3]{x+7} = y + 5$

move $+5$: $y = \sqrt[3]{x+7} - 5$

$$f^{-1}(x) = \sqrt[3]{x+7} - 5$$

no $\pm$ required because you are taking an ODD root.

Find the equation of the inverse for thos function.

$y = \frac{-4}{x + 7} - 8$

First switch x and y: $x = \frac{-4}{y+7} - 8$

Solve for y:

move -8: $x + 8 = \frac{-4}{y+7}$

mult by $y+7$: $(x+8)(y+7) = -4$

divide by $x+8$: $y + 7 = \frac{-4}{x+8}$

move $+7$: $y = \frac{-4}{x+8} - 7$

$$f^{-1}(x) = \frac{-4}{x+8} - 7$$

You can now finish Hwk #40

Practice Sheet Section 7-7