End Behavior of polynomial graphs.

Where are the ends of a graph found?

At the far left and far right

When you are asked to describe the end behavior of a graph you are really asked to describe how the graph is behaving at the very far left and right of the graph.

The graph of every polynomial will do only one of two things at an end:

It will either go up (Y increases...bigger positive)

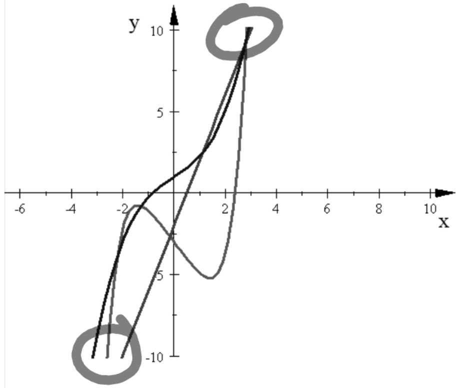
Or

It will go down (Y decreases...bigger negative)

Graph all three of these in a Standard Window:

$$Y_1 = 4x - 2$$
$$Y_2 = 0.25x^3 + x + 1$$
$$Y_3 = 0.1x^5 - 2x - 3$$

What do the graphs have in common?



$$Y_1 = 4x - 2$$
$$Y_2 = 0.25x^3 + x + 1$$
$$Y_3 = 0.1x^5 - 2x - 3$$

Same end-behavior

down on the left
and
up on the right.

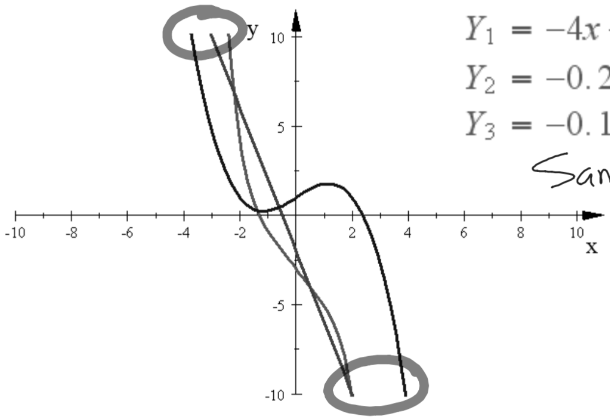
What do the equations have in common?

	Degree	Lead Coeff
$Y_1 = 4x - 2$	1	4
$Y_2 = 0.25x^3 + x + 1$	3	.25
$Y_3 = 0.1x^5 - 2x - 3$	5	0.1
	ODD	pos

All polynomials with an Odd degree and Positive leading coefficient behave the same at the left and right end.

Graph these by changing the Leading Coefficient to a negative.

$$Y_1 = -4x - 2$$
$$Y_2 = -0.25x^3 + x + 1$$
$$Y_3 = -0.1x^5 - 2x - 3$$



$$Y_1 = -4x - 2$$
$$Y_2 = -0.25x^3 + x + 1$$
$$Y_3 = -0.1x^5 - 2x - 3$$

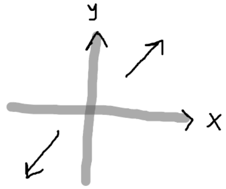
Same end behavior

up on the left
and
down on the right

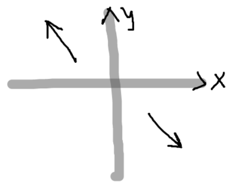
Functions with an ODD Degree

: Largest exponent is ODD when expanded
This is called the degree of the function.

Positive Leading Coefficient:
Moves from the third quadrant to the first quadrant.
Like a line with a Positive slope



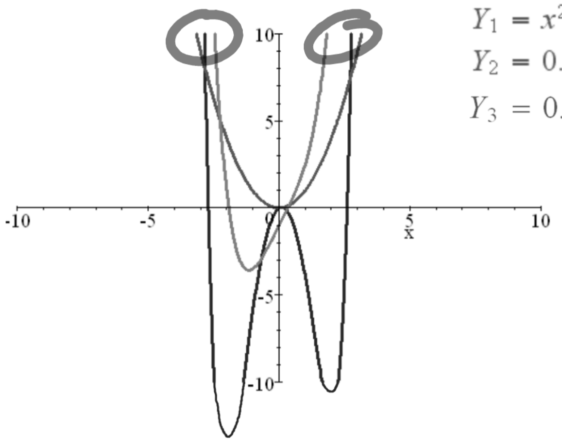
Negative Leading Coefficient:
Moves from the second quadrant to the fourth quadrant.
Like a line with a Negative slope



Graph all three of these in a Standard Window:

$$Y_1 = x^2$$
$$Y_2 = 0.5x^4 + 3x - 1$$
$$Y_3 = 0.1x^6 - 5x^2 + x$$

What do the graphs have in common?



$$Y_1 = x^2$$
$$Y_2 = 0.5x^4 + 3x - 1$$
$$Y_3 = 0.1x^6 - 5x^2 + x$$

Same end-behavior
up on both sides

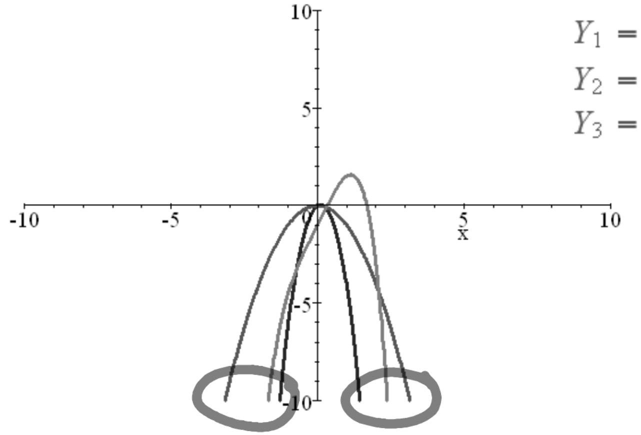
What do the equations have in common?

	Degree	Lead Coeff
$Y_1 = x^2$	2	1
$Y_2 = 0.5x^4 + 3x - 1$	4	0.5
$Y_3 = 0.1x^6 - 5x^2 + x$	6	0.1
	even	pos

All polynomials with an Even degree and Positive leading coefficient behave the same at the left and right end.

Graph these by changing the Leading Coefficient to a negative.

$$Y_1 = -x^2$$
$$Y_2 = -0.5x^4 + 3x - 1$$
$$Y_3 = -0.1x^6 - 5x^2 + x$$



$$Y_1 = -x^2$$
$$Y_2 = -0.5x^4 + 3x - 1$$
$$Y_3 = -0.1x^6 - 5x^2 + x$$

Same end-behavior

down on both sides

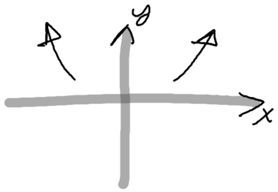
Functions with an EVEN Degree : Largest exponent is EVEN when expanded

This is called the degree of the function.

Positive Leading Coefficient:

Moves from the second quadrant to the first quadrant.

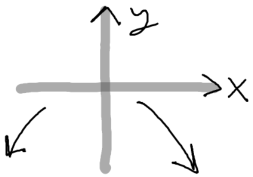
Like a parabola with a>0



Negative Leading Coefficient:

Moves from the third quadrant to the fourth quadrant.

Like a parabola with a<0



End-Behavior:

The behavior of the graph on the far left and the far right.

How the value of the function (y) changes as x becomes ...

larger negative LEFT END $x \to -\infty$

and

larger positive RIGHT END. $x \to \infty$

END BEHAVIOR

EVEN Degree

Positive Leading Coefficient:	Negative Leading Coefficient:	
(↖ , ↗)	(↙ , ↘)	Our Book
as $x \to -\infty,\ y \to \infty$ as $x \to \infty,\ y \to \infty$	as $x \to -\infty,\ y \to -\infty$ as $x \to \infty,\ y \to -\infty$	Other Authors & Mathematicians
as $x \to \pm\infty,\ y \to \infty$	as $x \to \pm\infty,\ y \to -\infty$	

Because all polynomials with an Even degree have the same behavior on both the left and right you can write one combined statement.

END BEHAVIOR

ODD Degree

Positive Leading Coefficient:	Negative Leading Coefficient:	
(↙ , ↗)	(↖ , ↘)	Our Book
as $x \to -\infty,\ y \to -\infty$ as $x \to \infty,\ y \to \infty$	as $x \to -\infty,\ y \to \infty$ as $x \to \infty,\ y \to -\infty$	Other Authors & Mathematicians

Since all polynomials with an Odd degree have the different behavior on both the left and right you CAN'T write one combined statement.

In fact, for Odd degrees, if one side goes up the other side will go down.

State the end behavior of each polynomial.

1. $y = \underline{4x^3} - 6x^2 + 11x - 93$ (↙ , ↗)
   pos odd
2. $y = 5x(x+2)(x-7)^2$
   $(5x)(x)(x)^2 \Rightarrow 5x^4$ POS EVEN (↖ , ↗)
3. $f(x) = 9x + 6x^2 \underline{-x^3} + 13$ (↖ , ↘)
   Neg odd
4. $y = (9x-7)(4-x)$
   $(9x)(-x) = -9x^2$ NEG EVEN (↙ , ↘)
5. $f(x) = -9x^2(3x-7)^2(8-2x)^3(1-4x)^5$
   $(-9x^2)(3x)^2(-2x)^3(-4x)^5$
   $(-x^2)(+x^2)(-x^3)(-x^5)$
   $= -x^{12}$ NEG EVEN (↙ , ↘)

You can now finish Hwk #22:

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Problems 1-10