-1 and 2 are solutions of this equation. The other two solutions are imaginary. Use division to help you find the other two solutions.

$$x^4 - x^3 + 2x^2 - 4x - 8 = 0$$

Do synthetic division w/ one of the zeros:

$$\begin{array}{r|rrrrr} -1 & 1 & -1 & 2 & -4 & -8 \\ & & -1 & 2 & -4 & 8 \\ \hline & 1 & -2 & 4 & -8 & 0 \end{array}$$

$x^3 - 2x^2 + 4x - 8$

DIVIDE THIS RESULT BY the other zero

$$\begin{array}{r|rrrr} 2 & 1 & -2 & 4 & -8 \\ & & 2 & 0 & 8 \\ \hline & 1 & 0 & 4 & 0 \end{array}$$

$x^2 + 4$

find the zeros of this

$x^2 + 4 = 0$

$\sqrt{x^2} = \sqrt{-4}$

$\boxed{x = \pm 2i}$

Use this division problem

$$\frac{x^3 - 5x^2 + 8x - 11}{x+3} = x^2 - 8x + 32 - \frac{107}{x+3}$$

What is the remainder? -107

What is the zero of the divisor? -3

Evaluate the dividend using the zero of the divisor.

$(-3)^3 - 5(-3)^2 + 8(-3) - 11 = -107$

What do you notice?

When you evaluate the dividend using the zero of the divisor you get the same value as the remainder when you divided!

Theorem	**Remainder Theorem**
If a polynomial $P(x)$ of degree $n \geq 1$ is divided by $(x - a)$, where a is a constant, then the remainder is $P(a)$.	

Find the remainder to this quotient: $\frac{2x^3 + x^2 - 7x - 10}{x - 2}$

$2(2)^3 + (2)^2 - 7(2) - 10 = \boxed{-4}$

This Theorem states: the remainder when dividing two polynomials can be found by substituting the zero of the divisor into the dividend.

What is the remainder of this quotient?

$$\frac{5x^2 + 8x - 11}{x + 3}$$

$$5(-3)^2 + 8(-3) - 11 = \boxed{10}$$

Or, you could just do the division to find the remainder.

Is $x + 2$ a factor of $2x^3 + 5x^2 + x - 6$

Only if the remainder is zero!

Use Division

$$\begin{array}{r|rrrr} -2 & 2 & 5 & 1 & -6 \\ & & -4 & -2 & 2 \\ \hline & 2 & 1 & -1 & -4 \end{array}$$

Use the Remainder Theorem

$$2(-2)^3 + 5(-2)^2 + (-2) - 6$$
$$= -4$$

The remainder is -4

Is X+3 a factor of $x^3 - 2x^2 + 10x - 21$?

Use Division

$$\begin{array}{r|rrrr} -3 & 1 & -2 & 10 & -21 \\ & & -3 & 15 & -75 \\ \hline & 1 & -5 & 25 & -96 \end{array}$$

Use the Remainder Theorem

$$(-3)^3 - 2(-3)^2 + 10(-3) - 21$$
$$= -96$$

The remainder is -96

The remainder theorem can be used to "check" to see if your answer when doing some polynomial division problems is reasonable.

Is this answer reasonable?

$$\frac{3x^3 - 7x^2 + 8x - 4}{x - 2} = \boxed{3x^2 - x + 6 \qquad R = 8}$$

$3(2)^3 - 7(2)^2 + 8(2) - 4 = ?$ 8

Since the Remainder Theorem gives the same remainder as my division this answer is reasonable.

You can only "check" this way if the divisor is linear with a leading coefficient of 1

Is the given answer reasonable?

$$\frac{-2x^3 + x^2 - 4x + 7}{x - 3} = \boxed{-2x^2 - 5x - 19 \qquad R = 50}$$

use the Remainder Theorem:

$$-2(3)^2 - 5(3) - 19 = -50$$

Since the Remainder Theorem gives a different remainder as my division this answer is NOT reasonable. I must have made a mistake.

Sec 6-4: Solving Polynomial Equations

- Solve by factoring
- Solve by graphing

Find all Complex solutions by Factoring.

$$96a^4 - 224a^3 - 40a^2 = 0$$

$$8a^2\left(\underbrace{12a^2 - 28a}_{} - 5\right)$$

-60

$+2 \quad -30 \quad \Rightarrow$

-28

	$2a$	-5
$6a$	$12a^2$	$-30a$
$+1$	$+2a$	-5

$$= 8a^2(6a+1)(2a-5)$$

$$a = 0, -1/6, 5/2$$

Find all Complex solutions by Factoring.

$$3x^7 + 6x^5 - 9x^3 = 0$$

$$3x^3\left(\underbrace{x^4 + 2x^2 - 3}_{}\right) = 0$$

-3

$+3 \quad -1 \quad \Rightarrow \quad (x^2+3)(x^2-1)$

$+2$

$$3x^3(x^2+3)(x^2-1)$$

$$= 3x^2(x^2+3)(x \pm 1)$$

$x^2 + 3 = 0$

$\sqrt{x^2} = \sqrt{-3}$

$$x = 0, \pm 1, \pm i\sqrt{3}$$