

Hwk #20

1. Monomial:

A real number, a variable,
or the product of a real
number and variables.

Variables may have exponents. a Term

Give three examples of a monomial:

7, x , $5a^2$

2. Polynomial:

A monomial or the
sum of monomials

A monomial is a polynomial
with just one term

Give two examples of a polynomial:

$$4x + 3$$

$$-7x^2 + 4x - 8$$

3. a. The exponents of monomials and polynomials must be what kind of numbers?

Whole Numbers

b. The coefficients of a polynomial must be what kind of numbers?

Real Numbers

Definition

Polynomial Function

$P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$ where n is a nonnegative integer
and the coefficients a_n, \dots, a_0
are real numbers.

$$y = (3x + 1)(x - 8)$$

$$y = 3x^2 - 23x - 8$$



This is called expanded form

$$y = 3x^2 - 23x - 8$$

expanding means to eliminate
parentheses by Distributive Property
then combine like terms.

This is called factored form

$$y = (3x + 1)(x - 8)$$

Factors are things being multiplied
together and that is what this problem
show. There are usually parentheses
when in factored form.

4. What does a polynomial in standard form look like?

Expanded, with terms in descending order according to their exponent (degree).

5. The leading coefficient of a polynomial is

The coefficient of the term with the largest exponent after it's been expanded.
If it's in Standard Form it will be the first coefficient.

6. The degree of a polynomial is

The largest exponent after it's been expanded.
If it's in Standard Form it will be the first exponent.

standard form of a polynomial. A one-variable polynomial in standard form has no two terms with the same degree, since all like terms have been combined.

$$P(x) = 2x^3 - 5x^2 - 2x + 5$$

Diagram labels for $P(x) = 2x^3 - 5x^2 - 2x + 5$:

- Polynomial**: points to the entire expression.
- Degree**: points to the exponent 3 in $2x^3$.
- Leading coefficient**: points to the coefficient 2 in $2x^3$.
- Cubic term**: points to $2x^3$.
- Quadratic term**: points to $-5x^2$.
- Linear term**: points to $-2x$.
- Constant term**: points to $+5$.

7. Complete these two tables by filling in the blanks.

| Degree of Polynomial | Name by Degree |
|----------------------|----------------|
| 0 | Constant |
| 1 | Linear |
| 2 | Quadratic |
| 3 | Cubic |

| # of terms in polynomial | Name by # of terms |
|--------------------------|--------------------|
| 1 | Monomial |
| 2 | Binomial |
| 3 | Trinomial |

8. Is each of the below a polynomial? If not give a reason.

a) $y = \frac{3}{7}x^2 + 3x - 14x^4 + 4$

Yes.

All exponents are whole numbers and all coefficients are real numbers

c) $y = 9\sqrt{x} + 3x^7 - x^{\frac{2}{3}}$

No.

There is a fractional exponent and \sqrt{x} means $x^{\frac{1}{2}}$

b) $y = 4x^{-2} + x^3 - \frac{8}{x}$

No.

There is a negative exponent and $\frac{8}{x}$ means $8x^{-1}$

d) $y = 9^x + 10ix^4 - 15$

No.

There is an imaginary coefficient and an exponent that is a variable.

9. a) $(x + 3)^2 - 1$

Standard Form: $x^2 + 6x + 8$

Degree: 2

Leading Coefficient: 1

Name by Degree: Quadratic

Name by # of terms: Trinomial

b) $15 + 6x^3 - 3(x^2 + 5) + x^3$

Standard Form: $7x^3 - 3x^2$

Degree: 3

Leading Coefficient: 7

Name by Degree: Cubic

Name by # of terms: Binomial

10. State the degree of each polynomial.

Polynomials in Expanded Form:

a) $7x^2 + 12 - 13x^4 + 8x$

b) $9x + 1$

c) 6

Degree: 4

Degree: 1

Degree: 0

Without expanding the whole polynomial you can find the degree and leading coefficient by multiplying the leading term of each factor.

Polynomials in Factored Form:

d) $\frac{(x+3)(2x-1)}{(x)(2x)} \rightarrow 2x^2$
 Degree: **2**

e) $\frac{(x^2)(x)}{(x-7)^2(x-5)} = x^3$
 Degree: **3**

Domain of all Polynomials is: $(-\infty, \infty)$

Definition

Polynomial Function

$P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$ where n is a nonnegative integer and the coefficients a_n, \dots, a_0 are real numbers.

Polynomials: Exponents must be Whole Numbers

This means exponents can't be:

- Negative
- Fractions (rational #'s)
- Decimals
- Variables

This also means that the variable X can't be

- In a denominator (neg exponent)
- Under a radical (fractional exponent)
- An exponent itself

Get a small white board, rag, and marker

State the Degree and Leading Coefficient of each.

$$1. y = \underline{2}x^5 - 9x^4 + 13 - \underline{3}x^5 + 7x^2 \quad 2x^5 - 3x^5 = -x^5$$

Degree: 5

Leading Coefficient: -1

$$2. f(x) = 14x^3 + x^2 - \underline{7}x^4 + 3x - 5$$

Degree: 4

Leading Coefficient: -7

Without expanding the whole polynomial you can find the degree and leading coefficient by multiplying the leading term of each factor.

State the degree and leading coefficient of each.

$$1. y = (\underline{7}x + 11)(\underline{9}x - 15) \quad \begin{array}{l} \text{Deg: } 2 \\ \text{LC: } 63 \end{array}$$

$$(7x)(9x) = 63x^2$$

$$2. y = (\underline{2}x - 7)(\underline{3}x + 1)(\underline{4}x - 9) \quad \begin{array}{l} \text{Deg: } 3 \\ \text{LC: } 24 \end{array}$$

$$(2x)(3x)(4x) = 24x^3$$

State the degree and leading coefficient of each.

$$3. y = (\underline{x} + 5)^2(\underline{x} - 3)^2 \quad \begin{array}{l} \text{Deg: } 4 \\ \text{LC: } 1 \end{array}$$

$$(x^2)(x^2) = x^4$$

$$4. y = (\underline{2}x + 3)(\underline{x} - 2)^3(\underline{x} + 6)^3 \quad \begin{array}{l} \text{Deg: } 7 \\ \text{LC: } 2 \end{array}$$

$$(2x)(x^3)(x^3) = 2x^7$$

State the degree and leading coefficient of each.

$$5. y = \underline{-2}x(\underline{2}x - 5)^3(\underline{3}x + 1)^2 \quad \begin{array}{l} \text{Deg: } 6 \\ \text{LC: } -144 \end{array}$$

$$(-2x)(2x)^3(3x)^2$$

$$(-2x)(8x^3)(9x^2) = -144x^6$$

$$6. y = (\underline{2}x + 9)^2(\underline{2} - \underline{5}x)^3(\underline{4} - \underline{3}x)^2 \quad \begin{array}{l} \text{Deg: } 7 \\ \text{LC: } -4500 \end{array}$$

$$(2x)^2(-5x)^3(-3x)^2$$

$$(4x^2)(-125x^3)(9x^2) = -4500x^7$$

What will be most important for Chapter 6 is whether the Leading Coefficient is either

Positive or Negative

What will be most important for Chapter 6 is whether the Degree of a Polynomial is either

Odd or Even

For each polynomial state if the:

Degree is EVEN or ODD

and

Leading Coefficient is POS or NEG

$$1. y = 4x^2(x + 3)^2(11 - 2x)(4x + 1)^3$$
$$(4x^2)(x)^2(-2x)(4x)^3$$

DEG: Odd/Even

Add the exponents:

$$2+2+1+3 = 8$$

LC: Pos/Neg

$$(+)(+)^2(-)(+)^3$$

$$(+)(+)(-)(+) = \text{Neg}$$

to determine if the Leading Coefficient is POS or NEG you only have to analyze the product of the sign of the leading coeff of each factor.

$$2. y = -5x(7x - 8)(2x + 3)^2(9x - 10)$$

$$(-5x)(7x)(2x)^2(9x)$$

DEG: Odd/Even

Add the exponents:
1 + 1 + 2 + 1 = 5

LC: Pos/Neg

$$(-)(+)(+)^2(+)$$

$$(-)(+)(+)(+) = \text{Neg}$$

$$3. y = -3x^2(5x - 6)^3(2 - x)^2(7 - 4x)(8 - 3x)^3$$

$$(-x^2)(+x)^3(-x)^2(-x)(-x)^3$$

DEG: Odd/Even

add exponents:
2 + 3 + 2 + 1 + 3 = 11

LC: Pos/Neg

$$(-)(+)^3(-)^2(-)(-)^3$$

$$(-)(+)(+)(-)(-) = \text{Neg}$$

$$4. y = 7x^4(4x - 9)^3(x^2 + 6)^3(9 - 2x)^2(x + 7)(3x - 5)^2$$

$$\begin{array}{ccccccc} (+x^4) & (+x)^3 & (+x^2)^3 & (-x)^2 & (x) & (+x)^2 \\ \hline (+x^4) & (+x^3) & (+x^6) & (+x^2) & (x) & (+x^2) \end{array}$$

DEG: Odd/Even

add exponents:
4 + 3 + 6 + 2 + 1 + 2 = 18

LC: Pos/Neg

$$(+)(+)(+)(+)(+)(+) = \text{Pos}$$

You can now finish Hwk #21 Sec 6-1

Practice Sheet