Finding zeros of a function:

Real Zeros of a function are x-intecepts of the graph.

One way to find zeros of a function is to
FACTOR the function and find the zeros of each factor.

If factorable, this will lead to ALL Complex Zeros.

Theorem **Fundamental Theorem of Algebra**

If $P(x)$ is a polynomial of degree $n \geq 1$ with complex coefficients, then $P(x) = 0$ has at least one complex root.

Corollary

Including complex roots and multiple roots, an nth degree polynomial equation has exactly n roots; the related polynomial function has exactly n zeros.

The Theorem states that a polynomial that is at least linear then it has at least one zero.

The Corollary states that the number of complex roots is exactly the same as the degree of the polynomial.
An example of multiple roots is: $(x - 3)^2$ this could be written as (x-3)(x-3), therefore, this represents two zeros, both are -3. You would write -3 only once but need to understand it actually is two zeros.

Theorem **Irrational Root Theorem**

Let a and b be rational numbers and let $\sqrt{b}$ be an irrational number. If $a + \sqrt{b}$ is a root of a polynomial equation with rational coefficients, then the conjugate $a - \sqrt{b}$ also is a root.

When finding the zeros of a factor like $(x^2 - 7)$ you would set the factor equal to zero and solve for x.

$x^2 - 7 = 0$

$x^2 = 7$

$x = \pm\sqrt{7}$ since 7 is not a perfect square this is an irrational root.

When solving with square roots you will always get two answers, one of them with a + sign and one with a - sign, conjugates. If you know one irrational root there is automatically another one just by changing the sign.

Irrational roots always come in pairs.

Theorem **Imaginary Root Theorem**

If the imaginary number $a + bi$ is a root of a polynomial equation with real coefficients, then the conjugate $a - bi$ also is a root.

When finding the zeros of a factor like $(x^2 + 9)$ you would set the factor equal to zero and solve for x.

$x^2 + 9 = 0$

$x^2 = -9$

$x = \pm 3i$ since you had to take the squar root of a negative the root is imaginary.

When solving with square roots you will always get two answers, one of them with a + sign and one with a - sign, conjugates. If you know one imaginary root there is automatically another one just by changing the sign.

Imaginary roots always come in pairs.

Find ALL Complex zeros of each function by factoring.

1. $y = 6x^6 - 48x^4 - 288x^2$

2. $f(x) = 648x^5 - 5000x$

See next two screens for the answers

1. $y = 6x^6 - 48x^4 - 288x^2$

$6x^2(x^4 - 8x^2 - 48)$

$6x^2(x^2 - 12)(x^2 + 4)$

-48, -12, $+4$, -8

$X = 0, \pm 2\sqrt{3}, \pm 2i$

$x^2 - 12 = 0$

$x^2 = 12$

$x = \pm 2\sqrt{3}$

$x^2 + 4 = 0$

$x^2 = -4$

$x = \pm 2i$

2. $f(x) = 648x^5 - 5000x$

$8x(81x^4 - 625)$

$8x(9x^2 + 25)(9x^2 - 25)$

$8x\ (9x^2 + 25)\ (3x \pm 5)$

$x = 0, \pm \frac{5i}{3}, \pm 5/3$

$9x^2 + 25 = 0$

$9x^2 = -25$

$\sqrt{x^2} = \sqrt{\frac{-25}{9}}$

How do you find the zeros of a function if you can't factor it?

By graphing

Finding zeros of a function with the graphing calculator:

Method 1: Finding ZEROS

$$y = x^4 + 2x^3 - 3x^2 - x + 3$$

Use the option on the graphing calculator to find zeros:
2ND TRACE
2: ZEROS → You will then be asked to surround the zero by moving the cursor to the left (Left Bound) of the zero and press Enter, then move the cursor to the right (Right Bound) of the zero and press Enter. Guess? asks you to move closer to the point in question and press Enter once more.

zeros are: -2.81 , -1

Zeros of a function are the values of x when y = 0.

Method 2: Finding Intersections

$$0 = x^4 + 2x^3 - 3x^2 - x + 3$$

Graph $Y_1 = x^4 + 2x^3 - 3x^2 - x + 3$
and $Y_2 = 0$

use the option on the graphing calculator to find points of intersection.
2ND TRACE
5: intersect → It will ask you to identify which two equations you are trying to find the intersection of:

First Curve? If Y_1 is the first equation press Enter.

Second Curve? If Y_2 is the second equation press Enter.

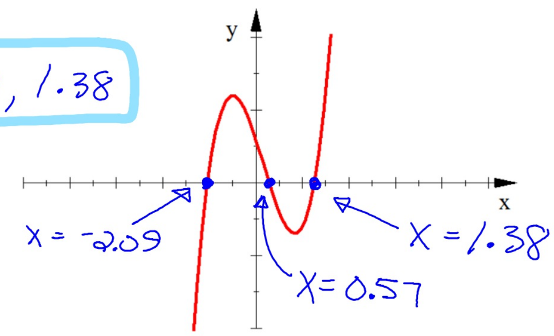
Guess? asks you to move closer to the point in question and press Enter once more.

zeros are: -2.81 , -1

When finding zeros by graphing you are only able to find the REAL zeros!

Find the real zeros of this function: $y = x^3 - x^2 - 5x + 3$

X = -2.09, 0.57, 1.38



What if you don't have a graphing calculator to find zeros?

Check out the Helpful Math Links on my blog!

You can now finish Hwk #24

Practice Sheet: Due tomorrow

Max's, Min's, & Zeros of Polynomials.