Factor this: $4x^2 - 25$

$= (2x \pm 5)$

Can you factor this? $9x^2 + 49$

NO, you can only factor the Difference of perfect squares.

Can you factor each?

$x^2 - 9$

$= (x \pm 3)$

$x^3 - 8$

Yes, but not the same way as factoring $x^2 - 9$. Neither x^3 nor 8 are perfect squares.

what are these numbers? 1, 8, 27, 64, 125, ...

$1^3, 2^3, 3^3, 4^3, 5^3, \ldots$

Perfect Cubes

Factoring the difference of perfect cubes:

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Factor the following:

1. $x^3 - 8 \Longrightarrow (x)^3 - (2)^3$

$a = x \quad b = 2$

$= \boxed{(x-2)(x^2+2x+4)}$

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Factor: $125x^3 - 216 \Longrightarrow (5x)^3 - (6)^3$

$a = 5x \qquad b = 6$

$= \boxed{(5x - 6)(25x^2 + 30x + 36)}$

You can't factor the sum of perfect squares, but there IS a way to factor the sum of perfect cubes.

$\boxed{a^3 + b^3 = (a + b)(a^2 - ab + b^2)}$

Factor the following:

$x^3 + 27 \longrightarrow (x)^3 + (3)^3$

$a = x \qquad b = 3$

$= \boxed{(x+3)(x^2 - 3x + 9)}$

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Factor the following: $64x^6 + 125$

$= (4x^2)^3 + (5)^3$

$a = 4x^2 \qquad b = 5$

$= \boxed{(4x^2 + 5)(16x^4 - 20x^2 + 25)}$

Find the next two rows of this pattern of numbers.

This pattern is called: Pascal's Triangle.

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

Each row starts and ends with a 1. The numbers inbetween are the sum of the 2 numbers above

Row 0 → 1

Row 1 → 1 1

Row 2 → 1 2 1

Row 3 → 1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

The Row Number in Pascal's Triangle is the 2nd Number in each row.

Sec 6-8: The Binomial Theorem

Expand each. Write answers in Standard Form.

1. $(a + b)^0 =$

$(a+b)^0 = \boxed{1}$

2. $(a + b)^1 =$

$(a + b)^1 = \boxed{1a + 1b}$

3. $(a + b)^2 =$

$(a + b)^2 = \boxed{1a^2 + 2ab + 1b^2}$

4. $(a + b)^3 =$

$(a + b)^3 = \boxed{1a^3 + 3a^2b + 3ab^2 + 1b^3}$

Do you see a connection to Pascal's Triangle and the results of expanding powers of (a + b) ?

$(a+b)^0 = 1 \longrightarrow 1$

$(a+b)^1 = a + b \longrightarrow 1a + 1b$

$(a+b)^2 = a^2 + 2ab + b^2 \longrightarrow 1a^2 + 2ab + 1b^2$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 \longrightarrow 1a^3 + 3a^2b + 3ab^2 + 1b^3$

The coefficients of each term are the numbers in the row of Pascal's Triangle that to corresponds to the exponent of (a + b).

Do you notice a pattern in the exponents?

Powers of a decrease from left to right

Powers of b decrease from right to left

For each term the powers of a and b add to the exponent of (a + b)

$$(a+b)^0 = 1$$

$$(a+b)^1 = a + b$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

What do you notice about how many terms each has?

$$(a+b)^0 = 1$$

$$(a+b)^1 = a+b$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

The number of terms is one more then the power on (a + b).